Children's perceptual judgement of area

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This paper proposes and justifies a program of research to extend knowledge of how children use judgements to determine area of a rectangle. It looks particularly at the relationship between perimeter and height + width rules used in making judgements. Children's responses to a variety of tasks in a pilot study are described and analysed and inferences drawn for further study.

Our interest in investigating area stems from the number of years spent by the first author teaching mathematics in a number of Queensland High Schools. Each year a new group of Year 8 students would arrive in the mathematics classroom and were able to demonstrate their abilities in calculating the area's of regular plane shapes, but were unable to distinguish between area and perimeter. It seemed that a lot of children had little or no understanding of the area concept.

The first explorations of the literature on area learning were based on the work of Piaget (Piaget, Inhelder & Szeminska, 1960). Piaget's theory of developmental stages, his proposals on conservation of area, and his notion that young children's quantitative concepts were largely one-dimensional in character attracted attention. Piaget's findings appeared to be widely accepted by developmental psychologists (e.g., Silverman & Paskewitz, 1988)

However, further exploration uncovered criticisms concerning the validity of Piaget's views and alternative theories. Piaget's theories appeared to ignore perception and did not provide easy application to instruction. A paper by Anderson and Cuneo (1978) proved to be very interesting. They investigated rectangular area judgements of 5 year olds. They concluded that "... most children in this age group can and do take account of both stimulus variables in making their judgements." (Anderson & Cuneo 1978, p.346). They employed a theory they called Information Integration Theory (IIT) and a functional measurement technique to find that young children's judgements of area followed two-dimensional rules: the additive height + width rule; and the multiplicative height x width rule. They showed that children as young as 5 years were using additive and multiplicative integration rules in judging area of rectangles.

The paper of Anderson and Cuneo (1978) influenced a series of studies that employed the methods of IIT and functional measurement to trace out the course of development of area concepts. These studies reported that adults and older children tended to follow the normative multiplicative rule for area while younger children tended to follow the additive rule.

Anderson and Cuneo (1978) considered that the additive height + width rule was a misuse of perimeter. In this, they were supported by the body of literature that followed them. They suggested that the height + width rule reflected a primitive integration process. They argued that the way in which children integrate both the height and width of a shape to form the area is crucial in children's understanding of area.

The mechanism which underlies the height + width and the height x width rules are not known. The form of integration in these rules (additive or multiplicative), and what stimuli cue this integration in 12 to 13 year-old children, has become the interest of the authors.

Research Questions

The aim of the research is to explore how 12 to 13 year-old children determine the area of rectangles (i.e., to identify and explain children's judgement rules for area). This exploration is based on the proposition that children determine area of a rectangle through three steps. First, values are assigned to the height and width of the test rectangle. Second, these values are added or multiplied depending on whether the subjects use the additive or multiplicative rule, respectively. Finally, the results of this operation are reported and can be measured on a response scale.

Purposes

Specifically, the research has the following purposes (which stem from its primary aim):

(1) to identify the way in which children integrate stimuli cues to determine area of a rectangle (i.e., to identify the function by which separate stimulus cues are integrated);
(2) to explore children's use of the specific additive or perimeter judgment rule, Area = Height + Width;
(3) to develop models and theories to explain use and misuse of judgement rules; and
(4) to draw implications for learning and teaching area.

Significance

Mensuration is a means of quantifying and describing the world around us. It provides us with a link between mathematics and the 'real world'. Measuring "... involves students in active participation in mathematics, and provides experience in handling and describing discrete and continuous quantities." (Department of Education, Queensland, 1987, p. 58).

Linear measurement is mensuration in a single dimension. Mensuration of plane and solid figures involves the use of formulae, and when considered in terms of plane figures, it becomes mensuration in two dimensions (Hubbard, Blakey, Elliott, Hubbard, Low, & Maher, 1972). Thus, the study of area is the study of mensuration in two dimensions. An understanding of geometrical figures existing in one plane is essential if a student is to understand mensuration in three dimensions.

As stated above, this research is based on the body of literature stemming from Anderson and Cuneo (1978) which has studied children's judgements using the height x width and height + width rules. It extends this body of literature by investigating the mechanism behind the Area = Height + Width rule, particularly any relation to perimeter. The literature considers that the perimeter and the height + width rule would produce similar responses thus rendering the two rules indistinguishable. The literature fails to address this aspect of children's thinking concerning area of rectangles. The study aims to fill this gap in the literature.

It should be noted that Silverman and Paskewitz (1988) have expressed concern against taking literally the height + width and height x width rules generated in the Anderson and Cuneo (1978) study of area. They agreed that the height + width rule may reflect the use of a perimeter rule. However, as the two rules, perimeter and height + width produce very similar responses, they expressed disquiet that the two rules may be different but not distinguishable in practice.

