When Does Student Talk Become Collaborative Mathematical Discussion?

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The potential role of student discussion in developing mathematical knowledge continues to interest researchers and teachers. This paper reports on a study that investigated patterns of student discussion and interaction in senior secondary school mathematics classrooms. Observations of one group of students are used to illustrate three factors that appear to influence the collaborative quality of mathematical discussion: students' orientation towards the task; their relative task-specific expertise; and the degree of challenge the task presents.

Introduction

Recent research in mathematics education has increasingly focussed on new ways of conceptualising mathematics teaching and learning. Informed by theories concerning the social context of learning and development, this research argues that learning mathematics involves participating in the activities of a community of practice, whose identity is forged by the adoption of the language conventions and ways of thinking valued by the wider community of mathematicians (Forman, in press; Lampert, 1990) .

In "community of practice" classrooms, students take on new roles that contrast sharply with those allowed within more traditional settings (Nickson, 1992). In traditional classrooms, mathematics is presented as abstract knowledge already discovered by experts, with the teacher as the sole source of knowledge and authority. Students are passive consumers of knowledge, their participation limited by the structure of the conventional recitation script. By comparison, in community of practice classrooms mathematical knowledge is constructed and tested by students as they work collaboratively under the guidance of the teacher. Student participation is therefore expanded to include discussion with peers in order to solve problems and assess their growing understanding.

This paper reports on the preliminary outcomes of a two year research project whose general aim is to investigate patterns of classroom social interactions that improve senior secondary school students' mathematical understanding, and facilitate a more accurate perception of the communal nature of mathematical knowledge. The specific purpose of the paper is to identify factors that influence the collaborative quality of student discussion.

Collaborative Mathematical Discussion

The role of peer discussion and collaboration in developing mathematical knowledge has been an issue of interest to researchers for some time. For example, previous studies have investigated the effectiveness, characteristics and occurrence of student-student talk (Hoyles, 1985; Pimm, 1987; Pirie, 1991; Pirie & Schwarzenberger, 1988) and the development of classroom norms for collaborative dialogue in small group interactions (Wood & Yackel, 1990). However, some care is needed in clarifying what is meant by *mathematical discussion*, and what makes the discussion *collaborative*. We find useful Pirie and Schwarzenberger's (1988) definition of mathematical discussion as purposeful talk on a mathematical subject, with genuine student contributions (whose input assists the talk or thinking to move forward) and interaction (indicating that ideas have been picked up by other participants). Further, the discussion is collaborative if the students explore each other's reasoning and viewpoints while working on a common activity, so that shared understanding evolves simultaneously for all participants (Granott, 1993).

The question then arises as to how mathematics teachers might create conditions that encourage students to engage in this kind of collaborative dialogue. One factor that appears to influence the quality of students' interaction and talk concerns the structure of the task on which they are working. Tasks that require students to interact about processes, such as planning and decision making, elicit more elaborated reasoning than tasks that only require students to interact about products or means, for example, answers, materials, learned procedures and algorithms (Hertz-Lazarowitz, 1989).

However, our study has identified a set of interacting factors, related to both task and student characteristics, that influence the quality of student discussion. These factors will be illustrated by reference to three lessons involving the same group of students.

The Study

Four mathematics classes (three Year 11 and one Year 12) in four secondary schools have participated in the first year of the study. All schools are co-educational; two are government high schools and two are independent schools. At the beginning of the year questionnaires and other written tasks were administered to obtain information on students' beliefs about mathematics, perceptions of classroom practices, and metacognitive knowledge (see Goos, 1995, for details). From March until September, one mathematics lesson per week was observed in each classroom in order to gather data on patterns of teacher-student and student-student interactions. At least ten lessons were videotaped for each class.

The initial purpose of our observations was to select in each classroom a group of students who were in the habit of working together and discussing their ideas in mathematics lessons. These groups became the focus of our observation and videotaping in subsequent classroom visits. We also used video-stimulated recall techniques (e.g. Frid & Malone, 1995) to interview each group of students on at least one occasion, to seek their interpretations of their own interactions with each other.

Although we observed and recorded many student discussions in the four classrooms, in this paper we have chosen to illustrate our findings by focussing on one group of students and their social interactions during three separate lessons.

The Students

Lincoln, Gary, Paul and Christopher (pseudonyms) were students in a Year 11 Mathematics B class in one of the participating schools. (Mathematics B is a prerequisite subject for students seeking entry to science based tertiary courses.) The boys were good friends, and their interaction and helping behaviour were similar for most of the lessons observed.

