Statistical Concepts of High School Students: Some Findings from Fiji

Sashi Sharma
University of Waikato

Growing interest in statistics and probability in schools has resulted in a change to developing concepts rather than carrying out calculations. However, statistical reasoning is not easily acquired by all students and common errors (misconceptions) reflecting difficulties have been identified in several studies. This paper presents the results of a study which explored form five (14 to 16-year-old) Fijian students' ideas of statistics and probability and how these related to their previous school and cultural experiences.

Introduction

Despite the growing use of statistics and probability in our society, there is substantial documentation of the systematic errors and biases that people display when attempting to interpret statistics in their daily lives. Kahneman and Tversky (1983) state that even highly educated professionals who use statistics and probability in their daily lives display biases when attempting to interpret the statistics they produce.

Research on students' understanding of probability and statistics provides evidence that students appear to have difficulties developing appropriate intuitions about fundamental statistical ideas (Brown et al., 1988; Bright and Hoeffner, 1993; Shaughnessy and Bergman, 1993). In a review of the literature, Garfield and Ahlgren (1988), give three reasons for some of the difficulties which have arisen in teaching statistics at the college level. First, students are generally weak in rational number concepts and proportional reasoning used in calculating, reporting and interpreting probabilities. For instance, the concepts of combinatorics and relative frequency both assume a grasp of proportionality. Secondly, many students have a distaste for probability and statistics through having been exposed to its study in a *highly abstract and formal way* (p. 47). Thirdly, statistical ideas often appear to conflict with students' experiences and how they view the world.

Shaughnessy (1992) reports that most of the research in probability and statistics has been done with elementary school children or with college students, resulting in gap in our knowledge about students' conceptions of probability and statistics at the secondary level. In order to help inform teachers and curriculum designers, it is crucial to carry out investigations at this level. He adds that to obtain a clearer picture of student conceptions, beliefs and misconceptions, it is essential to have teaching experiments and more long-term interventions with a strong clinical methodological component.

The purposes of my research were to see how students related their views of the world and prior experiences to statistics in the classroom, and to determine whether Fijian students were different from other students. This paper reports the results.

Method

Sample

The secondary school selected for the research was a typical Fijian Indian high school. The sample consisted of a class of 29 students aged 14 to 16 years of which 19 were girls and 10 were boys. According to the teacher, none of the students in the sample had received any in-depth instruction on statistics prior to the first interviews. The whole class participated in the first phase of interviews, and 14 students participated in the later phases during which the class teacher taught a unit on statistics and probability. This group of 14 was representative of the larger group in terms of abilities and gender.

Interview Process

Each student was interviewed individually by the researcher in a room away from the rest of the class. The interviews were tape recorded for analysis, and notes were made of student non-verbal behaviours observed during the interview. Each interview lasted about 40 to 50 minutes. Paper, pencil and a calculator were provided for the student if he or she needed it.

Interview Schedule

Open-ended interview questions and tasks were selected and adapted from those used by other researchers. The appropriateness of these interview tasks for the Fijian children was established by checking the tasks with the Fijian Ministry of Education Mathematics Syllabus (Ministry of Education, 1988).

Results and Discussion

Analysis of the interviews and children's written records indicates that the students used a variety of models for solving the statistical problems. The data also revealed that many of the students held beliefs and used strategies based on prior knowledge which, would tend to inhibit their development of statistical ideas. This paper focuses on two types of experiences— school and cultural ones. The aspects of cultural experiences discussed are religious and everyday experiences. Extracts from typical individual interviews are used for illustrative purposes. Throughout the discussion, I is used for the interviewer and Sn for the nth student.

School Experiences

Aspects of prior school experiences used inappropriately by the students related to sample, median, range, event, proportional reasoning, compound events, normal distribution and drawing graphs.

Sample: A key factor in selecting a sample is that it is a fair representation of the wider population. Representation is often achieved by picking a random sample or selecting a reasonably larger sample. The results of this study indicate that when students were asked to define the word sample or to select samples, they drew upon their prior knowledge. Criteria such as smartness, intelligence and ability to talk were used as the basis of selection, as the following interchange shows.

