Teaching Children to Draw Diagrams in Solving Word Problems: An Exploratory Study.

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This paper discusses the results of diagram drawing instruction on pupils' ability to understand and solve word problems. The results show that after instruction, diagrams drawn were more relational and resulted in better performance. There was a conscious attempt on the part of the pupils to illustrate the relationship among variables in their diagrams. The pupils also demonstrated positive attitude towards this approach of solving problems.

Introduction

In recent years, there has been a shift in Brunei Darussalam towards a greater emphasis on mathematical applications and problem solving in school mathematics. The new lower secondary mathematics syllabus issued by the Curriculum Development Department (CDD,1995) in Brunei clearly states that *"teachers should help students develop problem-solving skills besides striking a balance between concept development and mastery of basic skills."* (p.2). Translating this into practice involves the use of word problems as a means of engaging students in practical and realistic application of mathematical concepts and skills learned in class.

Analysis of solution strategies used by lower secondary school children in national mathematics examination in Brunei Darussalam(Veloo, 1995) reveals that, pupils solve word problems by first attempting to formulate algebraic expressions using information contained in the word problem. This is the most difficult step and a large proportion of pupils had little success at this stage. In another study on primary school children, it was noted that less than 1% of the pupils drew any kind of diagram in solving word problems (Lopez-Real, Veloo & Maawiah, 1992).

Many researchers have reported that an important facilitator in the process of solving a word problem is the translation of information and conditions given in the problem statement into a visual diagrammatic form. According to Larson (1985) a first step in understanding a problem is to draw and label a figure, diagram or a graph. Children have been found to perform better in mathematical problem solving situations when diagrams are used by teachers to elicit appropriate mental images (Yancey,V. et al,1989). Norman Webb (1979) reported that pictorial representation was one of the component scores that accounted for most variance in problem-solving scores.

Although diagrams have been recognised as useful in problem solving, not much has been reported on how to teach children to generate their own diagrams. This study is a follow up of two earlier studies (Lopez & Veloo, 1993; Veloo & Lopez,1994) conducted in Brunei Darussalam. In these earlier studies, it was found that (a) primary school pupils performed better on word problems after being asked to draw a diagram before solving the given problems (b) primary school pupils tended to draw diagrams that were concrete representations of the problem, whereas secondary school children were more inclined to draw diagrams that were abstract, (c) pupils generally have not been taught how to draw and use diagrams in solving word problems and (d) the diagram drawing strategy once understood was readily accepted by pupils. In these earlier studies no instruction on how to draw diagram was given.

This exploratory study examines the effectiveness of diagram drawing instruction on pupils' understanding and successful solution of mathematical word problems.

Methodology

Sample

The research was conducted in a secondary school in Brunei Darussalam. Fifty students from two secondary 4 classes, comprising 28 girls and 22 boys(average age of about 15 years) were involved in the study. The study consisted of three phases, a pre-test, an instructional phase and a post-test.

Instrumentation

A pre-test and a post-test consisting of word problems and open ended questionnaire were prepared for administration in this study. The word problems used in the pretest and posttest were obtained from a survey of school textbooks, past year examination papers and mathematics education literature. The open ended questionnaire consisted of 5 questions on diagram drawing and solving word problems was prepared and administered as part of the posttest.

The pre-test

Ten problems that were deemed suitable were selected for the study were selected for the pre-test. The test booklets were divided into two sets, Test A and Test B. The questions in both Test A and B were identical but differed only in the instruction given to the candidates. Pupils attempting Test A were asked to read the question carefully and then solve the problem showing all working in the space provided. Test B candidates were asked to read the question carefully, draw a diagram that illustrates the information given in the problem and then solve the problem showing all working. Pupils in both classes were randomly assigned either Test A or B. All students were given as much time as they require to complete test. All question booklets were collected after the test.

The post-test

The post test consisted of 5 questions, selected from the pre-test, whose facility levels were below 45%. The test was given to all pupils to determine the effect of diagram drawing instruction on pupils ability to understand and use diagrams to solve the word problems. The open ended questionnaire was administered as part of the posttest.

