Student Error Patterns in Fraction and Decimal Concepts

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This paper uses data from an international study of number sense in Australia, the United States, Sweden and Taiwan to investigate students' misconceptions and error patterns in fraction and decimal concepts. Selected items that were administered to two or three age levels were analysed. It was found that misconceptions about basic concepts were prevalent, and these persisted even among 14-year-olds. Results suggest the need for more meaningful treatment of fraction and decimal concepts, and some relocation of these topics in the curriculum.

The Main Study

Mental computation tests were developed and administered to students aged 8, 10, 12, and 14 in Australia, Japan, and the United States (McIntosh, Bana & Farrell, 1995a; McIntosh, Bana & Farrell, 1995b). In order to investigate number sense further, more extensive number test instruments were developed and administered to these same students in Australia and the USA. Many of these items were also given to Swedish and Taiwanese students. This international follow-up study of number sense is reported by McIntosh, Reys, Reys, Bana and Farrell (1997).

The items developed for the number sense tests were based on the number sense framework outlined by McIntosh, Reys & Reys (1992). Group tests ranging in length from 30 to 45 items were administered to sample sizes ranging from 110 to 160 students in each of the 8-, 10-, 12-, and 14-year-old student groups in Perth, Australia and Columbia, Missouri in the USA. Some of the items were modified for US students. Many of these items, often modified slightly, were administered to 10-year-olds and 14-year-olds in Gotheborg, Sweden. Some of the items, again with modifications in certain instances, were also given to 12- and 14-year-olds in Taiwan.

The major purpose of this study was to attempt to measure number sense over a large age range and in a number of countries. It was not the intention to compare countries' performances since the samples were not truly representative nor were they matched except in fairly crude terms. However, there was some matching of age groups as indicated above. In order to observe the development of number sense across age levels, many of the items were common across two or three different age groups.

Purpose of this Study

The particular purpose of the study reported here was to analyse a representative sample of the items from the main study of number sense relating to fractions and decimals to determine error patterns and prevalent misconceptions in these topics. The items used in the main study focussed on understanding rather than on factual knowledge or routine computation. This analysis should provide some guidance for teachers in effecting remedial strategies, and provide evidence as to whether or not some relocation of these topics in the curriculum may be necessary.

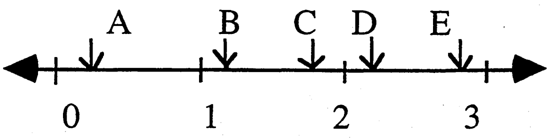
In order to undertake such an investigation the analysis below concentrates on items that were common across a number of age levels; thus providing a good overview of the development of fraction and decimal concepts through the students' schooling, and signalling any desirable re-locations of these topics in the school mathematics curriculum.

Results

Selected items with their results are presented below. Table 1 shows an item asking students to identify a point on the number line corresponding to 2.19. All items were presented in 'boxes' in this format, so that students wrote or selected responses alongside the item itself on the test paper. In this case a high percentage, especially among 10- and 12-year-olds, selected the point closer to three rather than to two. It seems that there is a lack of understanding of basic decimal numeration concepts here. Yet these same students who still have difficulty locating a decimal on a number line are expected to undertake computations using decimals. It would be interesting to ask these students whether $2.19 is closer to $2 or to $3. Presenting the item in context is likely to bring about a different result.

Table 1
Decimal locations on the number line

Item:



Which letter on the number line above best represents 2.19? ______________

Response % for age groups	Aus 10	Aus 12	Aus 14	USA 10	USA 12	USA 14	Swed 10	Swed 14
A, B or C	11	2	2	7	3	2	20	3
D	47	79	97	20	34	73	35	84
E	40	18	2	64	62	25	44	12
Misc Incorrect	0	0	0	3	1	0	0	0
No Response	1	0	0	6	1	0	2	1

The item in Table 2 asked students to identify the fraction representing the largest number. It is clear that many students selected the fraction with the greatest denominator (9) as representing the greatest number. This misconception was quite pronounced, even among the 12-year-olds, and persisted for 14-year-olds in Sweden and the USA. Again,

Table 2
Identifying the fraction for the largest number

Item: Circle the fraction which represents the largest amount.	A $\frac{5}{6}$ B $\frac{5}{7}$ C $\frac{5}{8}$ D $\frac{5}{9}$

Response %	Aus 10	Aus 12	Aus 14	USA 10	USA 12	USA 14	Swed 14
A	42	79	98	29	74	85	92
B or C	2	5	1	1	4	3	1
D	57	17	2	69	22	11	6
No Response	0	0	0	1	0	1	1

as is the case for decimals, it is clear that many students are having to carry out computations using fractions while still having misconceptions about the fractions themselves.