The work of Anderson and Cuneo (1978) is extended in this research by using a way of exploring children's thinking on area of rectangles which has not been taken up in the literature: the use of nonrectangular figures. Deleting a rectangular corner from a rectangle produces a figure of equal perimeter but less area. The removal of a scoop out of one side of a rectangle produces a figure with less area but greater perimeter. Children using a perimeter judgement rule might be unable to account in their judgements for variation in physical area, while a height + width mechanism might be able to do so (using similar arguments to Anderson & Cuneo, 1978). Any kind of height + width rule implies that responses to rectangles, rectangles with corners removed, scoops removed and right triangles of equal height and width will be proportional. Under the perimeter rule, the responses would show systematic deviations from linearity.

The intention of the research is, then, to seek to find the mechanism that underlies the Area = Height + Width rule. In this way, the research makes a contribution to the teaching of area in terms of differing ways in which children perceive area. This contribution has immediate implications for mathematics education in both primary and secondary schools. Theoreticians aspiring to understand children's perception and practitioners implementing area education can benefit from the outcomes. The results of the study could be directly implemented into both curriculum design and classroom practices in the domain of area.

Theoretical Background

The existing research into the concept of area is based on two theoretical frameworks.

Logical operational approach

The first framework belongs to Piaget who hypothesised how cognition developed, and he claimed that this developmental process is marked by a series of stages, the order of which is invariant although the age of the commencement and conclusion may vary. Piaget does not claim that a person functions solely at one stage, but that a person may function at one level for one concept and at a higher or lower level for another. Each stage represents a different way of dealing with a particular aspect of the environment, and as such, a child's thinking would reflect the stage he/she has reached.

Piaget is also responsible for a theory of the development of the knowledge illustrated by these characteristic stage behaviours. The theory is based on biological functioning which stresses that cognitive development is the result of the person's adaptation to the environment through accommodation and assimilation (Turner, 1975). In this theory, intelligence, at the operational level, is thinking, and both thinking and intelligence have to be distinguished from learning

Integrated Information Theory

The second theoretical framework is that of Information Integration Theory (IIT). IIT is based on the assumption that virtually any obvious response is the integrated combination of personal responses to different aspects of an object (Wolf, 1995). The methodological counterpart of the theory, called functional measurement, allows diagnosis, in simple algebraic terms, "... of the rules which govern integration of information about perceived stimuli." (Wolf, 1995, p. 49-50).

The guiding idea behind functional measurement is to use algebraic rules as the base and frame for psychological scaling. The algebraic rules provide the validity criterion needed to decide the controversy over rating and magnitude estimation. The algebraic rules provide a breakdown of the observed response into its functional components, as represented by the scale values and weights of the various pieces of information (Anderson & Cuneo, 1978).

Interview tasks

The interview tasks employed by Anderson and Cuneo (1978) were based on children's perceptions of the areas of different sized rectangles. Their study investigated ten children aged 5, 8, and 11 who were asked to judge the area of rectangular cookies. Nine of the cookies corresponded to factorial combinations of 7, 9, and 11cm. To obtain measures of the children's area judgements, the children were provided with a scale with two end points. Two special cookies of dimensions 5 x 5 cm, and 13 x 13 cm were used as end anchors. The children were asked to judge how happy a child would be to receive each of the 9 cookies to eat. The cookies, all of equal thickness, were presented individually, and judgement was expressed on a 19-point response scale with a smiling face at one end and a frowning face at the other. The presentation of the cookies was randomised, and a practice phase preceded the test phase.

Analysis of data

The children's responses are plotted against the length of one of the dimensions of the rectangles. If the resulting plot is parallel (a collection of parallel lines or curves), the students' judgements are considered to be additive. If the plot is fan shaped (expanding lines or curves), the students' judgements are considered to be multiplicative. Parallelism provides joint support for three conclusions: (a) it supports the additive model; (b) it provides validational support for the linearity of the response measure; and (c) it provides estimates of the subjective, psychological values of the stimulus variables on validated linear scales. In relation to the area studies, the parallelism theory illustrates the height + width rule. Similarly, fan shaped curves give rise to multiplicative rules. In an area example, estimated area of a rectangle would be expected to follow a height x width model. Statistical tests for both the models are available by use of analysis of variance. Through the use of within-subject design and numerical response, the integration approach makes feasible analysis at the level of the individual subject (Anderson & Cuneo, 1978).

Pilot study

The pilot study conducted was based on the body of literature stemming from Anderson's work in 1978.

Purposes

The purpose of the pilot study was to determine the following:

(1) that 12 to 13 year olds can perform the tasks;
(2) that the changes made to the size and shape of the rectangles to produce the nonrectangular shapes produce different behaviours from the rectangular shapes;
(3) that the instructions given to the children are easily understood and not at all confusing; and
(4) that Queensland educated children can show abilities to integrate both sides of a rectangle when judging its area.

Participants

Three boys were interviewed in the pilot study, David aged 8 years, and Rick and Simon, both aged 12 years. The individual interviews covered three separate experiments and lasted approximately 30 minutes.

Procedure

The boys were asked to pretend that they were very hungry. In order to get rid of their hunger, they were offered a rectangular piece of chocolate. The boys were asked to indicate on a wooden scale (the same as used by Anderson & Cuneo, 1978) how happy or sad the hungry child would be to receive that amount of chocolate in relation to the anchors. The expectation was that a small piece of chocolate may yield a sad response on the scale whereas a large piece may yield a very happy response.

