Number Plugging or Problem Solving? Using Technology to Support Collaborative Learning

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This paper reports on a study that examined student interaction and discussion while working on computer-based tasks in a senior secondary school classroom. Analysis of verbal and observational data suggested that the task itself was an important variable influencing the degree of collaboration between students, and that the teacher's intervention could change students' engagement with the task. The findings have implications for specifying the teacher's role in relation to the use of computer technology in the mathematics classroom.

Background

The current push for the incorporation of mathematically enabled technology and applications (META) into the pedagogical practice of school mathematics classrooms is mirrored in curriculum documents (e.g. Australian Education Council, 1990; National Council of Teachers of Mathematics, 1989), school mathematics syllabuses (e.g. Board of Senior Secondary School Studies, 1992), and classroom support materials (e.g. Faragher et al., 1994; Sjöstrand, 1994). This stands in stark contrast to the degree to which META have been adopted and adapted into the instructional design of school mathematics programs where "it almost seems that computing technology is to be held at bay for as long as possible perhaps in the vain hope that it will go away" (Willis & Kissane, 1989, p. 57).

Significant changes in the affordability of these technologies, the development of new hypermedia technologies and the convergence and packaging of mathematically important technologies into "super-calculators" or hand-held "Personal Mathematical Assistances" (Kissane, 1995), as well the ever increasing dependence of society on technological infrastructure means that "to expect that schools and teachers can continue to exist apart from serious technological support is hopelessly myopic" (Kaput & Thompson, 1994, p. 682).

At the same time little research has been conducted into the influence of META on the learning and teaching of mathematics (Kaput & Thompson, 1994) although a number of authors have attempted to define the territory (e.g. Kaput, 1992; Taylor, 1981).

Taylor (1981) has suggested three ways in which computers are used in education:

- as a tutor—in which the computer environment is programmed in such as way as to provide instruction on some topic within a program of study. Examples of this category include Computer Assisted Instruction (CAI), Interactive Tuition and Electronic page turning.
- as a tutee—in which the learner takes an active part in the programming of the computer environment and learns something about specific non-computer oriented content domains as a consequence. For example, learning to program in LOGO demands students deal with problem situations which are mathematical by nature.
- as a tool—in which students make use of the capabilities of a computer to perform mathematical activities they would otherwise have conducted in some other way or to perform tasks that may have been beyond their capabilities without the assistance of computer technology.

Willis and Kissane (1989), building on Taylor's commentary, have also added the category of The Computer as a Catalyst. In this mode the computing environment is used as a means of provoking mathematical explorations and discussion or to invoke the use of problem solving skills.

The purpose of this study is to examine the effect of using META (specifically spreadsheets), as both a tool and a catalyst, on the collaborative learning practices already established within a particular classroom environment (see Goos & Geiger 1995). We begin by presenting a theoretical rationale for using computer technology to promote peer interaction and discussion.

Collaboration and Discussion in Computer Environments

In the past, educational theory has been largely concerned with individual modes of thinking, and educational practice has been based on the prevailing view that learning is a solitary enterprise. However, individualistic models of learning fail to take into account the social and communicative processes through which students are initiated into the intellectual life of their culture. Recently, interest in Vygotsky's (1978) ideas on the interactive nature of learning has led to the development of a new theoretical framework for understanding social aspects of cognition. Sociocultural theory shifts attention from individual to social modes of thinking, and emphasises the role of language in learning, both as a tool for thinking and as a medium for communication. A corresponding shift from an individualistic to a communicative approach to teaching and learning requires some consideration of classroom practices that encourage social interaction and sharing of knowledge. Collaborative learning arrangements favour such interaction by offering students opportunities to think together through the medium of language.

It has been claimed that collaborative peer interaction is facilitated by *computer environments*, since the computer provides a concrete and public focus for students' joint mathematical activity (Kennewell, 1994). Observational studies in the US and UK have confirmed that computer tasks provoke spontaneous interaction and discussion between students (Light, 1993), and have identified some patterns of interaction that promote different types of computer-based learning. For example, Hoyles and her co-workers (e.g. Hoyles, Sutherland & Healy, 1991) analysed the way in which pairs of secondary school students generated hypotheses about mathematical patterns while working in EXCEL and LOGO environments. Examination of the students' verbal exchanges and the video record of the computer screen revealed critical points in the discussion that provided the bridge between specific cases and formal generalisations.