Procedure

A week following the pre-test, a two-week instruction on visualization and techniques of illustrating word problems pictorially was given to all pupils involved in the study by the researcher. Topics covered in the instruction included, *Fractions, Percentage, Ratio, Profit and Loss* and *Simple Algebra.* These topics were selected because problems involved can be easily represented by diagrams. Students were initially introduced to the *part-whole model,* the *comparison model* (Kho T. H.,1987) and *ratio model* of representing word problems pictorially. These models were further refined as word problems were discussed during instruction and problems solving exercises.

Results

Table 1 shows pupils' performance on the pre and post-tests on five questions whose facility indices fell below 45%. One mark was awarded for correct response and zero mark was awarded for incorrect response.

Table 1: Pupils' performance on pre and post-test Comparison of pupils' performance and use of diagrams in the pre and post tests (n=41)

Question	Q3	Q4	Q5	Q6	Q10	mean	SD	t
Pre-test Facility(%)	12.2	28.5	42.8	12.2	22.4	1.27	1.43	1.325
Diagram(%)	50	4.1	8.2	6.1	8.2			
Post-test Facility(%)	21.1	65.8	86.8	13.2	36.9	2.35	1.29	
Diagram(%)	92.1	78.9	92.1	84.2	73.6			

Table 2: Comparison of pupils' performance on 3 questions in the pre and post tests. (n=41)

Question	Q4	Q5	Q10	mean	SD	t
Pre-test Facility(%)	28.5	42.8	22.4	1.05	1.06	2.125*
Post-test Facility(%)	65.8	86.8	36.9	1.90	0.87	

(* significant at $p< 0.05$)

On all 5 questions that were re-tested in the post-test, pupils' performance was better. On three questions (4, 5 and 10) their performance was statistically significant ($p<0.05$), see table 2. A marked improvement in the quality of diagrams was observed. Diagrams drawn were more *relational* that is pupils tried to illustrate the relationship among variables in their diagrams.

Further analyses revealed that most of the diagrams drawn by pupils in the pre-test which the writer considered helpful were observed for questions 3, 8 and 9.

Question 3: *A motorist traveled from town A to B at an average speed of 90 km/h. After traveling one-third of the journey in 45 minutes, he continued to travel another 120 km to reach town B. Find his average speed for the second part of the journey.*

Question 8: *A 48 metre wire is cut into three pieces. The second piece is three times as long as the first. The third piece is four times as long as the second. How long is each piece?*

Question 9: *A long pole is planted through some water into the mud at the bottom of a lake. Half of the pole is in the mud, one third is in the water and the part above water is 1l\2 metres long. How long is the pole?*

A possible reason why more diagrams were observed for these three questions and not the others in the pre-test is that, these questions could be illustrated pictorially without much difficulty. For the other questions in the test, pupils either did not draw diagrams or attempted to draw *literal* representations of information given in the problem. An example of a *literal* diagrams is shown below.

Question 1: *Ali is 10 kg heavier than Lee Meng. Their total weight is 100 kg. What is Ali's weight?*

