An Analysis of Teacher Generated Whole Number Word Problems

Kevin J. Maguire
La Trobe University

In curriculum statements provided for teachers, teachers are encouraged to present to their pupils a rich variety of word problems. Teachers may draw their word problems from reference texts, "real-life" problem situations or generate their own word problems. An analysis of teacher generated word problems reveals that these word problems not only fail to provide as diverse a range of problems as suggested in the resources made available to teachers, but also reinforce some misconceptions about the four arithmetic operations. The implications of these findings are discussed and proposals advanced to overcome this problem.

Over the last few years there has been a considerable interest in word problems, both as a topic for research (Carpenter, Ansell, Franke, Fennema, & Weisbeck 1993; Carpenter & Moser, 1984; Carpenter & Moser, 1982; Cortes, 1995; English, 1996; in press a; in press b; Gillies, Walker & Bailey, 1995; Greer & McAnn, 1993; Greer, 1992; Mulligan & Mitchelmore, in press), and in mathematics statements throughout much of the English-speaking world. The National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989, p. 5) states, as one of the goals for students to achieve, *that they become mathematical problem solvers* and that they aim to *develop and apply strategies to solve a wide variety of problems* ... (p. 23). A similar approach has been taken in Australia both nationally (Australian Education Council and Curriculum Corporation, 1991) and in the states such as Victoria (Board of Studies, 1994), and New South Wales (New South Wales Department of Education. 1989) as well as in England and Wales (Department for Education, 1995). This point is considered below. Of particular interest is the focus upon solving "real life" problems. This point is taken up below. Such "real life" problems are often presented to students as word problems. Hembree and Marsh (1993, p. 152) define a problem as *a task or question stated in words wherein the person attempting its answer must select the operations.* They suggest that word problems can assist students refine their conceptual understanding of mathematics and learn to use appropriate strategies suited to the particular situation. Greer (1992) makes reference to misconceptions often implicit in teacher generated word problems. This point is considered below. The present investigation analyses arithmetic whole number word problems generated by a class teacher. The problems are matched with the general curriculum statements published for the guidance of teachers in Victoria. To provide the means of categorising word problems a system, based on semantic analysis of word problems (Carpenter & Moser, 1982; Maguire, in press), has been used in this analysis.

Categorisation of word problems

Additive Word Problems: Greer (1992) comments that the categorisation of word problems is a research tool and in no way indicates any solution strategy adopted by a child to solve the word problem. The source of students' "real life" problems can be a text book, the mathematics of a given situation itself or the class teacher. Silver (1992) has suggested that further research is needed into this latter source of problems. Carpenter and Moser (1982; 1984), and Carpenter et al. (1993) have distinguished six

categories of addition and subtraction word problems. These categories are based on the semantic features of the problems. Table 1 provides examples of each of these categories of word problems.

Table 1. Problem categorisation, representative word problem and semantic criteria used for the categorisation of word problem based on Carpenter and Moser (1982; 1984), and Carpenter et al. (1993).

Problem categorisation	Representative samples of word problem	Criteria of word problem categorisation
Joining	*Mary has 5¢ in her pocket. On the way home she found 10¢ in the grass. How much money does she have now?*	1. Active. 2. Two quantities are a sub-set of the third. 3. Increase.
Separating	*Richard had 15 marbles. He lost 7 of them in a game. How many marbles has he left?*	1. Active. 2. Two quantities are a sub-set of the third. 3. Decrease.
Equalising-add on	*Mary has 15¢. Susan has 35¢. How much more does Mary need to save to have the same amount as Susan?*	1. Active. 2. Compares disjoint sets. 3. Increase.
Equalising-take away	*Rowan has 25 marbles and Ellen has 17 marbles. How many does Rowan need to give away to have the same number of marbles as Ellen?*	1. Active. 2. Compares disjoint sets. 3. Decrease.
Part-part-whole	*In the purse there is a 10¢ coin and a 20¢ coin. How much is in the purse?*	1. Static relationship 2. Two quantities are a subset of the third.
Comparison	*In his bag Rowan had 25 small marbles and 16 large marbles. How many more small marbles are there than large marbles?*	1. Static relationship 2. Compare disjoint sets.

