Towards an understanding of numerical workplace knowledge

Clive Kanes
Griffith University

In this paper airline ticketing desk operations are studied in order to develop a greater understanding of numerical workplace knowledge as seen from a non-essentialist account of mathematical knowledge. In the first part of the paper alternative approaches to knowledge use are discussed and their epistemological groundings noted. Next, transcripts are analysed in order to ascertain the logic of ticket exchange task performance. Methods used for the study include those derived from linguistic philosophy; decisive use is also made of the Vygotsky inspired notion of artifact mediation within task performance. Finally, an attempt is made to find the place of numerical knowledge within the workplace knowledge of the episodes observed.

Background and aims of study

Doubt concerning the adequacy of essentialism as a theory of knowledge for the contexts of mathematics education has been expressed within the research literature over the previous fifteen years. Working from findings drawn from a variety of settings and theoretical and frameworks (anthropological studies: Gerdes (1985), Bishop (1988), Saxe (1991), Lave and Wenger, (1991), Nunes, Schlieman & Carraher (1993); sociological and linguistic theories: Bloor (1983), Lave (1988), Solomon (1989); micro-sociological analyses: Bauersfeld (1991), Voigt (1993); epistemological studies: von Glasersfeld (1995); discourse theories: Walkerdine (1988, 1994)) researchers have forged a widely held view that mathematics is a construction of some kind (as opposed to a pre-given entity) and one which at does not entirely (or at all) belong to the mental domain. Nevertheless, despite the solidity of these findings and their empirical base there currently remains a significant bias towards essentialist accounts in our thinking about mathematics within education circles. For instance, a recent pamphlet entitled 'Using mathematics ideas and techniques for practical purposes' (Australian Association of Mathematics Teachers, 1996), provides examples of mathematical ideas and techniques at work in practical situations. In writing about these, the pamphlet creates the strong impression of an essentialist bias in its description of mathematical knowledge. For instance, although it concedes that "required mathematical ideas and techniques will always be specific to a person's interaction with the context" it notes that "mathematical ideas and techniques are often not recognised as such". In making these statements, the pamphlet is not arguing that mathematical knowledge is somehow synthesised in the circumstances in which it is deployed, but that it is transferred to those sites. Indeed, a principal thrust of the publication hinges on the belief that a clear view of the problem situation will promote the effective use of mathematical knowledge precisely because in the presence of such clarity a recognition of "when and where mathematics could help" can be made. In short, mathematical knowledge is used precisely in those sites to which it can be transferred.

Given both the uncertain basis for essentialist thinking in mathematics education and, notwithstanding this, the presence of a strong bias towards it, analyses adopting non-essentialist perspectives are indicated. One such possible approach is reported in this paper. Workplace contexts - in this case from certain over the counter operations within the airline industry - were chosen as the site for investigation as these are frequently overlooked in studies relating to the construction and deployment of knowledge within the mathematics education research literature. Specific aims for the paper are, therefore, (i) to explore what a non-essentialist account of mathematics use in an actual workplace situation could look like, and (ii) to investigate how a task performance involving numerical knowledge within this site might be considered to operate. It is emphasised that as this paper is meant to be indicative of a possible line

for future investigation, findings, and indeed the form of the analysis itself, are to be regarded as preliminary.