Hoyles et al. (1991) also found that choice of software and choice of task were important decisions affecting students' style of interaction. The role of *software* is emphasised in the growing body of research concerned with the design of computer environments to support collaborative learning. Much of this work makes use of network communication tools or explicitly collaborative software to help students solve problems together in domains such as physics or engineering (e.g. Narayanan et al., 1995). However, collaboration may also be fostered by software not designed for that purpose, if the system possesses structural properties that encourage students to share information, ideas and program tools (e.g. diSessa, 1995).

In this paper we focus on both the social organisation of computer-based activity and the material basis for mediating collaboration. In contrast with the research mentioned above, the students who participated in our study worked collaboratively while using commercially available software whose structure neither explicitly nor implicitly encouraged collaboration. Our interest therefore centres on the second aspect of students' material environment—the nature of *tasks* that promote collaborative interaction at the computer.

Context

Participants in the study consisted of fifteen students, seven female and eight male, aged between 16 and 17 years of age, studying the second year (Year 12) of a two year senior secondary school course, their teacher (first author), and an observer (second author). The students were participating in the subject "Mathematics C" which formed part of their secondary accreditation and also contributed credit towards entrance into tertiary studies. The subject is considered an advanced course, being based on content areas applicable to students intending to pursue careers in engineering or the physical sciences. Students enrolled in Mathematics C are also required to study a parallel subject, Mathematics B, an introductory calculus and statistics course. Thus, 40% of their timetabled course time is devoted to the study of mathematics. Mathematics C students are typically able and highly motivated.

Instruction was based on written materials produced by the teacher. The materials defined the mathematical phenomena to be investigated over a series of lessons and offered guidance as to how to proceed during the course of the exploration. Students were strongly encouraged to interact with the teacher and each other when working through the materials and appeared to respond equally well to peer interaction as direct interaction with the teacher.

The topic being examined by students was part of an unit of work based on Chaos theory. This was a school option chosen and developed by the class teacher. The unit was chosen as a vehicle to:

- introduce students to an area of recently developed mathematics demonstrating the developmental nature of the discipline
- provide students with a learning experience based on the study of a topic within discrete mathematics, a branch of applicable mathematics of developing importance
- act as a vehicle for the natural use of computer technology as a means of exploring mathematical ideas.

Classes were conducted in the students' regular classroom. Fifteen Apple Powerbook 150 mobile computers were used, providing each student with access to the spreadsheet module of the software package Clarisworks. Students had gained familiarity with using these machines in junior secondary mathematics studies, Year 11 Mathematics C, and a number of topics in Year 11 Mathematics B. Thus they were relatively experienced users of spreadsheet software although it must be said they had had little experience in Year 12 in either subject.

This group of students had also participated in an earlier study during Year 11, when they worked on a similarly structured set of computer based activities on applications of Chaos theory (see Goos & Geiger, 1995). During the Year 11 observation period the students had displayed highly collaborative behaviour as they shared their ideas, sought help from their peers, and checked their understanding with each other. However, in the second series of lessons in Year 12 the students appeared to be working individually, with minimal interaction and discussion. The present study was conducted in order to investigate this difference in behaviour.

Data Analysis

Several lessons were observed during each unit of work on Chaos theory in Years 11 and 12. Here we present observations from one lesson in each year, to illustrate the differences in the students' interaction mentioned earlier. A small audio tape recorder was placed near one group of students during each lesson, and their talk was later transcribed. An observer also kept field notes that included a written record of the students' computer screens. While the discussion of results refers specifically to these target students, we consider that their behaviour was typical of that displayed by the whole class.

In our analysis of the verbal and observational data we examine three variables: the function of students' *talk*, the structure and focus of their *interaction*, and the type of *task* on which they worked. Student talk was classified according to the following functions (based on Kumpulainen, 1994):

Informative	Providing information
Organisational	Organising the task or the learning process, or controlling behaviour
Argumentational	Seeking and providing clarification, explanation and justification
Exploratory	Speculating, predicting, discovering, hypothesising
Metacognitive	Planning, monitoring progress, evaluating outcomes.