Carpenter and Moser (1982) have used the criteria of action/static, set inclusive and sub-set relationships, as well as whether there was an increase or decrease in a given quantity to categorise additive word problems.

In the joining problem: *Mary has 5¢ in her pocket. On the way home she found 10¢ in the grass. How much money does she have now?* the problem is active in that it involves the action of joining the two quantities in order to obtain a solution. The two quantities *5¢* and *10 ¢* are a subset of the solution quantity. The operation upon these two quantities produces an increase in the amount of money Mary now has. Each of the above word problems can be categorised according the semantic criteria proposed by Carpenter & Moser (1982).

Word problems provide information in the form of statements which are usually followed by a question. In the word problem *Mary had 5¢ in her pocket. On the way home she found 10¢ in the grass. How much money does she have now?* there are two informational statements - *Mary had 5¢* and that she *found 10¢*. These two statements are reference statements in that they provide a reference for the solution calculation. The question asks "*How much money does she now have?*". This is a solution statement. These points are further developed in the following section.

Multiplicative Word Problems: Many researchers (English, in press a; Greer, 1992; Mulligan, & Mitchelmore, in press; Silver, 1992; Vergnaud, 1983) working with children's categorisation of multiplicative word problems have identified a number of categories. A sample of these is presented in Table 2.

Table 2. Problem categorisation, and a representative example of word problem based on Greer (1992) and English (in press a).

Problem categorisation	Representative sample of word problem
Comparison multiplication	*Richard had written 12 lines of his story. Jessica had written three times as many lines as Richard. How many lines has Jessica written?*
Comparison division	*Paul has already written 46 lines of his story so far. This is twice as many as Tony has written. How many lines has Tony written?*
Cartesian product	*Nancy has three different summer skirts, four different tops and two pair of shoes. How many different outfits can she wear?*
Partitive division	*Mrs Jones bought 15 chocolate bars to give to the children at the party. There were 5 children at the party. If each child was given the same number of chocolate bars, how many chocolate bars did each child receive?*
Equal groups	*Mrs Jones gave three pieces of fruit to each of her four children. How many pieces of fruit did they have between them?*

This table does not include all possible multiplicative word problem categories. Table 2 is representative of word problems considered by Greer (1992) and English (in press a). The sample does not include quotition division, comparison division and non-routine word problems.

The above multiplicative word problems involve an implied operation. This operation can result in either an increase or a decrease in the solution quantity (for example the product or the quotient) without an increase in quantities of the subset or the comparison of the disjoint sets. Maguire (in press) has provided criteria which allow multiplicative word problems to be categorised according to semantic criteria. These criteria are different from those criteria developed by Carpenter and Moser (1984) and have been introduced by this author to classify multiplicative word problems. Multiplicative word problems involve the interrelationship between the three types of statements present in many word problems, viz an informational reference statement, an informational relational statement and a solution statement. Word problems generally consist of three main types of statements, two (or more) of which are informational statements. Informational statements may be reference statements in that they provide information necessary to solve the problem. For example in the word problem *Richard had written 12 lines of his story. Jessica had written three times as many lines as Richard. How many lines has Jessica written?* the reference statement *Richard had written 12 lines of his story* provides reference information about the number of lines written by Richard. The relational statement *Jessica had written three times as many lines as Richard* as well as providing information, establishes a relationship between the number of lines written by Jessica and Richard. The solution statement *How many lines has Jessica written?* identifies the unknown and provides a unit of quantity.

Mathematics Guide

Prior to the current analysis, a Victorian primary school had undertaken a review of its mathematics syllabus. The teaching staff developed a curriculum guide using the state's Mathematics Framework (Ministry of Education (Schools Division), 1983) as a resource. The guide was prepared to assist teachers to develop their individual class syllabi and included numerous references to the different approaches a teacher could take to encourage mathematics learning within a problem solving classroom environment. The guide advocated mathematics learning through the use of problem solving situations. This was reflected in the choice of resource material suggested. Not only was mathematics to be presented in a problem solving format but, in addition, it was suggested that students be given weekly exercises in the use of problem solving techniques.