Lincoln was a high achieving student who was confident in the knowledge of his own ability, but anxious that his performance should match his high expectations for himself. When working on mathematics problems with his friends he tried to maintain organisational control by eliciting their input and assigning them different subtasks. (However, it was our observation that the other students usually ignored him and continued to contribute in any way they chose.) Gary was also a capable student, with a more relaxed approach to his work. Both Lincoln and Gary also took Mathematics C, a more specialised subject that prepares students for further study of mathematics at tertiary level. This subject combination conferred a privileged status that was not shared by Paul and Christopher, whose additional enrolment in Mathematics A (which concentrates on mathematical applications for daily living) further marked them as lower status members of the group. Consequently, Paul's useful ideas and suggestions were often ignored by Lincoln and Gary. Christopher was the least able student in the group, and he rarely contributed to the discussion. Yet he appeared to follow his friends' reasoning as they talked, and kept track of progress with his calculator. When interviewed, he explained the benefit of his participation in the group as being able to learn from the others.

These students was chosen as a target group for a number of reasons. First, questionnaire data showed that the students themselves reported using discussion as a means of learning mathematics. In one questionnaire students were asked to indicate frequency of participation (always, often, seldom, never) in seven different learning activities. The four target students claimed that they were *always* or *often* likely to be *talking about maths to other students* or *listening to other students*, with none of them expressing a greater preference for any other learning activity (e.g. *listening to the teacher*, *working alone*, *copying notes from the blackboard*). Written responses were also sought to the open-ended question *How do you know when you understand something in maths?* Unlike the majority of students who measured their understanding in terms of their ability to obtain the correct answer to a problem, Lincoln and Gary reported that they knew they understood a mathematical idea when they could *explain it to other people*. The students' self-reports were confirmed by our observation that they worked and talked together during mathematics lessons, more so than other students in the class. Taken together, this evidence seemed to suggest a commitment to learning through peer discussion.

The Lessons

Describing Interaction and Discussion

The collaborative quality of the students' mathematical discussion is described in terms of two indices: the structure of their social interactions, and their use of explanations. Social interactions are structured according to the extent to which the activity is shared, and the degree to which students take direction from each other (Damon & Phelps, 1989; Granott, 1993). Thus we may distinguish between the following interaction structures:

Parallel activity	Students work separately on the same task, with little or no sharing of materials or ideas.
Peer tutoring	Students work on a common activity, but one student with superior competence controls the instructional agenda.
Collaboration	Students work jointly on a problem that none could solve alone.

Differences in the structure of interaction are associated with corresponding differences in the quality of the students' discussion, especially the kinds of explanations they use. During parallel activity students interact only intermittently, perhaps to check results with each other, and elaborated explanations are not heard. In peer tutoring, the more knowledgeable student teaches a partner how to carry out a task, and the student-tutor explains in order to impart knowledge that the partner does not yet possess. However, when students work collaboratively they share their ideas with each other, and it is through a process of mutual explanation that a common understanding is reached.

The differences between no-explaining (parallel activity), explaining-to-teach (peer tutoring) and explaining-to-understand (collaboration) are illustrated in Lessons #1 and #3. A further contrast is provided by Lesson #2, in which the students avoided interaction altogether.

Lesson #1. The Observatory Problem

> An observatory is in the shape of a vertical circular cylinder, of diameter 30 metres, surmounted by a hemispherical dome of equal diameter. It stands on horizontal ground. From a point on the ground 50 metres from the wall of the observatory, the angle of elevation of the apparent top of the building is 30°. Find how high the building is centrally.

The Observatory Problem was presented as a practice exercise to prepare students for an assessment task to be given the following week. Students were allowed to work together in class to clarify the task and discuss strategies, as they would for the assessment task, but were required to produce individual written solutions. However, in this lesson the four target students worked through the entire problem and solved it as a group.

After clarifying the aim of the problem the students began working with an incorrect diagram, in which the tangent to the dome extended only to the point of contact. (The correct diagram is shown in Figure 1.) Assuming that the tangent nevertheless represented the hypotenuse of a right angled triangle with sides of 65 and $(x + 15)$ metres, the boys "solved" the problem by calculating the height of the building as 65 tan 30°, or 37.53 metres. Although they were suspicious of the apparent ease with which they had reached a solution, the boys called on the teacher to verify their answer. His suggestion that they should extend the tangent alerted the students to their error, and they quickly constructed the correct diagram.

Realising that the height of the building was $(37.53 - y)$, the students then explored various ideas for finding the unknown distance y (see Figure 1). Their discussion centred on ways of modifying the diagram in response to Paul's suggestion that "there's a triangle here, in here somewhere". The following excerpt from the videotape transcript illustrates the collaborative quality of their interaction as they shared and tested ideas.