Knowledge in use: transfer, construction, participation

When thinking about mathematics knowledge use, three traditions have had a strong influence on the research literature most recently. Of these, the approach drawing on the cognitive science and information processing literature is the one which has adopted an essentialist model for mathematical knowledge most completely. Characteristic of this approach is the concern for internal cognitive processes and their coordination with an external environment. Knowledge is coded representationally and held by the user; use, conceptualised as knowledge transfer, is determined by the capacity of this agent to elicit correspondence between the problem situation and internal representations (Singley & Anderson, 1989; Anderson, 1990). Correspondences can, for instance, be accessed by analogical reasoning and can be promoted by metacognitive activity (Brown, 1989; Reimann & Spada, 1996). In a second approach, knowledge in use is not considered to be transferred to a site of deployment, instead it is constructed by the agent who utilises internal cognitive operations to modify, extend, replace, attune prior knowledge in relation to the problem presented within the given situation (Confrey, 1990; Hiebert & Carpenter, 1992). Within this paradigm, knowledge is not given essentially as an abstract universal, rather it takes the form of a viable set of situationally attuned procedures within the terms of the problem space (Cobb, Yackel, Wood, 1992; Cobb, 1994). When considering the viability of subjective knowledge structures, criteria relating to constraints within the physical, conceptual and social domains are pertinent (von Glasersfeld, 1995). Within the third approach, knowledge in use does not lie within the head of the user so much as being distributed within a collective practice relating to a particular set of work or other culturally related tasks (Brown, Collins & Duguid, 1989; Rogoff, 1990). Knowledge use is seen as equivalent to participation within a designated "community of practice" rather than as the far transfer of specific content and procedural knowledge or the subjective construction by the individual of locally pertinent knowledge (Lave & Wenger, 1991). Critical in this approach, therefore, is the social domain - its habits, customs, values, conventions, together with its mediating artifacts, technologies of production and social organisation (Wertsch, 1985; Engeström, 1991; Engeström, Engeström, & Kärkkäinen, 1995).

In this paper the theoretical framework adopted has been influenced most strongly by the third of these approaches. Unlike much of this literature, however, less attention has been given to the nature and characteristics of the domain of practice related to the workplace site under observation, and correspondingly more attention has been given to the microgenetic formation of numeracy related task competencies. A discussion of what methods were engaged in this exploration can be found below.

An empirical inquiry

Data for the current study is drawn from a much larger exploration of over the counter operations within the airline sector of the transport industry. In the larger study data was collected within multiple sites. In the analysis reported here, however, specific attention is given to a body of data relating to operations on the ticketing desk within the Qantas domestic terminal at Sydney airport. Transactions among sales assistant staff (coded SA1, SA3), and other airline employees, customers, and researchers (coded RA) were recorded by a conveniently placed video camera over a period of about six (6) hours duration. Interactions recorded between airline staff and researchers were designed to prompt staff to elaborate discursively on workplace tasks as either they were in progress or as soon as was practically possible thereafter.

Review of the transcript corpus (supported by the corroboration of respondents) identified ticket exchange as being a regular feature of the work of site observed. Operations associated with this task are illustrated in the transcript below. The structure of the episode is as follows: a passenger (C45) approaches a sales assistant (SA3)

seeking to exchange a ticket for one currently held; this leads to the performance of a process which concludes with the completion of the transaction; a research assistant (RA) then prompts the sales assistant to discuss the context of the preceding interaction.

*SA3 to C45:
Hi
*C45 to SA3:
I just need to pick up a ticket for Harvey
*SA3 to C45:
Harvey, is it?
*C45 to SA3:
Yep.
*SA3 to C45:
Where're you going to?
*C45 to SA3:
Ah, Canberra. I'm on the eight-fifteen flight//
*SA3 to C45:
((inaudible))
*C45 to SA3:
Yeh, well they're going to use this ticket as a credit towards the/
*SA3 to C45:
Oh sure, no problems. ..What time are you flying to Canberra?
*C45 to SA3:
Eight fifteen
*SA3 to C45:
Eight fifteen
*C45 to SA3:
The booking's all changed now?
*SA3 to C45:
Yeh.... OK, so this ticket's valued at three eighty and your ticket's six hundred and eighteen dollars forty so you just want the difference to go to the Amex?
*C45 to SA3:
Yeh, you want the Amex?
*SA3 to C45:
No its' OK, the number's already in there... OK, it's two hundred and thirty eight forty the difference - and there we go - oh, sorry, actually if you can sign it please, might help ... just ah, down the bottom there, just anywhere there... great. There you go. It's all yours.
RA to SA3:
Can you just explain what happened in the last transaction there?
SA3 to RA:
This one here? He was umm, using a ticket that he had, that he wanted that money toward a new ticket. So we do an exchange ticket. So the ticket he had was worth three hundred and eighty. The new ticket was six hundred and eighty dollars. So he put the additional money through to Amex.
RA to SA3:
So each step that you went through? - like he paid ((inaudible))
SA3 to RA:
He wanted a ticket, yeh, but he already had a ticket (so thus I knew that) he either wanted to change his ticket - normally if they have this ((inaudible)) it can be just re-validated pay the additional money.