Patterns of student-student interaction were analysed with the aid of a two-dimensional framework developed by Granott (1993). The first dimension refers to relative expertise between students, while the second dimension represents the degree of collaboration. As the target students were of similar expertise, it is the second dimension that is of interest. Highly collaborative interactions are characterised by shared activity and continuous communication, while in less collaborative situations the activity is mostly independent, with occasional interaction to observe another participant or exchange information. Although both the relative expertise and collaboration dimensions are continuous, for our purpose it is helpful to label high and low collaborative interactions as *mutual collaboration* and *parallel activity* respectively.

Similarly, tasks may be conceptually defined at two levels according to the quality of interaction they elicit (Hertz-Lazarowitz, 1989). In *low-collaborative* tasks students interact about means (such as how to use materials), or about products (for example, the results of a calculation). In *high-collaborative* tasks, students interact about the process of doing the task, and their discussion focuses on planning or decision making. Task structure also influences the level of verbal elaboration in peer interaction, with high-collaborative tasks producing higher levels of elaboration (evaluation, application) than low-collaborative tasks (characterised by information exchange). These levels of elaboration can be matched with the functional categories of talk outlined earlier, so that high-level elaboration is heard in argumentational, exploratory and metacognitive talk, and low-level elaboration in informative and organisational talk. Table 1 shows the relationship between talk, interaction and task that guided our investigation of students' computer-based activity.

Table 1. Framework for Analysis of Talk, Interaction, and Task

Type of Task	Focus of Interaction	Structure of Interaction	Type of Talk
Low-collaborative	Products (e.g. results)	Parallel activity	Informative
	Means (e.g. materials)		Organisational
High-collaborative	Process (e.g. planning or decision making)	Mutual collaboration	Argumentational Exploratory Metacognitive

As well as observing peer interaction and discussion during lessons we also interviewed the class towards the end of Year 12, to seek their views on the benefits of collaboration. The next section presents our analysis of students' activity during the two sample lessons referred to earlier, together with the students' perceptions of the differences we observed.

Results

Mutual Collaboration on a High-Collaborative Task (Year 11)

In Year 11 the students were introduced to iterative processes as a means of examining exponential growth and decay, for example, in compound interest or animal populations. As explained earlier, a high degree of interaction and discussion was observed during this initial work on computer-based tasks involving chaos theory, even though the students had individual access to computers. In the lesson we have chosen to illustrate this type of activity the target students (given the pseudonyms Belinda, Rob and Louise) were working on the following task.

Nga is offered the following terms with two different financial institutions:
Institution 1 12.5% compounded annually
Institution 2 12% compounded monthly.
Nga knew that in the short term the conditions offered by Institution 1 were superior but suspected that in the long term Institution 2 might be the best proposition. For what period of time would she need to invest with Institution 2 before she realised a better return on her investment?

Students were expected to create two spreadsheets in order to compare returns from the institutions.

The following excerpt from the audio tape transcript shows how the task elicited the kind of continuous communication and sharing of ideas that characterises mutually collaborative interaction. Rob has incorrectly used Institution 2's annual rate of interest instead of converting to the monthly rate of 1%. Belinda points out the error and then explains her own spreadsheet strategy (which is also incorrect). The interaction focuses on the *process* of justification, and the talk has both *argumentational* and *metacognitive* functions as the students challenge and evaluate each others' reasoning.

R: OK, how come if you're working the interest out, for three years compounding *annually*, for 12.5, you get that? (pointing to computer screen) That's my problem. And at two years, that's all you get? Or am I doing it wrong?

B: Because it's compounding interest. [inaudible] That's years and that's months. (confidently) *This* is compounding annually so it doesn't matter. *This* one is compounding monthly so you have to take into account the number of times you compound it in a year.
R: (doubtfully) Yeah ... (flash of understanding) Oh, this is interest rate per *year!*
B: (simultaneously) —*Year!* So you've got to work it out with your calculator then do the interest—
R: —So you divide 12 by 12—
B: No you've got to do point one two divided by 12 which is point zero one, and then that's—
L: It's not years, it's 12 months.
B: Twelve months?
L: If I do two years, that's 24 spaces (referring to rows in spreadsheet), and then—
B: I did 20 years and I had 240 spaces.
R: But why?
B: (to R) Why? Because the only reason you didn't have to do any calculations for this one is because [inaudible] when it's only calculated annually.
R: ... So then put the zero one—no problem. (after a pause) What interest rate did you use for the second one?
B: Point zero one. It's not actually point zero one, it's [inaudible].
R: How did you work that *out*?
B: See, compound interest, you've got to add one to it.
R: Is that it? That it, one? What for?