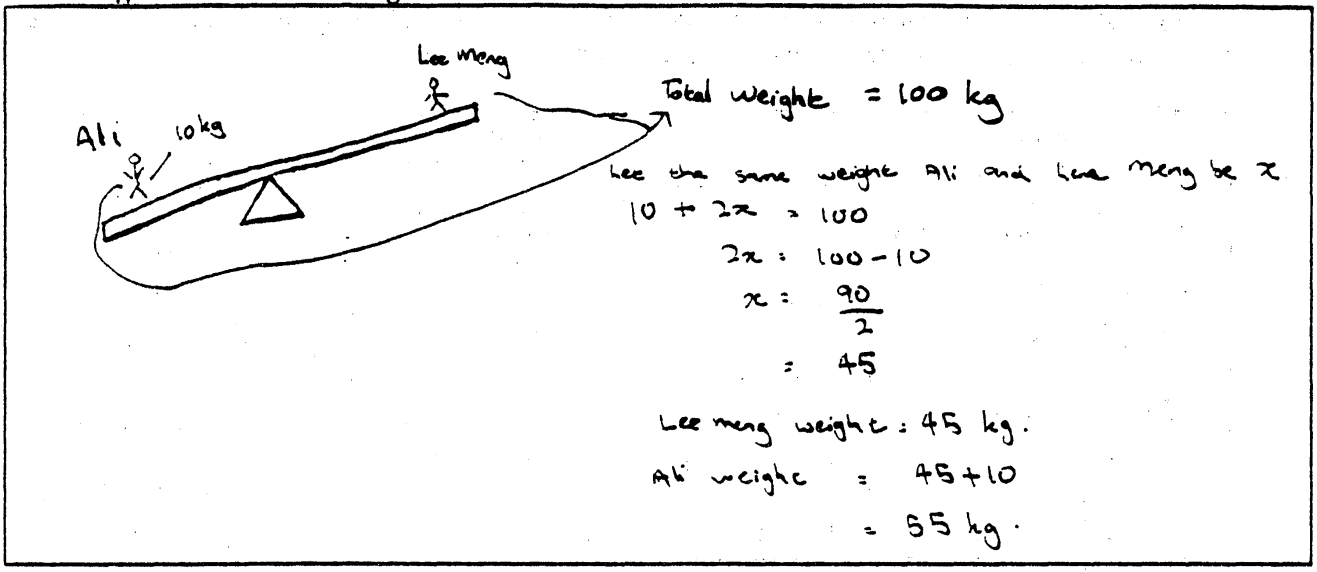

Fig 1

The following are examples of diagrams observed for questions 4, 5, 6 and 10 in the posttest.

Question 4: *Two brothers A and B together had a total of $80. If A spends one-third of his money and B spends $15, they would have the same amount of money left. How much did each have at the start?*

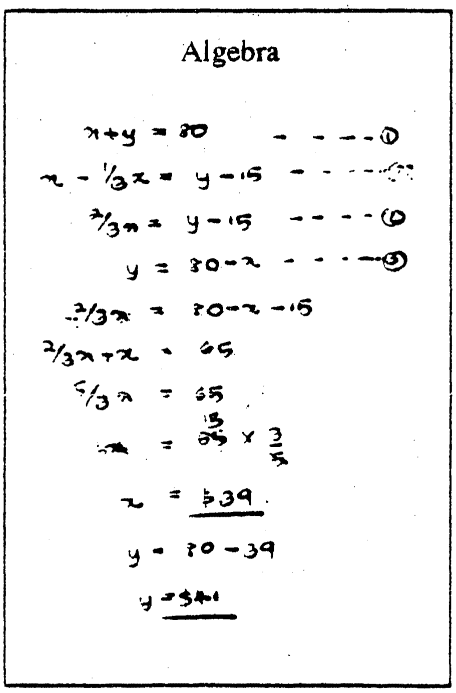

Fig 2(a)

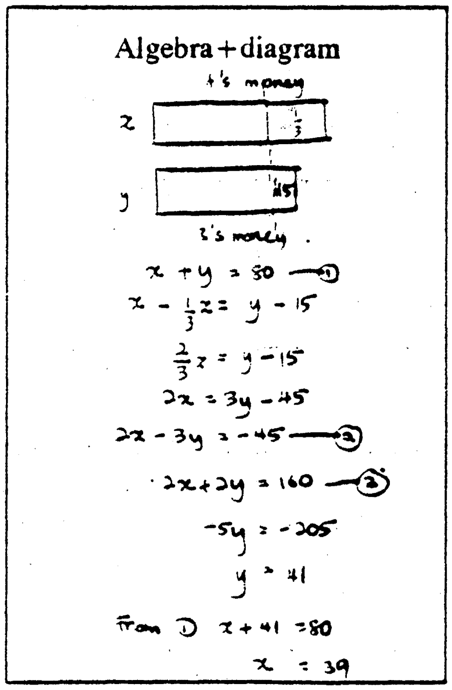

Fig 2(b)

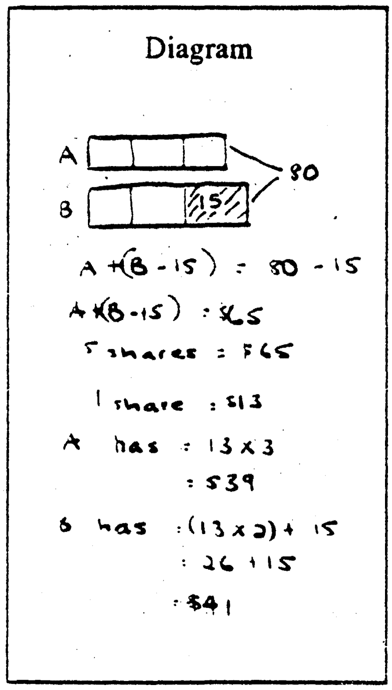

Fig 2(c)

Question 5: In a class three-fifth of the pupils were girls and the rest were boys. If the number of boys were doubled, and 6 more girls joined the class, there will be equal number of boys and girls. How many pupils were in the class at the start?