Analysis

The present analysis seeks to analyse some 282 word problems posed by the Year 3/4 teacher. The teacher was one who wished to provide for his students a thorough understanding of those class/age appropriate arithmetic concepts underlying the four arithmetic operations (Ministry of Education (Schools Division), 1983). Anecdotal evidence suggested that many students within this class had commenced the year with poor mathematical understanding and experience. The children had been taught by a teacher whose mathematics syllabus was not in accord with the curriculum guide provided.

This analysis deals only with positive integers in mainly single-step arithmetic word problems involving the four operations. The term "quantity" therefore generally refers to a positive whole number and not a fraction. Initially, in Years 3 and 4, the children were presented only with partition and quotition division word problems which resulted in whole number quotients. Later, after the children had gained more experience in using multiplicative concepts (Year 4), quotients with remainders were introduced and later, involving money, quotients with decimal fractions were included.

The teaching situation: The class was a composite class of both Grade 3 and 4 students. Throughout the mathematics sessions both grades were set separate class activities. Each student was provided with worksheets which s/he was to complete by the end of the school week. The teacher planned that one section was to be completed each day. Each section of the worksheet dealt with a different aspect of the class mathematics syllabus. At the beginning of each mathematics session the teacher would introduce new work to the Grade 3 students whilst those in Grade 4 read through their assigned work or completed any unfinished work from the previous session. During this time the Grade 3 children were required to look through the activities whilst the teacher provided an overview. Occasionally some examples were worked through as a class exercise before the children commenced their worksheets. Children were encouraged to work cooperatively in groups of two or three students. After checking that every child had commenced work the teacher moved to the Grade 4 children. When all children had commenced work, the teacher moved from group to group listening, talking with children, challenging them and helping them to resolve any difficulties they were experiencing. Towards the conclusion of the session both grades were brought together for a class discussion of the work completed.

The word problems: The word problems selected for analysis come from the class syllabus for both the first and second semester of 1993. The first of these problems (Grade 3) were devised to extend the children's understanding of addition of two digit numbers together with a selection of subtraction, and introductory multiplication and division word problems. For these problems children were encouraged to use calculators (initially) and manipulatives (MAB) were freely available. The teacher commented that he was more concerned with the children learning to read the problems and map them to an appropriate operations than actually calculating the "correct" answer. The focus of these lessons was the understanding of the underlying mathematical concepts. Both multiplication and division operations were introduced concurrently as the teacher wished the children to understand the interrelationship of these two operations.

The four processes: Whilst the problems given to the Grade 3 children were straight forward, those presented to Grade 4 were generated to extend their ability to reason mathematically. The focus of these questions was not to access the children's ability to

calculate but their ability to select an appropriate operation to solve the problem. Calculator usage was permitted for many of these word problems. The initial selection of word problems was included in this analysis as they were generated by the teacher as a revision activity. The second selection gives an example of a longer complicated group of interrelated problems for which the children were permitted to use calculators.

Diversity of Word problems: As shown in Table 3 the number of word problems of different categories differ quite notably in their representation. Whilst some categories are not represented (Equalising subtraction, Comparison multiplication and division, and Cartesian product), there is a tendency to present certain categories (Joining, Separating, and Equal Groups) to a greater extent than others (Equalising Addition, and Comparison). Far from providing "a wide variety of problems", the problems are quite limited in range and often not relevant to the students' out-of-school activities. Addition and subtraction are represented almost entirely by Joining (24.8%) and Separating (23.1%) word problems, with Comparison a mere 2.1% of the total. Multiplicative word problems are almost all represented by Equal Groups (24.8%), Quotition Division (9.6%) and Partition Division (12.4%).

Table 3. Problem categorisation, representative sample of problem type and the number and percentage of such problems evident in the analysis.