Experiments

The first experiment contained 16 rectangular pieces of chocolate corresponding to all the possible combinations 3, 6, 9, and 12 cm. The second experiment used identical rectangular pieces of chocolate to the first experiment, but with a rectangular corner 'bitten' off producing a figure of equal perimeter, but less area. The dimensions of the 'bitten' off corner were all one third of the length and one third of the width of the rectangular stimulus. The third experiment again used identical rectangular pieces of chocolate but this time had a 'bite' out of one side producing a figure with less area but greater perimeter. The semicircular 'bite' was to be centred along the longest dimension, with the radius of the 'bite' one third of the length of the shortest dimension. Two trials were conducted for each child.

Results

The tasks were appropriate for the two older boys and provided interesting data. The younger boy found verbalising his thinking difficult.

The analysis of the results was two fold. Initially, factorial plots were drawn. These were parallel and indicated that all the three boys had utilised an additive rule in their judgements of area in all three experiments. However, these factorial plots failed to differentiate between the additive height + width rule and the perimeter rule. The second method of analysis was a simple linear regression of the responses. The responses were the dependent variables, while either the actual area, perimeter, half perimeter (height + width), or half perimeter using altered corners or sides, were the independent variables. The equation of the regression line was determined for all possible combinations for all experiments. These equations were then compared with the regression line equations generated by using the actual maximum and minimum values of the chocolate pieces, thus yielding a 'true' regression equation (correlation coefficient of 1.00). The closer a child's regression equation was to the true equation, then it was assumed that the child was using that method to judge the area of the chocolate pieces.

The true regression equations are given below along with the equations best matched from the subjects.

Experiment 1 - Rectangles. True Area is given by $y = 0.54x$, true Perimeter by $y = 0.42x - 3.6$ and true height + width by $y = 0.83x - 3.6$. David is represented by $y = 0.46x - 7.5$ (perimeter rule), Simon $y = 0.33x + 0.96$ (perimeter rule) Rick $y = 0.48x - 4.81$ (perimeter rule).

Experiment 2 - Corner Bites. True Area is given by $y = 0.6x$, true Perimeter by $y = 0.42x - 3.6$, true Height + Width by $y = 0.83x - 3.6$, and true Height + Width (corner sides) by $y = 1.3x - 3.8$. David is represented by $y = 1.0x - 7.8$ (height + width rule), Simon by $y = 0.69x - 0.90$ (height + width rule), and Rick by $y = 1.4x - 4.9$ (height + width rule).

Experiment 3 - Side Bites. True Area is given by $y = 0.66x$, true Perimeter by $y = 0.42x - 3.6$, true Height + Width by $y = 0.83x - 3.6$, and true Height + Width (bite plus adjacent side) by $y = 1.25x - 2.8$. David is represented by $y = 0.48x - 8.1$ (perimeter rule), Simon by $y = 0.82x - 3.9$ (height + width rule), and Rick $y = 0.42x - 4.7$ (perimeter rule).

Following the presentation of the three experiments to each child, the interviewer asked the child what method they had used to judge each piece of chocolate. David was not able to express himself in terms of a rule that he was following, but his comments suggest that he was looking at how many times the width of the test piece could be superimposed along the width of the largest end anchor, and likewise for the test piece length.

Simon indicated that he was looking at both height and width of the test pieces:

> I used the rule length by width times 2. No wait. I mean length plus width. Like if this is 6 cm and this is 3 cm then 6 + 3 = 9, so the area is 9 cm.

This indicates that Simon was using the Area = Height + Width rule, as the linear regression equation suggests. Rick informed the interviewer that the area rule for the rectangle was length plus width times two.

> You get a value for the length and a value for the width ... add 'em together then times the lot by two.

When queried about the missing sections he said you do the same for them, then subtract these from the original. Rick was using the perimeter rule, which again supports the results of the linear regression equations.

Discussion

The pilot study indicated that the research process contemplated could succeed as linear regression equations in the pilot study was able to extend the work of Anderson and Cuneo (1978) by differentiating between students using the Area = Height + Width rule and the Perimeter rule for judging area.

Implications for Proposed Study

The pilot study has shown that the proposed research has potential in two ways. First, the method of estimating area by using hunger, happiness and sadness appeared to work and provide insight into area judgement uncued by reference to area formulae. Second, the use of altered shapes, and altered areas and perimeters, appeared to provide the data from which it was possible to discover more of the nature of the integration cues for determining rectangular areas. In this, the pilot was particularly interesting in that the nonrectangular tasks enabled differentiation between the perimeter and length + width rules.

A large proportion of the research into the area concept has been conducted within the theoretical framework of the logical-operational approach. The logical-operational approach has two significant limitations - the ignoring of perceptual judgment, and a methodology which tests the supplementary abilities as well as the logical abilities of the child. This study utilises an alternative approach which considers the perceptual judgment of the subject, and has a functional measurement methodology, which has the specific purpose of assessing stimulus integration rules.

The primary aim of this research is to identify the function by which separate stimulus cues are integrated in determining the area of a rectangle. The pilot indicated that the functional measurement methodology developed by Anderson could be highly appropriate for this aim.

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