The Table 3 item tests the notion of an infinite set of numbers between any two given numbers expressed as decimals - in this case, 1.52 and 1.53. The results show that the majority of students have the misconception that there are no, one, or a few numbers between 1.52 and 1.53. Even among 14-year-olds there was a lack of understanding. However, Taiwanese students in this age group performed quite well, although they were not required to support their responses with examples.

Table 3
Awareness of an infinite set of numbers between any two given numbers (decimal form)

Item:	
How many different decimals are there between 1.52 and 1.53? Circle your answer and then fill in the blank.	A None. Why? ____________ B One. What is it? ____________ C A few. Give two: ________ and ______ D Lots. Give two: ________ and ______

Response %	Aus 12	Aus 14	USA 12	USA 14	Swed 14	Taiw 12	Taiw 14
A	42	23	55	45	23	36	5
B	33	11	23	8	14	10	5
C	6	2	9	0	2	5	3
D & examples	16	62	3	40	51	*	*
D no examples	3	2	2	4	6	32*	78*
No Response	0	0	8	3	4	17	9

*Taiwanese students were not asked to give examples

Students were asked the same question as in Table 3 regarding the fractions $2/5$ and $3/5$. The results of this item are presented in Table 4. Approximately half the 12- and 14-year-old students said there is no fraction between $2/5$ and $3/5$. Many others said there is one,

Table 4.
Awareness of an infinite set of numbers between $2/5$ and $3/5$

Response %	Aus 12	Aus 14	USA 12	USA 14	Swed 14	Taiw 12	Taiw 14
A	57	29	57	53	47	60	40
B	29	21	14	9	14	8	4
C	4	7	8	5	5	6	3
D & examples	7	40	1	7	12	*	*
D no examples	3	3	11	15	13	11*	35*
No Response	0	0	10	11	8	15	18

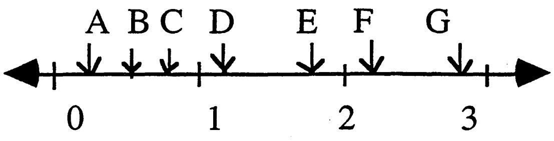
*Taiwanese students were not asked to give examples or a few fractions. Performance on this item was considerably lower than for the decimals item in Table 3. It is very likely that students focussed on the numerals or fractions rather than on the numbers they represented, and this would help explain the results. The form of the question might have led to students considering only fifths as possible answers. That is, many students probably concentrated on the fact that there are no fifths between the two given fractions, ignoring the fact that there are many numbers represented by other fractions.

Table 5 gives results for an item that required students to identify a point on the number line representing a fraction whose numerator would be slightly greater than the denominator. Overall, only about a quarter of students were successful in this item. Even among 14-year-olds in all three countries, only a third of them showed understanding. Significant numbers of students in all age groups chose points F or G, which represent numbers greater than two. There is no obvious explanation for this misconception. Once again, we must question whether these students who have difficulty with basic fraction concepts can be expected to undertake meaningful computations with fractions.

Table 5
Fraction locations on the number line

Item:

In the fraction $\frac{5}{8}$, 5 is the numerator and 8 is the denominator.



Which letter on the number line above names a fraction where the numerator is slightly more than the denominator?

____

Response %	Aus 12	Aus 14	USA 12	USA 14	Swed 14*	Taiw 12*	Taiw 14*
A	6	7	15	10	6		
B	2	5	7	3	1		
C	11	6	11	10	7		
D	14	37	15	30	19	14	32
E	10	7	6	11	17		
F	34	20	13	10	16		
G	13	12	14	6	8		
Misc. Incorrect	0	0	8	0	11	25	44
No Response	10	6	11	19	16	61	24

*For Swedish and Taiwanese students the introductory sentence was omitted from the item

The item shown in Table 6 required students to estimate 87 x 0.89 in relation to the number 87, and they were not very successful on this item. The importance of estimation has been well documented (Sowder, 1992). On this item, many students thought that the product 87 x 0.89 would be more than 87, again indicating significant misconceptions about decimals and their use in computation.