I: *How would you pick this sample so that it is a fair representation of the whole class?*
S3: *Those students who have been to a meeting.*
I: *The students haven't been to the meeting yet. You are to select the students.*
S3: *Select the students who are talkative. They can talk and have some confidence in them.*
I: *Isn't this sample biased towards the students who are talkative in class?*
S3: *Eh ... They might go in the meeting and start shivering.*
I: *Would you say that it is a fair sample?*
S3: *It is a fair sample because some students in our form hardly talk and when they go and talk in the front the voice is so slow that we can't hear. We are sitting in front, still we can't hear and they hardly talk.*

The students did not seem to realise that their samples were biased towards bright and talkative students and not typical of the population. The idea of representation had no meaning for them. Instead they appeared to be deeply convinced that, in selecting a sample, it is impossible to disregard the personal ability of the individuals. They did not realise that the names could have been drawn out of a hat, or random number tables could have been used.

Median: While 12 students could calculate the median from a set of data, only five students could calculate it when data were presented as a frequency distribution. Often, prior school mathematics experiences distorted students' thinking. For example, when asked to find the median from a frequency distribution, one student drew upon prior knowledge of sets and decided that identical scores should not be listed more than once!

Range: The range is the difference between the highest and lowest value of the variable but when asked to find the range for the task in Example 1, two students used their prior experience of relations and functions to conclude that the range was the first element in the data set.

Example 1: Weight problem

A small object was weighed on the same set of scales separately by nine students in a science class. The weights (in grams) recorded by each student are:

6.3 6.0 6.0 15.3 6.1 6.4 6.2 6.15 6.3.

What is the range?

This is evident in the following interchange:

I: *What is the range?*
S14: *6.3.*
I: *Why do you think that 6.3 is the range?*
S14: *The range I know of is the domain and the range. If there is 2.5 and 3.5, range is the first number and domain is the second number.*

Event: An event is a subset of the sample space. It is the outcome that is of our interest, or the desired outcome which is being investigated in the experiment. For example, tossing two coins and checking the possibility of getting two heads. The event is {HH}. A number of students in this study did not have a clear understanding of this term. While constructing their meanings for the term event, four students drew upon their prior sports experiences as indicated in the following interview.

I: *What does the word event mean to you?*
S22: *It is a happening that has to take place, say a match that has to take place tomorrow, that is organised.*
I: *Where have you used this word before?*
S22 *In athletics where there are track events and field events.*
I: *What does the word event mean in statistics?*
S22: *Event that takes place.*
I: *Can you give an example?*
S22: *When we toss a coin and find the probability.*
I: *So what is the event, tossing the coin or calculating the probability?*
S22: *Tossing the coin.*

Proportional Reasoning: The ratio concept is crucial to a conceptual understanding of probability. The results of the present study indicate that this concept is not well understood by the students. Rather than attending to proportionality information given on a marble task, (10 white and 20 black marbles versus 20 white and 60 black marbles) some students based their reasoning on previous sports experience. The following discussion illustrates this misunderstanding:

I: *Now Meena says that the game is not fair because in Ronit's box there are more white marbles than in her box. What is your opinion about this?*
S20: *Eh ... So if you play a game it should be equally in number.*
I: *What do you mean by equally?*
S20: *Like soccer if you are playing there should be 11 players aside. Then you will be able to have a win eh. So here it is like marble it is 10 white and 20 black marbles. So the game should be equally.*

Compound Events: In some situations, successive outcomes may be involved. For example, tossing two sixes in a row with a dice. If two dice are rolled, the probability of getting the same number on both is one-sixth and the probability of getting different numbers is five-sixths. The answers would be the same if one dice was rolled a second time, provided no particular order of numbers was required. In addition, theoretical probabilities of compound events can be analysed by using a table or a tree diagram.

For the dice problem (see Example 2), student 21 said that the probability of getting a 5 and 6 was more than the probability of getting a 6 on both because a six is hard to get.

Example 2: Dice Problem

If I roll two dice, is it more likely that a 5 and 6 will turn up, or that there will be a 6 on both dice? Or is the probability the same in both cases?

The student did not work out the sample space but based his reasoning on previous experience with board games.