This style of interaction continued after Belinda recognised that the answer her spreadsheet produced for Institution 2 did not make sense. She clarified her thinking through discussion with others, rejected explanations she did not understand, and eventually succeeded in resolving the impasse caused by her flawed strategy.

Parallel Activity on a Low-Collaborative Task (Year 12)

The topic of growth and decay was extended in Year 12 to include mathematical models for dealing with fluctuating populations. The task was centered around an investigation of the behaviour of the logistic equation, L(x) = bx(1 - x), when the parameter b is varied. Students were asked by examine how the equation behaved graphically when iterated (generating an orbit) for a range of values of b. This results in either stability after an initial increase (Figure 1) or decrease (Figure 2) in the values of L(x), periodic behaviour (Figure 3), or chaotic behaviour.

Iterate	L(x)
1	0.18
2	0.2952
3	0.41611392
4	0.48592625
5	0.49960386
6	0.49999969
7	0.5
8	0.5
9	0.5
10	0.5

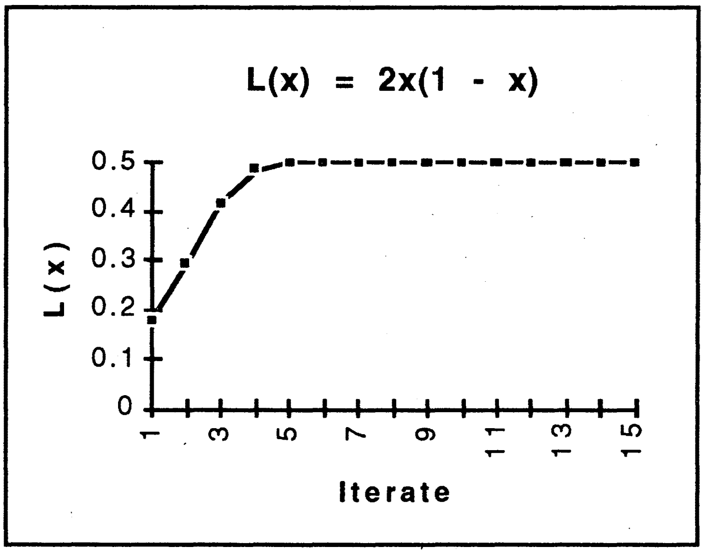

Figure 1. Spreadsheet and Chart for b = 2

The materials guided students through a sequence of activities aimed at assisting them to discover a relationship(s) between the values of b and the iterative behaviour of L(x). By using a spreadsheet to calculate and chart successive iterates of L(x) from an initial value, it was anticipated that students would find different behaviours manifest themselves between specific ranges of values of b. It was also hoped that students would discover the phenomenon of period doubling, in which a two cycle orbit splits, or bifurcates, into a four cycle oscillation. This in turn can also bifurcate into an eight cycle orbit, then into a sixteen cycle, and so on. However, instead of following instructions to compare the resulting graphs and search for patterns, the students became so immersed in the process of generating the data that there was very little interaction and talk.

Iterate	L(x)
1	0.045
2	0.0214875
3	0.01051289
4	0.00520119
5	0.00258707
6	0.00129019
7	0.00064426
8	0.00032192
9	0.00016091
10	8.0442E-05
11	4.0218E-05
12	2.0108E-05
13	1.0054E-05
14	5.0269E-06