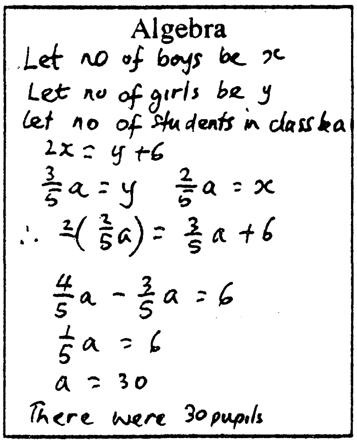

Fig 3(a)

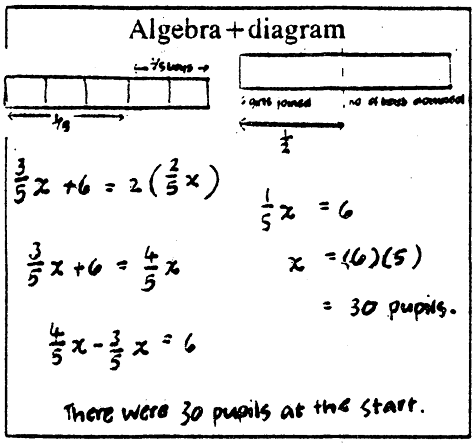

Fig 3(b)

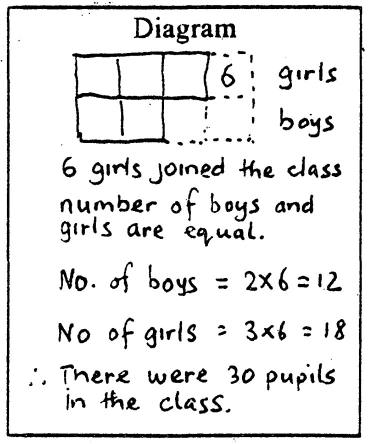

Fig3(c)

Question 6: In a class of 40 pupils, 25% were girls. When some new girls joined the class, the percentage of girls increased to 40%. How many new girls joined the class?

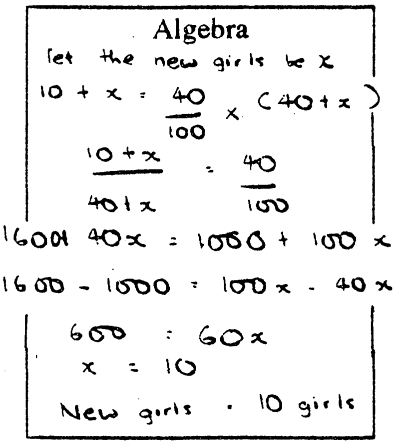

Fig 4(a)

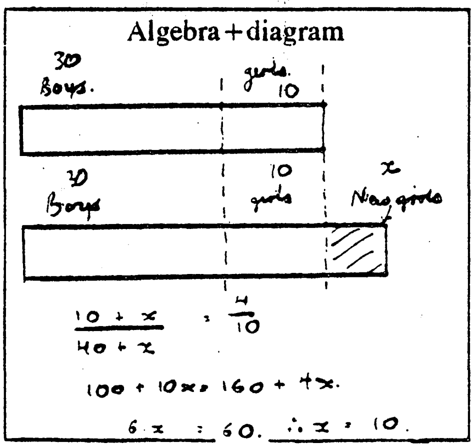

Fig 4(b)

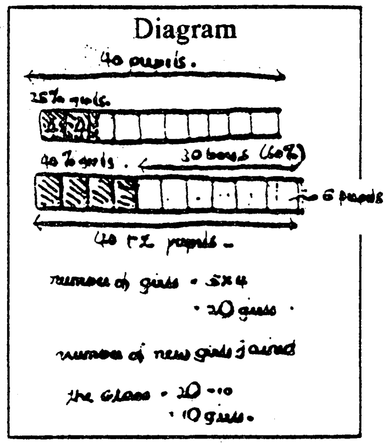

Fig4(c)

Question 10: Three tired and hungry men went to sleep with a bag of apples. One man awoke, ate one-third of the apples, and then went back to sleep. Later a second man awoke, and ate one-third of the remaining apples and he went back to sleep. Finally the third man awoke and ate one-third of the remaining apples. When he finished there were 8 apples left in the bag. How many apples were in the bag at the start?