As indicated previously, a number of alternative accounts of knowledge in use can be given. As a point of departure in exploring aspects of these, it will be useful first to consider the episode above from a knowledge transfer perspective.

In a transfer approach then, workplace knowledge is interpreted essentially as a rendition of numerical knowledge applied to a practical situation. An existing ticket is valued at $380, the desired ticket costs $618.40, and the subtraction of one sum from the other leaves a total of $218.40 for the customer to pay. Thus, in identifying the application of numerical concepts of subtraction (or its inverse, addition), this episode can be interpreted as an instance of knowledge transfer in which certain general procedures relating to numerical operations have been applied to a particular situation in a real life context. Certainly, a strength of this interpretation is its apparent simplicity. In appealing as it does to notions of a unitary mathematico-logical structure, knowledge of the details relating to the particular instance of the interaction are down played

(Walkerdine, 1994). At most, consideration of workplace knowledge under transfer is concerned with the problem of how to validly select and deploy appropriate numerical operations. From an epistemological point of view, numerical knowledge is thus regarded as both pre-given and mentalistic.

How does such an account stack up against the full range of evidence provided in this episode? One way to make this assessment would be to look for the presence of good analogical representations of numerical operations such as might be expected from the application of cognitive transfer theory (Brown, 1989; Singley & Anderson, 1989; Reimann & Spada, 1996). Evidence to be advanced later in the paper will indicate, however, that no such analogical relationships appear to exist in this case. This finding echoes the work of Cobb (1988) and others, and tends to weaken the case for transfer of numerical knowledge in this instance.

Towards numerical workplace knowledge: the logic of task performance

In this section the aim will be to outline a view of ticket exchange which does not make essentialist assumptions concerning the nature of numerical knowledge. The analysis will constitute an approach to determining the logic of actual task competency, that is, how workplace knowledge was actually organised and implemented in the events observed. Identification and stratification of component tasks with respect to their significance for the ticket exchange operation is the first step in the analysis. Next, the linguistic elements of task performances are analysed in order to explore relationships among them and to evaluate the place of the numerical within the workplace knowledge displayed. Wittgenstein's concept of the language-game (Hunnings, 1988; Wittgenstein, 1991) proved useful in obtaining a focus on the form of the interactions observed rather than their putative content. This was important as a non-essentialist epistemological paradigm makes problematic the notion of communicational content as a pre-given.

Focus on 'exchange'

In commencing the analysis, the first step was to review the use of the sign 'exchange' and its cognates (eg 'exchanging') within the transcript corpus. When this was completed two families of usage corresponding to language-games employing these signs appeared. In the first, typical instances were the exchange of one ticket for another or the exchange of a ticket for credit (eg "exchange a ticket to upgrade"). In instances of this kind the sense of effecting an actual swap of ticket items or entitlements was characteristic. In the second language-game, "exchange" was found to take its meaning from items in the following list: a ticket or coupon (an "exchange coupon"), or stamp (an "exchange stamp"); a software protocol (coded 'in exchange') where this was associated with the workplace procedure used to manage the ticket exchange process; the workplace ticket exchange procedure itself (as in, "do an exchange ticket"). In comparing these language-games it becomes noticeable that in the first, primary activity is directed to the resolution of the customer's service consumption needs and preferences in relation to constraints imposed on service provision by the airline itself. Whereas in contrast, the focus of the second language-game revolves around the organisation and management of legitimate task performance within the domain of service production. Component tasks found within each of these are summarised below.