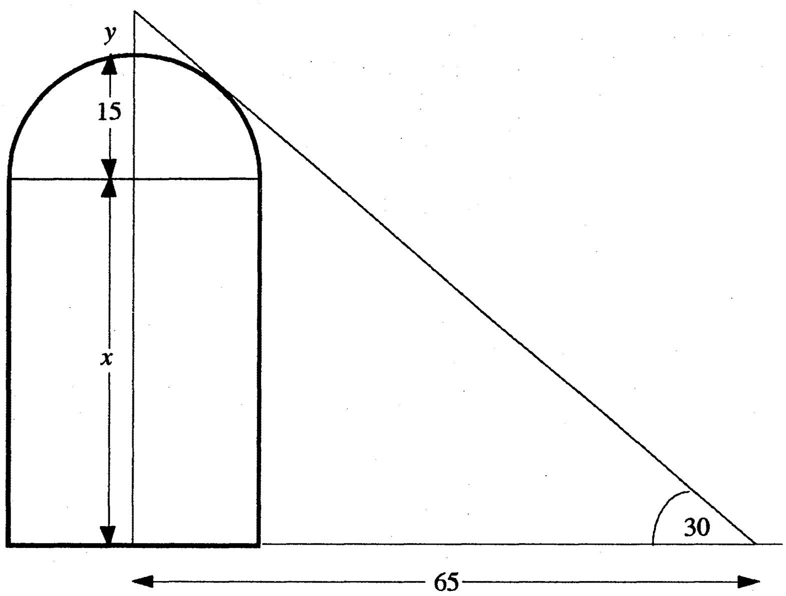

Figure 1. Diagram for the Observatory Problem

L: So where are you saying we draw the triangle, Paul?
P: Here could be good.
L: But how does that help us?
P: Oh no ...... draw it in there, I guess.
G: If you calculate x (the height of the cylinder) it'll help you.
L: X?
G: Yeah.
L: How?
G: If you calculate x you should be able to —
P: But we can't calculate x if we don't know the—
L: You can't draw a triangle as a tangent down there, can you? And if that is a right angle—
P: How will that help us? We don't want to know that.

Although progress was slow at first, a method for constructing similar triangles was eventually recognised in the following exchanges, and a solution was found soon afterwards:

L: OK well lets have a look. What's that there? (i.e. the distance marked as $x + 15 + y$ in Figure 1)
P: That's ... 37.5.
L: So if we know that up to there is 37.5 ... Does that help us?
P: And we've got this triangle here (constructs a radius perpendicular to the tangent) and we know that's 30 degrees (angle subtended by the radii).
L: (looking at diagram) How?
P: Because it's ... that's 90 degrees (i.e. radius is perpendicular to tangent), and—
G: Similar triangles.
L: And that's 60 degrees because that there's 90, OK?
P: (pushing L's hand out of the way) That's 30, so that there has to be 30.
L: Why?
G: Because that there is a similar triangle.
P: Because isn't that 60—
L: (responding to G) Is it?
G: And that's 60. (P and G both explaining at once.) See that whole angle there's 60—
P: And that's 30, and that's 30 60.
L: OK. But how does that help us?
G: Well we can work that out with similar triangles ...

The collaborative interaction between Lincoln, Gary, Paul and Christopher was similar to that observed in previous lessons when group tasks were given. They tackled the task, and the obstacles it presented, by generating ideas that were tested, challenged and justified in discussion with each other. In doing so, they explained their way towards understanding and solving the problem.

Lesson #2. The Theatre Design Problem

One week later the students were given the assessment task for which their group work on the Observatory Problem had prepared them. They were provided with a diagram representing a small picture theatre in cross section, together with information on the height of the screen, its distance from the floor, and the height of the audience's eye level. The students were to determine the horizontal distance x from the screen that would give the viewer the maximum viewing angle θ. As the class had not yet studied calculus, they were given a table of x values for which to calculate corresponding values of θ. The second part of the task asked them to investigate the effect of a sloping floor on the viewing angle.