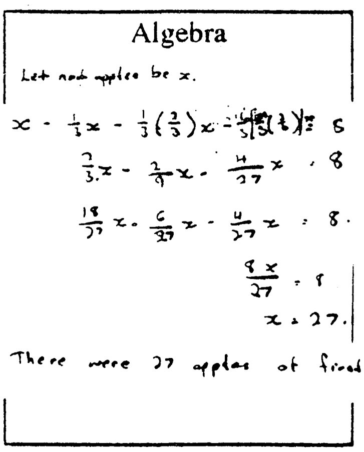

Fig 5(a)

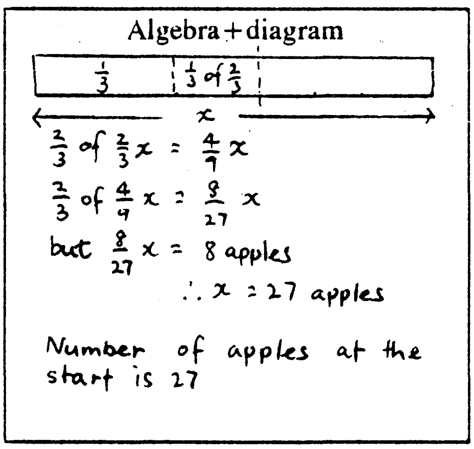

Fig 5(b)

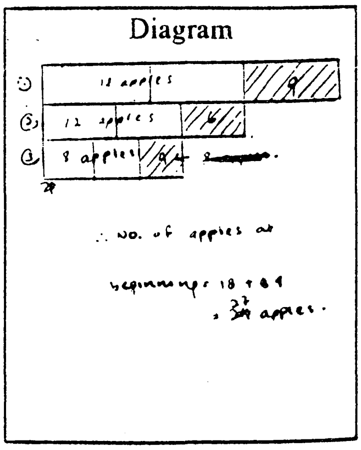

Fig5(c)

Discussion

Effect of instruction

The post-test result shows that pupils' performance on all questions retested improved and in three questions, Q4, Q5, Q10, their performance showed significant improvement. For these three problems, solutions could be obtained from the diagram itself once an accurate relational diagram is drawn. This is further discussed under the section on solution strategies.

Pretest results and discussion with pupils immediately after the pretest, revealed that pupils prefer and feel more comfortable with the algebraic approach to solving word problems. There was resistance on the part of the pupils in using diagrams as an initial step in solving problems. This was particularly so among the more able pupils who said in response to a question in the questionnaire, that drawing diagrams tend to *slow them down*. They would *draw diagrams only for the more difficult questions*. However, during and after the instructional phase, most pupils showed a general willingness to apply this strategy in solving word problems. Pupils learned the technique readily and some drew relational diagrams from which the solution to the problem could be easily obtained. Examples of such diagrams are shown under the sub-heading *diagram strategy* for Q4, Q5, Q6 and Q10. There is also evidence from the study to show that an application of diagram drawing approach results in better performance particularly among pupils whose conceptual understanding in weak.

Pupils' general attitude towards this approach can be described as positive. Students initial reluctance to drawing diagrams was mainly due to the fact that they were unfamiliar with this technique. The following is a summary of pupils' responses to a question posed in the questionnaire.

Q. Does drawing diagrams help you in solving mathematical word problems?(Yes/ No) Give reasons for your response.
"Diagrams helps me to understand the question better."
"The question turns out to be easier and I can check the answer by looking at the diagram."
"The diagram makes the problem clearer and I can understand the question better."

