Assessing the Major Trends and Directions of Research Into Students' Judgements of Area

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This paper highlights two major research approaches into studying how students judge area, logical-operations and information processing. The logical-operations approach focuses on determining the order in which students acquire area knowledge based on a framework developed by Piaget. The information processing approach focuses on perceptual judgement. The paper assesses the two approaches in terms of appropriateness for future research.

There are two basically different approaches which have guided the research into students' judgements of area. The first approach emphasises logical operations within a conceptual framework developed by Piaget. The other approach emphasises the role of perception and judgemental processes. Both approaches are considered in this paper.

The paper begins by reviewing studies based on the logical-operational approach to research into children's judgement of area. These studies are considered in terms of pre and post 1975; 1975 seems to be key year in which the nature of research changed from being concerned with the order in which area knowledge is acquired to being concerned with conceptions (understanding) and misconceptions of area, and the form of research changed from interviews to teaching experiments. Then, the role of perception and judgemental processes is considered in a section which outlines the nature of information processing techniques. Here a body of literature contradicting the findings of Piaget is considered in relation to students' perceptual judgement of area. The limitations of the logical-operational approach are then presented, thus giving rise to the appropriateness of the alternative approach and its focus on perceptual judgement. The final section of the paper recommends a possible direction for future research.

The Logical-operational Approach

The logical-operational view originates from initial research into students' understanding of measurement conducted by Piaget and his associates in 1960 (Piaget, Inhelder, & Szeminska, 1960). Piaget's description of the development of area concepts has been the subject of a wide variety of studies attempting to replicate, disprove, extend or explain his conclusions. These studies covered the development of the major concepts of conservation and transitivity and they dealt with the following four basic concerns:

(1) difficulties with validating the reality of individual cognitive operations and describing the stages of their development - this issue has led to studies which have replicated (with slight variation) some of Piaget's procedures and materials;
(2) difficulties in investigating the relationships between different cognitive skills - this second issue has been addressed by researchers by administering a series of different tasks to the same sample of students to establish a developmental hierarchy;
(3) misconceptions students have relating to area - information on this issue was obtained by analysing incorrect answers provided by students; and

(4) difficulties with identifying the nature of the progression between stages of development - this has generally been dealt with through training studies.

If these four types of research questions were dealt with simultaneously, the resulting study was considered to be a fifth type. Often this fifth type of study involved the application of information processing techniques.

Pre 1975

Area research issues gradually changed over time. Piaget and his associates published their findings in 1960. During the next 15 years, most area research centred around replicating and extending Piaget's work. A number of studies (Beilin & Franklin, 1962; Goodnow, 1968; Lovell, Healey & Rowland, 1962; Wagman, 1975) followed the design of Piaget and the Geneva group and focused on determining the ages at which students acquire an understanding of measurement concepts. Most, if not all, studies confirmed those of Piaget, Inhelder and Szeminska (1960) in identifying definite stages through which students pass. Piaget Inhelder and Szeminsk (1960) divided the development of measurement into four stages, the middle two stages being further divided into two substages. The defining feature of each stage was the presence or absence of set operations or mental actions.

Stages I and IIA. For children at this cognitive stage, the measurement of area is not feasible. The longest linear measurement is used to make area judgements. Children can not demonstrate an understanding of a complete covering with a constant unit of measure. Children (up to the ages of 5 or 6 years) are also unable to conserve area or understand the notion of transitivity.

Substage IIB. At about 6 - 7 years of age, children have an obscure perception of conservation and transitivity. Some area measurement is possible, but children fail to understand the importance of the size of the unit. Fractional units are often counted as whole. They equate two quantities which have the same number of covering units, even though the size of the units vary.

Substage IIIA. Children 7 - 8 years of age conserve interior area but not complementary area. They are able to apply middle term transitivity as long as it was as large as one of the objects being measured. Children recognise that a quantity is the sum of its unit covering, but they still have no notion of unit size and completeness.

Substage IIIB. From 8 - 10 years, change of position and subdivision are coordinated and measurement through unit iteration is possible. Usually the unit covering needs to be physically applied. At this level, conservation is generalised to cover complementary areas.

Stage IV. This is the logical operations stage. Beginning at 11 - 12 years of age, development of measurement is complete. A child develops the notion that space consists of an infinite and continuous set of points. Children are also able to calculate area from linear dimensions. An older child would "conserve", recognising that the area remains constant despite the perceptual change.

Other studies focused on whether the development of students through the stages of acquisition of measurement knowledge can be accelerated through training studies. In reviews of the literature, both Beilin (1971) and Carpenter (1976) concluded that most training procedures accelerate acquisition of conservation and transitivity, but found little evidence for the preeminence of a single set of training procedures. Carpenter (1976) argued that training is able to extend existing operations, but not create new logical operations.

Post 1975

After 1975, replicating studies were rarely seen, with the exception of Lu's work (Lu, 1981, 1991; Lu, Zhang, & Cheu, 1985) from China. It is interesting to speculate as to why Lu is the only researcher studying the order of acquisition since 1975. Perhaps Lu has not accepted Goodnow's (1968) findings that nationality has very little influence on conservation ability.