- *Exchange as swapping* (swapping tickets in order to satisfy consumer preferences and requirements): -

  - providing information to customers re transaction sums for customer and/or sales assistant generated ticket swapping scenarios

  - obtaining compliance with airline mandated ticket exchange regulations

- generating ticketing arrangement strategies in order to meet customer preferences and airline mandated compliance requirements

- making reasoned judgments relating to the appropriate application of airline mandated requirements.

- *Exchange as procedural management* (facilitating service transfer): -

    - warranting and validating ticket exchange

    - determining airline mandated ticket exchange regulations

    - processing sums relating to ticket exchange

    - managing and implementing the overall ticket exchange transaction process.

Thus far, the focus has been on what constituted ticket exchange task performance in the episodes observed. What a study of the logic of task performance now requires is an assessment of how these components were actually implemented. When approaching this question from a socio-cultural perspective, the mediating role of artifacts within the locality of the problem situation becomes decisive (Vygotsky, 1978). Paradigmatic examples of such artifacts are tools (physical and psychological) and signs (language and symbolic systems) associated with task performance. For later theorists (eg Leont'ev, 1981; Wertsch, 1981; Engeström, 1994) this list is also extended to include social elements of the problem space such as the organisation of tasks, rules and protocols within this situation, etc. Instances of mediating artifacts associated with the ticket exchange task listed were found to include:

- specific tools (such as concrete instruments eg exchange tickets, coupons and stamps)

- signs (such as particular words and symbols associated with exchange)

- language-games (including procedural rules for standardised software protocols, and authorised procedural protocols intended within specific and general levels of operation).

Given the limited scope of this report it will not be possible to treat each of these at any depth here. Analysis of the use of certain language-game will be attempted, however. In the following analyses, two complex language-games associated with the performance of 'exchange as swapping' (centred on the use of the word 'difference') and 'exchange as procedural management' (centred on the use of the work 'additional') will be explored. In a concluding section of the paper, this material will be used to move toward an understanding of how numerical knowledge becomes involved in the workplace activities observed.

Focus on 'difference' and 'additional'

Referring to the episode cited above, the sign "difference" at first performs the task of indicating a discrepancy between the value of the presented ticket and the value of the ticket sought by the customer. Later, this differential value is associated with an amount in dollars - "OK, it's two hundred and thirty eight forty, the difference" - and the denotation of this sum is understood to serve as a basis for a successful ticket exchange transaction.

Similar usage for the words 'difference' and 'different' can be found elsewhere within the corpus of the transcript. For instance, when a customer made an inquiry concerning the possibility of upgrading from economy class the following dialogue was observed.

*C61 to SA1:
Yeh, what's the difference between business and first class
*SA1 to C61:
OK - Business Class is eight hundred and fifty one, Economy is six hundred and fifty three, so what's that - a hundred and ninety eight dollars.
*C61 to SA1:
For?
*SA1 to C61:
And first class is nine hundred and seventy nine - that's about three hundred and ... twenty six dollars or something like that
*C61 o SA1:
Alright.. Business has the comfortable, more comfortable seats
*SA1 to C61:
Yeh, the seating's virtually the same, it's just a little bit different in the food presentation
*C61:
Oh, so seating's the same?
*SA1 to C61:
Yeh, virtually.. I think they're the same width and everything, but they could be a fraction longer, but..
*C61 to SA1:
OK
*SA1 to C61:
I think they've got velour on the first class seats and its just nice cloth on the other one
*C61 to SA1:
Oh right - if you can umm.. ((inaudible)) in first, `cause I just want some room and I want to be able to sleep
*SA1 to C61:
Oh OK. How would you like to pay the difference?