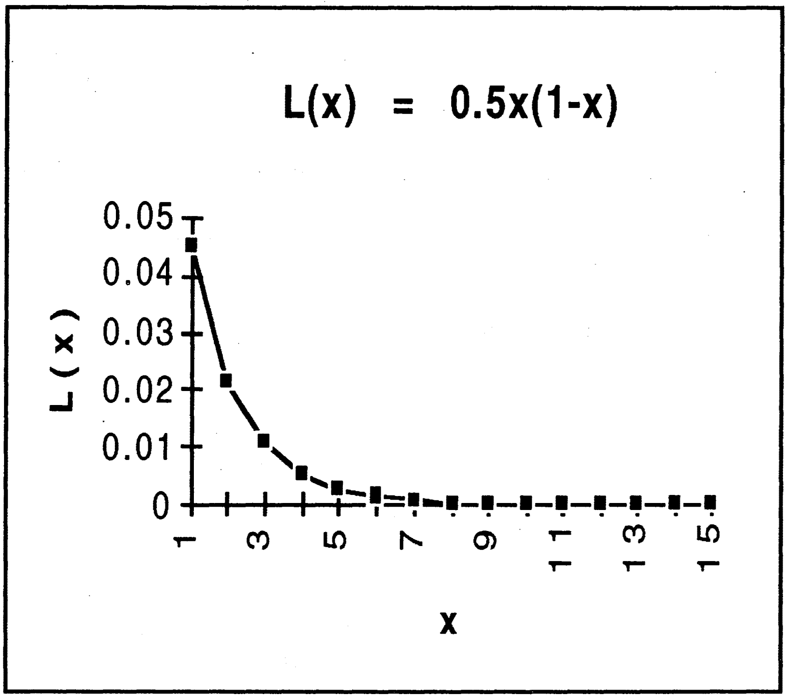

Figure 2. Spreadsheet and Chart for b = 0.5

The target students for the lesson discussed here, Rob, Ben and Duncan (pseudonyms), were chosen for observation because they usually worked collaboratively on classroom tasks. However, although there was a small amount of exploratory talk (e.g. speculating whether the graph would reach the same limit if a different initial value had been chosen), the students worked in parallel for most of the lesson and interacted to exchange information about *products*, such as spreadsheet cell values and the general shape of the graphs, and *means*, for example, procedures for managing the software. Thus the two main functions of their talk were *informative* and *organisational* respectively (see examples in Figure 4).

Examples of Informative Talk

1. R: Is that what you got?
2. B: Did you write anything on these graphs?
3. R: Did you get 0.32 for the initial value?
4. B: I get the same graph for 0.81 as for 0.8. But x_0=1.2 gives a different graph.

Examples of Organisational Talk

1. R tells B how to use keyboard shortcut -M to make a chart.
2. B asks how many rows are available in a spreadsheet.
3. Discussion of how to use the copy-paste-edit commands to make new charts.

Figure 4. Examples of Informative and Organisational Talk

Teacher Follow Up—Changing the Task

After confirming with the observer that there appeared to be a change in the quality of student interaction, the teacher decided to change the task to one that had a clearly defined goal at the beginning of the activity, but was less directive; that is, the desired outcome was made public but not the means of achieving it. The chosen task (below) was a natural extension of the ideas on which students had already been working, and thus was accessible to all students, but was not accompanied by detailed directions on procedure, as was the case in earlier tasks.

In your investigation of periodic and chaotic points in the growth equation f(x)=bx(1-x), you examined transition values for period doubling between 1 cycle, 2 cycle, 4 cycle, and 8 cycle behaviour. Can you find the point at which the 3 cycle begins? Does this begin another series of period doubling behaviour?

Procedural uncertainty made the new task more challenging for the students, and their interaction and discussion once again took on a collaborative character.

Orbit Values
0.1
0.288
0.6561792
0.7219457
0.6423682
0.7351401
0.6230691
0.7515327
0.5975401
0.7695549
0.5674884
0.7854250
0.5393042
0.7950565
0.5214131
0.7985327