For the two dice problem, students were asked whether rolling the two dice together or separately would make a difference to the results. The aim of this task was to check whether the students could extract the identical mathematical structure from the two practically different tasks. Two main types of unequal probabilities were mentioned by students. Some students considered that by successively throwing the die they had a higher chance of obtaining different numbers, while others thought that by throwing two die simultaneously they had a higher chance of obtaining different results. The explanations provided by the students indicate that they based their reasoning on their physics experiences. As one student explained,

S22: *If you roll once it might stop here because of the friction force the other one if the surface is smooth, it might stop somewhere here.*

These findings concur with the findings of Fischbein et al (1992). They used similar problems and found that only half the children in their study could see that the two procedures lead to the same results. The researchers concluded that the children in their study thought that outcomes can be controlled by the individual because the *mathematical probabilistic structure has not been detached from the concrete circumstances and considered in its abstract generality* (p. 530). In this study, this confusion seems due to the same difficulty.

Normal distribution: The total area under the normal curve is 1. Using mean and standard deviation, this area can be approximately sub-divided. For instance, approximately 68% of scores fall within one standard deviation from the mean. Some students completely ignored this information and based their reasoning on the activities they had done in other domains. When asked to find the values for $x_1, x_2, ..., x_6$ on a normal curve, given a mean of 50 and a standard a deviation of 5; two students based their calculations (wrongly) on directed numbers graphs which they learnt in Form Three. The students used positive numbers above the mean and negative numbers below the mean. For instance student 22 substituted 1 for x_1, 2 for x_2, 3 for x_3, -1 for x_4, -2 for x_5 and -3 for x_6. When asked to explain, he provided the following justification:

> *If the standard deviation is five, then we have to divide this part* [points to area above the mean] *into five parts. So x_1 will be 1. In line graphs., think of this* [indicates the mean] *as zero. Then you have positive numbers on this side* [gestures right of the mean] *and negative numbers on this side* [motions to left of the mean].

Another example of students bringing other school experiences inappropriately to bear on the normal distribution idea occurred in relation to a pulse rate item. Rather than considering the item statistically, nine students based their reasoning on activities they had done in science classes. For example, they wanted to talk about various pulse rates, which they had recently learnt in a biology class, not the number of adults (out of 400) who might have a pulse rate between 68 and 76, given a mean of 72 and a standard deviation of 4.

In their biology classes, the students had been investigating the effect of exercise on the heart beat. The conclusions from these activities were applied to the statistical problems.

Drawing graphs: The bacteria growth problem (see Example 3) was used to explore students' understanding of graphs.

Example 3: Bacteria growth problem

> A bacteria population cultured in a laboratory showed the growth pattern with respect to time, t, in minutes, as shown.
> (i) Show this information on a suitable graph.
> (ii) Why did you choose this graph?

t Minutes	0	1	2	3	4	5	6	7	8	9	10
Number (100s)	2	2.3	3.1	4	4.8	5.7	6.5	8	8.5	8.7	8.8

Although the students could draw a graph of the information given in the table, they could not give mathematical reasons for drawing a line graph or selecting a particular graphical representation. Three students said that they drew a line graph because they drew line graphs in their science lessons.

Cultural Experiences

In addition to basing their thinking on school experiences, some students based their reasoning on their cultural experiences. The cultural experiences discussed are religious and everyday experiences. The religious aspect is evident in independence and prediction tasks and the everyday experience in sample size and interpreting tables tasks.

Independence: The students thought God controls everything that happens in the world. For the baby problem (see Example 4), some students thought that one can not make any predictions because the sex of the baby depends on God.

Example 4: Baby Problem

The Singh family is expecting the birth of their fifth child. The first four children were girls. What is the probability that the fifth child will be a boy?

As one student explained:

> *We can not say that Mrs Singh is going to give birth to a boy or a girl because whatever God gives you have to accept it.*

Prediction Problem: Strong influences of religious beliefs were also apparent when students were asked to comment on an advertisement which claimed that for a fee of $20 and a sample of the mother's hand writing the sex of the baby could be predicted (with a money-back guarantee). Even when challenged to consider how the people placing the advertisement could make money, the students could not see that roughly half the babies born would be girls and half would be boys, that anyone could expect to be right in half the number of cases just by guessing. Rather than seeing that a clear profit could be made on 50% of the all the $20 sent in, powerful religious beliefs influenced the students' thinking, for example,

> *As I have told you before that God creates all human beings. He is the one who decides whether a boy is born or a girl is born.* (S17)

These findings of students drawing upon their religious beliefs concur with those of Amir and Williams (1994) who assert that culture plays a significant role in the building of a child's probabilistic thinking. It is certainly the case that the Indian culture in Fiji has a religious orientation.