Now in this case the sign 'difference' was being used quite differently by customer and sales assistant. For the customer the matter is seen to turn on the size of the seats and the type of cloth used in their upholstery. However, for the sales assistant, the sign "difference" seemed to suggest a relationship between values, and between various numbers associated by protocol with these (viz ticket prices). Note that the customer's interpolation of "For?" in response to the sales assistant's expression of quantitative values was set aside by the sales assistant and a new set of numerical figures reiterated. This observation tends to confirm the view that the customer and the sales assistant were participating in quite different language-games. A second, related occurrence of the sign 'difference' was to indicate on what financial terms a transaction could in fact successfully transpire ("OK. How would you like to pay the difference?"). To summarise: in a variety of representative instances, 'difference' was used by the sales assistant (i) to express a relationship between abstract rather than concrete entities and (ii) to indicate an amount which customers are asked to pay for the service sought. It is concluded from this analysis that the language-game involving the sales assistant's use of the sign 'difference' could be regarded as a mediating artifact in the 'exchange as swapping' language-game.

Analysis will now turn to the sign 'additional'. Review of the transcript corpus shows this term is most often used by the sales assistant in the context of an offered explanation to the researcher regarding events associated with procedures relating to the issue of an exchange ticket (see transcript above). Like the sign 'difference' discussed previously, the sign 'additional' is used to refer to the differential value between the current and proposed ticket. But as a sign, 'additional' is used in quite a dissimilar way. In this case, the value amounts depicted are those which connote the accumulation of a certain stock - money. Later in the transcript, when the same sales assistant is recorded discussing software protocols for the automated regulation of procedures for ticket upgrading, the following pronouncement concerning procedure is recorded: "Upgrade to J or F is permitted at any time subject to payment of additional fare." Here again, use of the sign 'additional' connotes the accumulation of a stock whereas, in contrast, the sign 'difference' is used more abstractly to suggest a relationship between values. It is concluded firstly that this language-game functions as a mediating artifact within the

language-game of 'exchange as procedural facilitation'; and secondly, that the language-games associated with the signs "difference" and "additional" are quite dissimilar.

Towards an understanding of numerical workplace knowledge

In the analysis so far, progress has been made towards determining the structure of the logic of the ticket exchange task performance. First, ticket exchange was seen to be the conflation of two kinds of language-games, the first involved 'exchange' as swapping tickets and/or entitlements, the second involved 'exchange' as the procedural management of exchange operations. Next, a more detailed study of artifacts mediating the operationalisation of these ticketing tasks was undertaken. This lead to the identification of an additional range of language-games deployed in the service of task performance. For instance, a language-game revolving around the use of the sign 'difference' was seen to underpin the operation of exchange as swapping; whilst another quite different language-game, related to the use of the sign 'additional' was found to underpin the operation of exchange as procedural management. What then, by way of conclusion, do these findings suggest for the position of the numerical within the workplace knowledge observed?

From an essentialist perspective it would appear that this involvement is easy to state: the use of 'additional' and 'difference' is suggestive of an analogical representation of the inverse relation which exists between addition and subtraction within the domain of numerical knowledge. However, the evidence adduced in the preceding analysis against such an account is quite compelling. Based on the essentialist account one would expect to find 'additional' and 'difference' within the same language-game - however, on the empirical evidence available, this was not the case. What then are the implications for the place of numerical knowledge within the ticket exchange task performance? I will conclude the paper with the following suggestions.

First, the workplace situation itself has primacy within the organisation and implementation of workplace knowledge - thus any approach to workplace task competency should proceed from a detailed knowledge of the particular occupational situation, not from an abstract set of numerical knowledge statements. Second, numerical knowledge does not enter the workplace as a pre-given entity. Third, numerical knowledge is not used as a unitary mathematico-logical structure within workplace sites. Analysis based on the evidence advanced in this paper suggests that numerical workplace knowledge is better seen as a body of fragmented knowledge. Fourth, and lastly, the logic of this fragmentation is governed by the character of mediating artifacts engaged within task performance.