Solution Strategies

A variety of strategies was used by pupils in solving the word problems. These can be classified broadly into three categories. The first is the *Algebra strategy*. The second is the *Diagram + Algebra strategy*, where pupils drew diagram as well as presented an algebraic solution to the problem. The third cagegory is the *Diagram strategy*, in which pupils presented a solution to the problem from an analysis of the diagram drawn.

Algebra strategy

Despite the two week instruction on diagram drawing why do some pupils still prefer the *Algebra* approach? It is the method of solution taught in schools and emphasised by the teachers. It is, according to some teachers, the method preferred by the examination board. Pretest results and discussion with pupils immediately after the pretest, revealed that pupils preferred and felt more comfortable with the algebraic approach. The following pupils' responses provides further evidence of what takes place in the mathematics classroom.

Q. Do you normally draw diagrams when solving mathematical word problems? (Yes/No) Give reasons for your response.

A. *"No, because we do not normally use this method in the class. We are more familiar with, writing equations to solve problems."*
"I usually go ahead and calculate without drawing, It is much faster."
"I never felt the need to draw diagrams as it wasn't the practice."

Another reason why the algebra strategy is popular is that it involves the formulation of suitable algebraic expression using information contained in the word problem. There are no fixed rules that a pupil can apply to translate a word problem into algebraic expressions. Analysis of pupils' answer scripts in public examinations showed that this was the stage, where most students fail in the problem solving process(Veloo, 1995). Similar findings were reported by Newman in her study of errors made by sixth grade pupils on written mathematical tasks.(Newman,1977). After this translation stage, not much thinking is involved in the next stage of the problem solving process which involves the application of routine procedures to solve for the unknown. The apparent simplicity of the algebra strategy may be the reason why it is popular and it provides success only to those who are skilled in the techniques of algebra.

Diagram + Algebra strategy

This was the most popular strategy. Diagrams appear to serve as an intermediate step between students concrete understanding of a problem and abstract algebraic model of the problem. The diagrams drawn by pupils in the post-test for questions, 4, 5, 6 and 10 clearly indicates increased understanding of the relationship among variables in the problem. A substantial number of pupils used this strategy to solve the word problems. Once a successful *relational* diagram is drawn, it appears that the next step of translating the problem into a suitable algebraic model becomes easier.

Diagram strategy

This approach was used by some pupils. Problems were solved entirely from the diagram. To successfully use this strategy the pupil must first understand the problem well. Second a relational diagram containing the important information in the problem must be drawn. The diagram helps in clarifying relationships between variables. When students use a diagram to illustrate the information given in a problem, they use a representational system that they are familiar with. The very act of drawing a diagram causes students to focus on relationships among variables in a problem. This stragety was most commonly used by students to answer Questions 4, 5 and 10 although not so common for Q6. While this method may have limited application it is one which requires thinking and analysis of the information contained in the word problem. It challenges one to think and consider the problem as a whole rather than in parts. It can be argued that if pupils can solve word problems using this strategy, then they can easily be helped to see the connection between this form of representation and algebraic representation.

Unsuccessful attempts

Despite drawing diagrams many pupils were still unsuccessful particularly in questions 3 and 6. Analysis of diagrams together with written solution offered by these students clearly showed the flaw in their reasoning. For question 3 pupils have not understood the concept of average velocity. For question 6 the most common incorrect response was 6 was obtained by calculating the total number of girls in the class *after* the increase to be 40% of 40 which is 16. Hence the number of new girls was 6. This incorrect interpretation of the question is clearly shown in the diagrams drawn by these pupils. This information will be of great benefit to the mathematics teacher, who will be in a better position to help these pupils correct their mistakes.

Conclusion

Drawing diagrams can be an important strategy in problem solving but one that is not fully exploited in the classroom. The desire to push children too quickly into abstract manipulation and algorithms may be one reason for this. It is also unfortunate that not many teachers draw diagram as part of the process of solving word problems in the mathematics classroom. Teachers must make a conscious attempt to learn this technique and must challenge themselves and their pupils to solve problems using diagrams. Once pupils develop competence in using diagrams to solve problems, they need to experience the connection between this form of representation and algebraic representation. Burns(1979) describes this as an often overlooked intermediate step. This study has shown that diagram drawing techniques can be taught to children in school. Further work needs to be done to assess long term effect of diagram drawing instruction on pupils ability to solve word problems particulary those involving algebra.

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