The teacher again explained the procedure, which was the same as that for the earlier lesson: sharing of ideas about possible strategies, followed by production of individual written solutions. However, with assessment now at stake, the target students were unwilling to discuss the problem in the way observed in previous lessons. Lincoln, in particular, was reluctant to share his thoughts, and busied himself with his own calculations until he had completed the first part of the problem. Towards the end of the lesson Paul attempted to initiate some interaction:

P: So what'd you get for [x =] four?
L: (Covers his working.) Noo!
P: Oh you can show me! ...
L: You've got to write it up yourself.
P: Was that right, what I had?
L: I don't know.
P: (Smiling) Show me! (L grins and places his fist firmly on his closed book. P grabs book from him and inspects L's working. L doesn't protest. P looks disappointed when he sees L's results.) So it goes ... (L retrieves his book) So three metres is the best, you reckon? (L puts his hand over his results; P tries to lift it.)
L: Then you do it each ten centimetres. (referring to values of x to use in the table)
P: Do you? Do you have to do that all the time? (L nods.) Geez! Are you going to do it like that?
L: I've done it.
P: (Impressed) Have you worked it out?
L: Yeah.
P: You're joking!
L: No. Now I've got to do the second half when I get home.

Despite the similar task demands and teacher instructions, collaborative interaction was not observed during this lesson.

Lesson #3. Binomial Expansions

In this lesson, which occurred about one month after Lesson #2, students were instructed to work in groups to complete a worksheet on Binomial Expansions. They were to explain the number patterns in Pascal's Triangle, and compare them with the patterns of coefficients and powers obtained by using the distributive law to carry out expansions from $(x+y)^0$ to $(x+y)^4$. The students were then asked to use the patterns they had discovered to write instructions for expanding an expression in the general form $(x+y)^n$. The teacher emphasised that the most important part of the task was for the students to be able to explain and communicate their ideas to each other.

Although this task appeared to present an ideal opportunity for collaborative discussion and explanation, the teacher's intentions were foiled by Lincoln and Gary, who had already covered this work in Mathematics C. Ignoring Paul and Christopher, they worked in parallel through the first part of the task, and used Pascal's Triangle instead of the distributive law to quickly expand the given expressions. Because explanations were unnecessary, their talk focussed on checking their results with each other:

L: What's x to the third?
G: X cubed—
L: Yeah.

G: —plus three ... stop, stop. (reaches for eraser)
L: Plus what?
G: (erasing) Um ...
L: Three x squared y ...
G: Yup, and then—
L: Plus three, y squared x?
G: (writing) Plus three x y squared. (to L) In the first one you have x squared, so in the second one you have y squared—
L: Yeah yeah yeah, plus y cubed.
G: Yep.
L: Too easy.

The structure of the interaction changed when Lincoln and Gary reached the part of the worksheet that required written explanations of the patterns of coefficients and powers. Because they already understood the connection with Pascal's Triangle, Lincoln and Gary formed peer tutoring pairs with Paul and Christopher respectively, in order to explain to the less knowledgeable students how the patterns worked. However, the tutors were more interested in demonstrating their expertise with the number patterns, than in helping the other two boys work through the preliminary algebraic expansions in order to discover the pattern for themselves. The tutors' explanations were purely procedural in that they emphasised what to do, rather than why.

The peer tutoring interaction precipitated by the Binomial Expansions worksheet did not provide opportunities for the students to exchange ideas, or to use mutual explanations to generate an understanding of the mathematical ideas around which the task was designed. The inadequacy of Lincoln's "expert" explanation is shown in the following exchange:

L: You see, the main numbers are just those down here, 1 3 3 1 (pointing to Pascal's Triangle), 1 3 3 1 (pointing to coefficients). And then the powers have to add up to, what the term is. So you've got 3, then you've got 2 plus 1, then 1 plus 2, and then there's 3. Do you get it?
P: No. I don't understand.

In the three lessons described above we have shown how differences in the structure of students' social interactions gave rise to differences in the quality of their discussion (Table 1). We now consider some factors that may be implicated in these differences.

Table 1. Summary of Interaction Structures and Discussion

Lesson	Interaction Structure	Explanations
#1 Observatory	Collaboration	Explain-to-understand
#2 Theatre Design	Interaction avoided	No discussion
#3 Binomial Expansions	Parallel activity	No explanations
	Peer tutoring	Explain-to-teach

Factors Promoting Mathematical Discussion

From our observations of students' talk and social interactions, we have identified three related factors that influence the occurrence of collaborative mathematical discussion: students' perception of the purpose of the task, the relative expertise of the students, and the degree of challenge the task presents (Figure 2).

Students' Orientation towards the Task: Learning vs Performance

We observed the operation of collaborative social norms within the target group of students when the task on which they worked had a learning purpose (e.g. Observatory Problem). However, these norms were suppressed when they were given a task that was to be individually assessed (Theatre Design Problem). The students' orientation towards the task—learning versus performance—appears to influence how they respond to it and to each other.