References

Bauersfeld, H (1991) The structuring of the structures. In Leslie P. Steffe (Ed.) *Constructivism and Education*. Hillsdale, NJ: Lawrence Erlbaum Associates

Bishop, A (1988) *Mathematical Enculturation*, : Kluwer

Bloor, D (1983) *Wittgenstein : a social theory of knowledge*. New York : Columbia University Press

Brown, Ann L. (1989). In Vosniadou, S. & Ortony, A. (eds.) *Similarity and analogical reasoning*. Cambridge : Cambridge University Press

Brown, J., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. Educational Researcher, 18(1), 32-42.

Cobb, P (1988) The tension between theories of learning and instruction in mathematics education. *Educational Psychologist*, 23(2), 87-103

Cobb, P (1994). *Learning mathematics: Constructivist and interactionist theories of mathematical development*. Dordrecht: Kluwer

Cobb, P, Yackel, E. and T. Wood (1992) A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education*, 3(1), 2-33

Engeström, Y (1991) Non scolae sed vitae discimus: toward overcoming the encapsulation of school learning. *Learning and Instruction*, 1, 243-259

Engeström, Y (1994) Teachers as collaborative thinkers: Activity-theoretical study of an innovative teacher team. In G. Handal & S. Vaage (eds), *Teachers' minds and actions: Research on teachers' thinking and practice*. London: Falmer

Engeström, Y., Engeström, R., & M. Kärkkäinen (1995). Polycontextuality and boundary crossing in expert cognition: Learning and problem solving in complex work activities. *Learning and Instruction*, 5, 319-339

Gerdes, P (1985) Conditions and strategies for emancipatory mathematics in underdeveloped countries. *For the Learning of Mathematics*, 5

Hunnings, G (1988) *The world and language in Wittgenstein's philosophy*. Albany, NY: SUNY Press

Lave, J. (1988) *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge: Cambridge University Press.

Lave, J., & Wenger, E. (1991) *Situated learning: Learning as situated peripheral participation*. Cambridge: Cambridge University Press.

Leont'ev, A. N. (1981) *Problems of the development of the mind*. Moscow: Progress

Nunes, T., Schlieman, A. D., & Carraher, D. W. (1993) *Street mathematics and school mathematics*. New York: Cambridge University Press.

Reimann, P & H. Spada (Eds) (1996) *Learning in humans and machines : towards an interdisciplinary learning science*. Oxford, England : Pergamon Press

Rogoff, B. (1990) *Apprenticeship in thinking: cognitive development in social context*. Oxford: OUP.

Rogoff, B. (1995) Observing sociocultural activity on three planes: participatory appropriation, guided participation, apprenticeship. In Wertsch, J, del Rio, P, .Alvarez, A (Eds), *Sociocultural studies of mind*. Cambridge: Cambridge University Press.

Saxe, G. (1991) *Culture and cognitive development: Studies in mathematical understanding*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Singley, Mark K & John R. Anderson (1989) *The transfer of cognitive skill*. Cambridge, Mass.; London: Harvard University Press

Solomon, Y. (1989) *The practice of mathematics*. London: Routledge

Voigt, J (1993) Ascribing mathematical meaning to empirical phenomena. Paper presented at the Conference "The Culture of the Mathematics Classroom: Analyzing and Reflecting Upon the Conditions of Change", Osnabrück, October 11-15

Vygotsky, L. S. (1978) *Mind in society: the development of higher psychological processes*. Cambridge, Mass; London: Harvard University Press

von Glasersfeld, E. (1995) *Radical constructivism : a way of knowing and learning*. London ; Washington, DC. : Falmer Press, 1995.

Walkerdine, V (1988) *The Mastery of Reason*, London: Routlege

Walkerdine, V (1994) Reasoning in a post-modern age. In P. Ernest (ed) *Mathematics, education and philosophy: An international perspective*. London: Falmer

Wertsch, J. (ed) (1981) *The concept of activity in Soviet psychology*. Armonk, NY: Sharpe

Wertsch, J. (1985) *Vygotsky and the social formation of mind*. Cambridge: Harvard University Press.

Wittgenstein, L (1991) *Philosophical investigations* (3rd Edition). Oxford: Basil Blackwell