Since 1975, area research generally focused on investigating relationships between different cognitive skills. Bell, Hughes and Rogers (1975) were followed by Carpenter, Corbitt, Kepner, Lindquist and Reys (1981), Foxman, Ruddock, Badger and Martini (1982), and Hart (1981), in undertaking large scale assessments of the performance of students on a variety of area tasks. These large scale studies appear to influence many smaller studies relating to the understanding of area conducted throughout the 1980s to the present.

Taken together, the conclusions of the studies indicate the following:

(1) students find the pre-formula area concepts easier than later area concepts;
(2) students use superimposed-grids for determining linear measurements which are then multiplied as well as for unit counting;
(3) most students are able to use A = l x b as a rule for finding area;
(4) students are often introduced to procedures before they fully understand the concepts of area;
(5) very few students have any knowledge or understanding concerning the relationship of the square and triangle area formulae to that of the rectangle;
(6) in estimating area, students tend to underestimate if the concept is presented in a 'real-life' context but overestimate if the same problem is presented in a mathematical context;
(7) students' test performance is dependant upon the means of testing; and
(8) nonconservers have shown area conservation skills if the transformation of the area is made to appear accidental.

Many students, and adults, as shown by Baturo and Nason (1996 in press), lack an understanding of basic area and measurement concepts. This lack of basic understanding means that their area knowledge is limited, disjointed and difficult to apply It also means that errors and weaknesses with area problems does not necessarily indicate a total lack of area knowledge or understanding; rather, the person may not be able to apply their knowledge or understanding in the given situation.

It is not surprising that with the availability of a large amount of data relating to students' understanding, that the 1980s saw the emergence of studies concerned with student misconceptions. Hirstein (1981) and Hirstein, Lamb and Osborne (1978) highlighted the need to investigate students who were failing the tasks an experimenter was administering. As a result, studies since the early 1980s have tended to focus on the misconceptions displayed by their subjects.

The research on misconceptions has shown that students' problems in understanding the area concept relate to a confusion between area and perimeter. In calculating either area or perimeter, the process of obtaining the shapes' measurements was also found difficult revealing errors concerning which dimensions to consider as well as how to count the units along the selected dimensions. It has also been shown that these errors are more conceptual than arithmetical.

As well as on misconceptions, studies have also focused on effective instruction of area concepts. Van Hiele (1986) claimed that instruction is most successful if it is directed at the student's level. The ideas of van Hiele (1986) are partially based on the notion that geometric growth takes place in identifiable levels of understanding. These levels were developed as a result of "perceived deficiencies with the views of Piaget" (Pegg, & Davey, 1991). Recent studies (Sayeki, Ueno & Nagasaka, 1991; Zaslavsky, 1991) have included teaching experiments which aim to provide novel methods for increasing the levels of understanding in the area concept. Their findings were that area instruction which employed novel tools or methods (such as concrete materials and everyday examples) achieved greater student understanding of the area concept than traditional methods. The conclusion drawn was that the use of concrete materials and everyday examples makes the information more relevant to the students, thus increasing their attentiveness and understanding.

Information Processing Techniques

Since 1978, alternative approaches to the research methods employed in the logical-operational studies have emerged. According to Langford (1979, p. 1), this has been in response to the belief that Piaget "... may have missed whole realms of concepts and modes of thought.". In particular, Anderson and Cuneo (1978) argued that, as perceptual processes and judgement theory were largely passed over in the logical-operational approach, they provided an alternative information-processing basis for the study of students' judgement of quantity. They proposed a theory that provided a combined treatment for both physical quantities and social quantities.

Information-processing techniques, based on those proposed by Anderson and Cuneo (1978), have been found to offer the best opportunity to explain the process of area concept development in students and, therefore, have been used in many recent area studies. Although these information processing techniques emerged from the theories posed by Piaget and the Geneva group, they took almost exactly the opposite stance towards Piaget, "... accusing him of insufficient use of mathematics and formal logic." (Langford, 1979, p. 3). Contrary to Piaget's proposals, which were sometimes vague, the information-processing techniques aimed to be quite precise about how area concepts are used in making decisions and guiding actions in specific situations. The evolving body of literature based on these techniques focused on perceptual processes and judgement-theory. It was based upon the assumption that virtually any response is the integrated combination of inner, or subjective, responses to different aspects of a target object (Wolf, 1995).

Anderson and Cuneo (1978) and Wilkening and Anderson (1982) developed a research methodology, titled the functional measurement methodology, which assessed students' quantitative judgements and allowed diagnosis, in simple algebraic terms, "... of the rules which govern integration of information about perceived stimuli." (Wolf, 1995, p.49-50). They developed a theory called Information Integration Theory (IIT) which places importance on problems of stimulus integration and multiple causation. According to IIT, all behaviours reflect a blend of stimuli, and a response is the consolidated resultant of multiple causal forces. In IIT, analysis attempts to scrutinise an observed response into its causal components, while synthesis determines the integration function combining the causal components.