Sample Size: In general, the larger the size of the sample taken, the more confidence one has that a sample statistic is a close estimation to the population parameter. People who rely on prior experiences, however, tend to estimate the likelihood of events by neglecting the sample size and by placing undue confidence in the reliability of their experiences. Rather than attending to sample size in a consumer report, two students in this study, in responding to the item about whether Mr Singh should buy Honda or Toyota (see Example 5) said that the *life* of a car depends on how one keeps it.

Example 5: Consumer Report task

Mr Singh wants to buy a new car, either a Honda or a Toyota. He wants whichever car will break down the least. First he read in *Consumer Reports* that for 400 cars of each type, the Toyota had more break-downs than the Honda. Then he talked to three friends. Two were Toyota owners, who had no major break-downs. The other friend used to own a Honda but it had lots of break-downs, so he sold it. He said he would never buy another Honda. Which car should Mr Singh buy?

They did not apply the idea of representativeness in this instance where it is really appropriate to do so. For example, student 29 explained:

> *He should buy any of the cars Honda or Toyota; it depends on him how he keeps and uses the car ... Ah Because it depends on the person, how he follows instructions then uses it. My father used to*

own a car and he kept it for ten years. He sold it but it is still going and it hasn't had any major breakdowns.

Interpreting Tables: The task comparing temperatures of Ba and Sigatoka (Example 6) was used to elicit student ideas in interpreting tables.

Example 6: Comparing temperatures of Ba and Sigatoka

Temperatures (in degrees C) were taken from Sigatoka and Ba on six consecutive days. Look at the temperatures from both the towns and decide if Ba is warmer than Sigatoka. How do you know? What else do these figures reveal about the temperatures in Sigatoka and Ba?

Example 6: Table comparing temperatures of Ba and Sigatoka

Day	1	2	3	4	5	6
Sigatoka	25	24	21	20	23	24
Ba	28	27	26	25	29	30

Analysis of the interview data showed that five students based their reasoning on their everyday experiences. These students made unwarranted topic assumptions when trying to read the table. The students said Ba was warmer than Sigatoka. When asked to explain their answer, they talked about the everyday weather conditions of Ba and Sigatoka. For example, student 21 explained,

Each day the temperature for Ba is greater than the temp for Sigatoka. And normally Sigatoka is called a valley. They are producing fruits; it rains there. My one uncle lives here. He mainly plants Chinese cabbages; because of the rain it grows so well there.

When asked to explain what else the figures revealed about the temperatures in Sigatoka and Ba, student 9 continued to talk about Sigatoka being warmer than Ba because everyday Sigatoka temperature was lower. The other four students continued to base their reasoning on everyday experiences.

Although this study provides evidence that reliance upon experience can result in biased, **non-statistical** estimates, in some cases this strategy may provide useful information. For example, the students knowledge of geography may have been reasonable.

Conclusion

The results indicate that certain misconceptions noted in the literature take a strong and common form in the thinking of the 14 and 16 year-olds' involved in the present study. There were also instances where individual reasoning was influenced by beliefs and experiences about the world and events and that this reasoning remained impervious to traditional instruction.

The strategies used by the students can be explained in terms of constructivist theory. The theory emphasises that children construct their own meaning on the basis of their existing knowledge rather passively receiving knowledge from the teacher. This implies that learning is a process of adapting to and organising one's existing structures of knowledge, not absorbing ideas imposed by others. It can be inferred that in the present study, children generated links with other domains while solving the statistical problems. Both anticipated and unanticipated learning occurred as children tried to generate links between the different domains.

It should not be taken for granted that children understand spontaneously the meaning of the terms sample, event, equally likely and range. Children have to be helped to distinguish between how these terms are used in statistics and in other domains. Trivial text book definitions such as *An event is a subset of the sample space* is not of much help.

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