Problem categorisation	Representative sample of problem type	Number of problems sampled
Joining	*Gloria skipped seventy-three skips on her first try. On her second try she was tired and skipped only another twenty-six skips. How many skips did she skip on her two tries?*	70 (24.8%)
Separating	*Steven taped a TV show for us to watch. It was eighty-five minutes long. If we have already watched twenty-eight minutes, how many more minutes have we still to watch?*	65 (23.1%)
Equalising-add on	*The computer game Steven wanted to buy cost $48.00. If he had only $23.00, how much more had he still to save until he had enough money to buy the game?*	1 (0.4%)
Equalising-take away		0
Part-part-whole		0
Comparison	*Sarah had seventy-eight stickers in her book. Gloria had only started saving stickers and had only thirty-one stickers in her book. How many more stickers had Sarah than Gloria?*	6 (2.1%)
Comparison multiplication		0
Comparison division		0
Cartesian product		0
Equal Groups	*Each day Helen's mother walked five kilometres. How far did she walk in ten days?*	70 (24.8%)
Quotition division	*Jack had twenty-five marbles. He gave each of them five marbles. With how many friends did he share the marbles?*	27 (9.6%)
Partitive division	*As a special treat Mrs. Jones bought twenty-two chocolate bars. She shared them with the children at the party. How many chocolate bars did each of the eleven children get?*	35 (12.4%)
Non-routine	*During the holidays I had $700.00 to spend. I bought a pair of trousers for $85.00, a jacket for $250.00; four pair of socks for $2.50 each, and a pair of shoes for $125.00. How much did I have left over?*	8 (2.8%)

Real-life problems: Word problems of the type *Peter had 150 chocolate bars. If he ate five chocolate bars each evening, how many evenings would it take him to finish eating all the chocolate bars?* may initially interest some children but could not be considered within the children's reality. It is unlikely that a child would have so much chocolate and would no doubt be quite ill or would lose their appeal well before they were all eaten. Problems not included among those generated by the teacher such as *Two pizzas are*

divided equally among eight people. How much pizza does each person get? could well be relevant to the children and within their experience. This author's observation of the word problems suggest that many problems were not within the children's experiences and may not be considered relevant to the children.

Misconceptions: Greer (1992) suggested that elementary school teachers share a number of misconceptions regarding multiplicative operations, viz

- that multiplication results in an increase whilst division produces a decrease in a given quantity
- that when dividing, one divides the larger number by the smaller

Observation of the representative word problems indicate these teacher generated word problems reinforce these misconceptions. In those cases involving multiplication the product was larger than any of the quantities in the reference or relational statements. A similar situation is found in division calculations where the quotient was always smaller than the dividend or the quotient. In no division calculation was the divisor greater than the dividend.

Implications

There are two main characteristics of these teacher generated word problems that invite consideration, viz

- lack of breadth
- reinforcement of misconceptions

The limited range of word problems represented in the analysis indicates that the children did not experience as diverse an range of problems as suggested in the resource literature (Board of Studies, 1994; Australian Education Council and Curriculum Corporation, 1991). It is this author's contention that the range of word problems provided was not encompassing enough to extend the children's arithmetical experience of the world in which they live.

The analysis raises two major questions which require further investigation, viz

1. whether primary school children, when presented with a limited range of word problems, are able to cope with problems which deal with rational numbers, decimal fractions or proportion. This question is in agreement with those raised by Silver (1992).
2. whether the misconceptions evident in the analysis
   a) that the multiplication of one quantity by another always results in an increase whilst division produces a decrease and;
   b) that when dividing one divides the larger number by the smaller (Greer, 1992), may cause young students difficulty in adjusting to operations of allocating a smaller number of articles amongst a larger number of groups.

Word problems such as *Two pizzas are divided equally among eight people. How much pizza does each person get?* require the division of the smaller quantity by the larger giving either a fraction or decimal representation of the quantity.

One method of overcoming the problem of a limited range of teacher generated word problems may be for mathematics educators to become involved with a cluster of primary schools to conduct continuing professional development in mathematics. This author regards such involvement as including the presence of a mathematics educator in cluster schools to work alongside class teachers, learning with them in an action research situation (Kemmis and McTaggart, 1990). In the action research paradigm the mathematics educator and the teachers work together as equal partners. The role of the

mathematics educator could be to introduce new research findings; to act as a conduit for ideas; and to enable the sharing of behaviours and experiences between teachers.