In IIT studies, students were provided with different rectangular shapes and asked to place their area on a linear scale. This scale position were plotted against one of the sides (length) of the rectangles. If the resulting factorial plot was a series of parallel curves, then the students was considered to be using an additive model (the length + width rule). If the factorial plot was "... a diverging fan of straight lines." (Anderson & Cuneo., 1978, p. 339), the students were considered to be using a multiplicative model (the length x width rule). Statistical tests for both the models are available by use of analysis of variance. Through the use of within-subject design and numerical response, the integration approach makes feasible analysis at the level of the individual subject (Anderson & Cuneo, 1978).

The results of the information-processing research have indicated that the Piagetian conclusions relating to development maybe inaccurate. The studies of Anderson and Cuneo (1978) and Wilkening and Anderson (1982), and those following them, revealed findings that strongly contradicted the assertions of Piaget "... that young students' judgements of quantity are necessarily or generally one-dimensional in character" (Silverman & Paskewitz, 1988, p.75).

A number of investigators (Anderson & Cuneo, 1978; Lautrey, Mullet & Paques, 1989; Rummele, Kuhn & Zoeke, 1990; Schlottman & Anderson, 1994; Silverman & Paskewitz, 1988; Wilkening & Anderson, 1982; Wolf, 1995) found that students' judgements of area obeyed two-dimensional rules. In other words, they found that young students (3 to 5 years old) used additive integration rules in which they judged area in terms of a height + width rule, while older students (by 11 years of age) obeyed the normative, multiplicative rules. They argued that 8 year old students were "... in a 'transitional stage' between the height + width rule and the height x width rule." (Silverman & Paskewitz, 1988, p.77). The studies of both Lautrey, Mullet and Pasques (1989) and the Silverman and Paskewitz (1988) found that not all preschool students follow the height + width rule. Silverman and Paskewitz (1988) had students rate the areas of rectangles, triangles and ellipses. Their findings indicated that only a minority of preschoolers use a one dimensional rule. This strengthened the Anderson and Cuneo (1978) argument for rejecting Piaget's contention that young students can judge a stimulus only according to a single, salient dimension.

Limitations of the Logical-operational Approach

A large proportion of research into the area concept has been conducted within the theoretical framework of the logical-operational approach. The logical-operational approach has two significant limitations that are of concern. The first is that the approach ignores perceptual judgement. In Piaget's writings, perceptions and operations are segregated (Flavell, 1963). Wolf (1995) considered the processes of perception a central component to almost all behaviour, including that of conservation of area tasks. It is doubtful, then, that an acceptable theory can be produced if perceptual judgement is ignored.

The second limitation is related to methodology. A correct response in the standard Piagetian task demands supplementary abilities besides the logical abilities in question. For example, the conservation tasks test not only logical capacity, but control of attention, estimation skills and correct semantics. Information-processing researchers (e.g., Anderson & Cuneo, 1978) argue that the all-or-none character of choice data in logical-operational research has fostered all-or-none conceptualisations of what may actually be continuous concepts.

As a result of these limitations, many generalisations have become accepted in logical-operational research that may be incorrect. An experimental methodology has evolved in this research approach that is inadequate to determine many of the substantive concerns that it faces.

Recommended Future Research

The evidence from the literature (Anderson & Cuneo, 1978; Lautrey, Mullet & Paques, 1989; Rummele, Kuhn & Zoeke, 1990; Schlottman & Anderson, 1994;

Silverman & Paskewitz, 1988; Wilkening & Anderson, 1982; Wolf, 1995) is that some students judge area using a height x width rule while others use a height + width rule. However, the mechanisms that underlie these area judgement rules is unknown. At this time, the best approach to find these mechanisms would appear to be to study the way separate stimulus cues are integrated to determine the area of a rectangle and to identify the function by which this integration occurs. Because the functional measurement methodology and Information Integration Theory (IIT) developed by Anderson use algebraic rules to provide the validity criterion needed for rating and magnitude estimation, they appear to be a highly appropriate approach to studying area judgement. The algebraic rules provide a breakdown of the observed response into its functional components, as represented by the scale values and weights of the various pieces of information (see Anderson & Cuneo, 1978).

One of the interesting aspects of the IIT studies is the relationship between the additive length + width rule and the perimeter rule. The literature considers that the perimeter and the height + width rule may produce similar responses thus rendering the two rules indistinguishable (e.g., Silverman & Paskewitz, 1988). It may be possible to investigate this relationship by using nonrectangular figures (Anderson & Cuneo, 1978; Silverman & Paskewitz, 1988). For example, deleting a rectangular corner from a rectangle produces a figure of equal perimeter but less area and removing a scoop out of one side of a rectangle produces a figure with less area but greater perimeter. In these cases, students using a perimeter judgement rule may be unable to account in their judgements for variation in physical area, while a height + width mechanism may be able to do so.

In summary, the problem of stimulus integration in students' area judgments remains largely unanswered. IIT research has made some advances with the Area = height + width and Area = height x width rules. There seems to be a possibility for further advances in continuing IIT research with nonrectangular figures. The development of models to explain the mechanisms underlying stimulus integration appears to be a fruitful aim of this research.

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