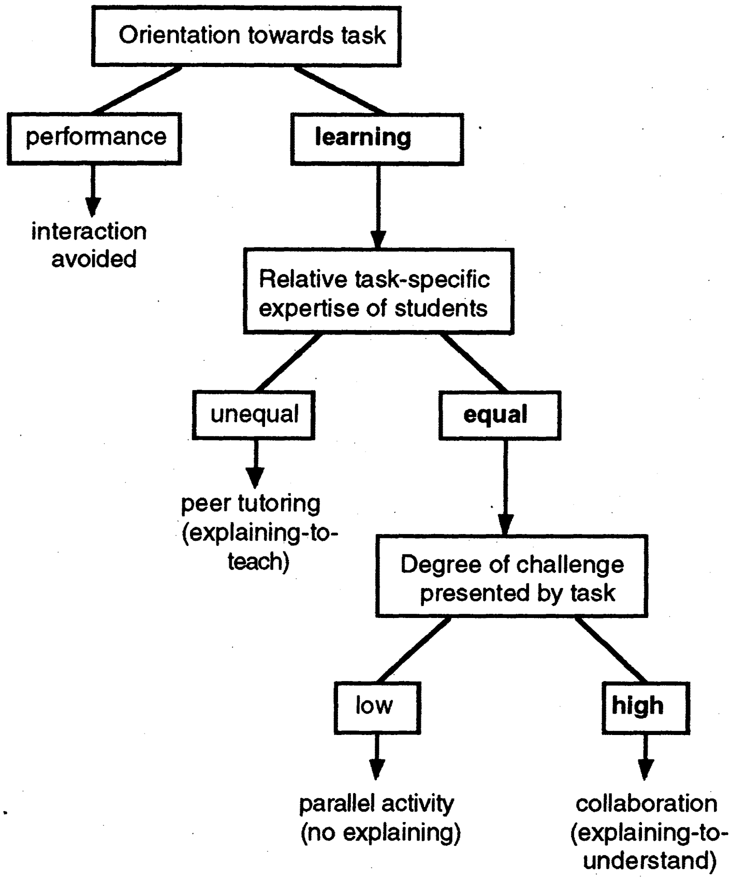

Figure 2. Factors Influencing the Collaborative Quality of Mathematical Discussion

Relative Expertise of the Students: Equal vs Unequal

Given that students approach the task with the purpose of learning, the next factor that influences the quality of their interaction is their relative expertise with respect to the task itself. Here, expertise depends not on previous mathematics achievement, but on relevant task-specific prior knowledge. If some students know more than others, then the interaction becomes one of peer tutoring rather than collaboration. This kind of interaction was observed during the Binomial Expansion activity, where Lincoln and Gary's previous experience with the topic gave them an advantage over the other members of the group. The purpose of the more expert students' explanations was to demonstrate procedures, rather than to share their understanding.

Degree of Challenge: High vs Low

If students come to a task with similar background knowledge, then the degree of challenge the task presents will also have an effect on their interaction. Collaborative interaction occurred when no one in the group knew how to attack a task, or when an obstacle prevented progress (e.g. Observatory Problem). Here, explaining was a mutual process arising from the need to clarify and justify ideas to the satisfaction of one's peers.

If the task was less challenging, the structure of the interaction changed to parallel activity, as observed during the Binomial Expansion lesson. Because the first part of the task proved to be a routine exercise for Lincoln and Gary, there was no need for them to test their understanding by explaining their thinking to each other. They merely exchanged information on results.

Conclusion

In this paper we have presented a sample of our extensive observations of senior secondary mathematics classrooms, made over the course of one school year. As a result of analysing patterns of student interaction and talk, we have identified three conditions that seem to favour collaborative mathematical discussion:

1. students have a *learning orientation* towards the task;
2. students have roughly *equal task-specific expertise*;

3. the task presents a *challenge*.

These findings are tentative and subject to confirmation through observation of additional classrooms in the second year of the study. Nevertheless, they have implications for mathematics teachers who wish to encourage collaboration and discussion in their classrooms.

Previous research has found that task design is an important variable in peer learning, but our study suggests that it is the interaction between student and task that influences the character of their talk. A task that is challenging for one student may be less so for another, so that the question of who works on which task with whom has a bearing on the form of social interaction, and the kind of talk, that will be observed. In addition, students whose task goals are related to performance, rather than learning, may be unwilling to work in collaboration with their peers. For some students, learning goals are further compromised if collaborative work on a task is to be individually assessed. Careful thought is needed in designing assessment programs that encourage students to tackle tasks with their classmates, yet allow teachers to measure the understanding that individuals gain as a result of collaborative effort. The problem of satisfying the individualised assessment requirements of senior secondary schooling while developing a collaborative classroom ethos is one which deserves further attention if the "community of practice" ideal is to be realised.

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