References

Australian Education Council and Curriculum Corporation. (1991). *A national statement on mathematics for Australian Schools,* Carlton, Victoria: Curriculum Corporation.

Board of Studies. (1994).Curriculum & Standards Framework: Draft for Consultation. Carlton, Victoria: Board of Studies

Carpenter, T. P., Ansell, E., Franke, M. L., Fennema, E. and Weisbeck L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education, 24,* (pp. 428-441)

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15* (3), (pp. 179-202)

Carpenter, T. P., & Moser, J. M. (1982). The development of addition and subtraction problem-solving skills. In T. P. Carpenter, J. M Moser and T. A. Romberg (Eds.). *Addition and subtraction: A cognitive perspective* (pp. 9-24). Hillsdale, NJ: Lawrence Erlbaum

Cortes, A. (1995). Word Problems: Operational invariants in the putting into equation process. In L. Meira & D. Carraher (Eds.). Proceedings of the 19th Psychology in Mathematics Education Conference, Reclife, Brazil. Volume 2 (pp. 58-65).

Curriculum Corporation (1994a). *Mathematics - a curriculum profile for Australian schools*. Curriculum Corporation.

Curriculum Corporation (1994b). *Mathematics - work samples*. Curriculum Corporation.

Department for Education. (1995). *Mathematics in the National Curriculum*. Department for Education. London, HMSO.

English, L. D. (1996) Children's reasoning processes in classifying and solving computational word problems. In L. D. English (Ed.). *Mathematical reasoning: Analogies, metaphors, and images.* Hillsdale, NJ: Lawrence Erlbaum.

English, L. D. (in press a). Children's problem posing and problem-solving preferences. In J. Mulligan & M. Mitchelmore (Eds). *Research in early number learning.* Australian Association of Mathematics Teachers.

English, L. D. (in press b). Children's problem posing with formal and informal computational contexts. *Journal for Research in Mathematics Education.*

Gillies, R. L., Walker, R. A. & Bailey, M. (1995). *The effects of metacognitive strategy and attributional interventions on students' ability to solve mathematical word problems.* Paper presented at the annual Australian Association for Research in Education Conference, Hobart.

Greer, B. (1992). Multiplication and division of models of situations. In D. A. Grouws (Ed.). *Handbook of Research on Mathematics Teaching and Learning* (pp. 276-295). New York: National Council of Teachers of Mathematics and Macmillian.

Greer, B & McAnn, M. (1993). Children's word problems matching multiplication and division calculations. In F. Furinghette (Ed.) *Proceedings of the Fifteenth PME Conference* (pp. 80-87). Assisi, Italy.

Hembree, R., & Marsh, H. (1993). Problem solving in early childhood: building foundations. In R. J. Jensen. (Ed.). *Research ideas for the classroom: Early Childhood Mathematics* (pp. 151-170). National Council of Teachers of Mathematics and Macmillan New York.

Kemmis, S. and McTaggart, R. (1990). *The Action Research Planner*, 3rd edition. Deakin University: Deakin University Press.

Maguire, K. J. (in press) *Towards a categorisation of multiplicative word problems.* Paper to be presented at the annual conference of the Mathematics Education Research Group of Australasia, University of Melbourne.

Ministry of Education (Schools Division). (1983). *Mathematics Framework P-10.* Ministry of Education (Schools Division), Victoria.

Mulligan, J. T. & Mitchelmore, M. C. (in press). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education.*

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

New South Wales Department of Education. (1989). *Mathematics K-6.* New South Wales Department of Education, Sydney.

Silver, E. A. (1992). Referential mappings and the solution of division story problems involving remainders. *Focus on Learning Problems in Mathematics, 14* (3), 29-39.

Vergnaud, G. (1983). Multiplicative Structures. In R. Lesh and M. Landau (Eds.). *Acquisition of Mathematics concepts and Processes* (pp. 127-174). New York: Academic Press.