Table 6
Estimation of a product involving decimals

Item: Without calculating the exact answer, circle the best estimate for: 87 x 0.89	
	A a lot less than 87
	B a little less than 87
	C a little more than 87
	D a lot more than 87

Response %	Aus 12	Aus 14	USA 12	USA 14	Swed 14
A	51	82	16	50	65
B	12	8	18	17	16
C	30	8	28	20	10
D	7	2	35	8	8
No Response	0	0	2	4	1

The quotient estimation shown in Table 7 is a similar item in that students were asked to relate the result to a benchmark - this time to 29 - given that the quotient to estimate was 29 ÷ 0.8. Over half the students, even among 14-year-olds, thought that the quotient was less than 29. We had previously found that many of these students showed significant misconceptions in mental computation with fractions and decimals (Bana, Farrell & McIntosh, 1995).

Table 7
Estimation of a quotient involving decimals

Item: Without calculating the exact answer, circle the best estimate for: 29 ÷ 0.8	
	A less than 29
	B equal to 29
	C greater than 29
	D Impossible to tell without calculating

Response %	Aus 10	Aus 12	Aus 14	USA 14	Swed 14
A	64	73	49	55	51
B	4	2	1	3	4
C	20	21	49	36	37
D	11	4	1	5	4
No Response	0	0	0	2	5

Understandings of basic properties of the operations were assessed in the number sense tests. For example, the multiplication property of one as it relates to equivalent fractions was tested in the item detailed in Table 8. However, it could be argued that this item is assessing understanding of equivalence rather than the multiplication property of one, since students are required to recognise that $^1/_2 = ^3/_6$, so indicating that the missing number is one. The results show that approximately a third of the students across all three countries gave three as the missing number to make the sentence true. To these students, three lots of a half is three sixths, which was possibly reasoned by multiplying both the numerator and denominator by three, which indicates significant misconceptions.

Table 8
The multiplication property of one and equivalent fractions

Item: Circle the number you can put in the box to make this sentence true: $\frac{1}{2} \times \square = \frac{3}{6}$	A $\frac{2}{4}$ B $\frac{2}{3}$ C 1 D 3

Response %	Aus 12	Aus 14	USA 12	USA 14	Swed 14
A	8	10	19	14	16
B	28	13	21	28	22
C	28	46	16	28	21
D	34	31	43	27	33
Misc Incorrect	0	0	0	0	1
No Response	2	0	1	4	7

Discussion

The use of items that probed understandings rather than routine procedures led to the identification of significant misconceptions about fractions and decimals. These misconceptions relate to the basic notions about fractions and decimals, and they persist through the age groups. Thus even among 14-year-olds we found that many students struggled to come to grips with concepts that we consider to be fundamental.

Despite these significant misconceptions, students are still expected to deal with a great deal of mathematics content involving fractions and decimals. For example, in Western Australia, the mathematics curriculum requires students to carry out addition and subtraction with decimals in Stage 4, which corresponds to Year 4 for most students (Ministry of Education of Western Australia, 1989). But it seems clear that students in this age group are unable to handle decimals in a meaningful way.

Thus the curriculum location of topics involving fractions and decimals needs to be carefully reviewed. It seems that all too often students are being required to compute using fractions and decimals before they have any real concept of the numbers and numerals involved. This is likely to result in instrumental rather than relational approaches, simply to cover the given content. We recommend that much more emphasis be placed on meaningful treatment of these topics in the classroom through the provision of relevant experiences for students. We also urge teachers to delay using fractions or decimals in computation until fundamental concepts are well established.

There is still much scope for further research in this area. We could only speculate on possible reasons for students' incorrect responses, as there were no follow-up interviews. Further work is needed through clinical interview procedures to determine the precise nature of these very prevalent misconceptions about fractions and decimals

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