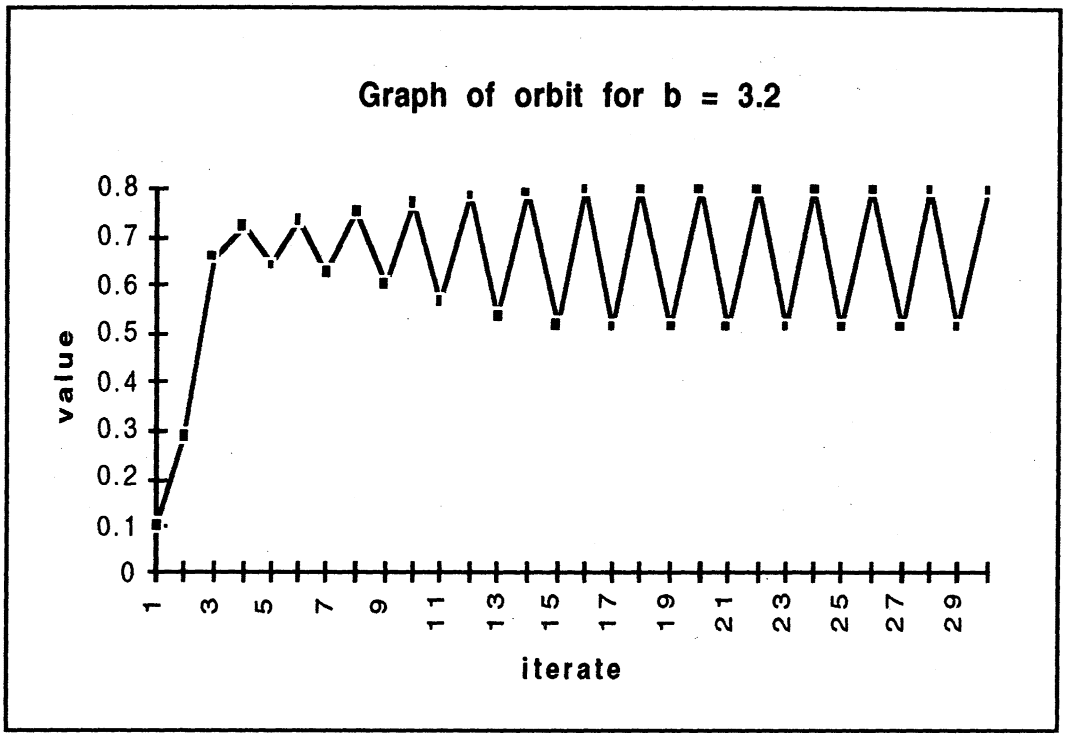

Figure 3. Spreadsheet and Chart for b = 3.2

Student Perceptions of the Tasks

Towards the end of Year 12, the students were interviewed on the general topic of how peer discussion "fits in with learning". When they were reminded about their lack of discussion during the second series of lessons on Chaos theory, they explained that the task was boring and repetitive because it had merely involved "number plugging":

D: We did talk, but it was about "what did you get?"
B: We only talked at the end when you said we hadn't been writing things about the graphs.
R: Yeah. And maybe we all knew what we were doing, so we didn't have to talk.

When the teacher explained how the task was then modified to make it more problematic, Rob responded:

R: Well, then we didn't know what to do! So we had to talk.

These comments seem to confirm that the task is a critical variable affecting peer interaction.

Discussion

This study has investigated the social and material mediation of computer-based learning. The computer was intended to act as both a tool, in enabling students to generate and manipulate data, and as a catalyst, in provoking exploration of the patterns that emerged from the data. However, the extent to which such exploration occurred depended on the type of task the students were given. Differences in the social organisation of students' work, identified in the function of their talk and the structure of their interaction, were associated with differences in task focus, with a focus on process, rather than products or means, producing collaborative discussion. Our results show that computer environments do not automatically facilitate peer interaction, and that tasks need to be carefully structured if they are to elicit high level verbal reasoning.

These findings have implications for teachers wishing to integrate META into school mathematics programs. Task design is clearly an issue of importance: the purpose of the computer task is for students to achieve new mathematical understanding, not simply procedural facility with the hardware or software. However, as students will need to generate some data in order to begin exploring ideas, there is a danger that the initial process of "number plugging" will conceal the problematic nature of the task. Hence, the teacher's intervention is needed to contextualise the task, focus students on the conceptual goal, and steer their discussion away from information exchange towards argumentation and hypothesising. Once students have completed the task, the teacher can stimulate further discussion and reflection on the solution process by conducting a whole class review in which students are encouraged to share their learning.

The role of the computer itself as a partner in the learning process should not be overlooked. When used as a tool or catalyst, computer technology has the potential to reorganise interactions between students and enrich their understanding of mathematics; but the computer does not replace the teacher. The extent to which computer environments allow students to articulate and reflect on their growing understanding depends on the teacher's expertise in designing appropriate tasks, and in orchestrating students' interaction with the computer and each other.

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