Problem Solving in Algebra: Ability Grouping, Affective Factors And Gender Differences

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Delineation of the quantum of influence of *extra-cognitive auxiliary inputs* like affective factors may help to explain gender differences by illuminating the mechanics of cognitive functioning in early adolescents engaged in mathematics problem solving. Part of the data from a semi-longitudinal study is analysed in this paper to explore the relationship between algebraic problem solving ability and affective factors based on the differential problem solving abilities among junior high school students. The results show that there is a difference in this relationship for female and male students. A *transient refractive affective state* of high achieving girls could be the reason for compromising any advantage over their male counterparts.

Background

Recent research in problem solving has identified the affective domain as one of the important areas where gender differences are noticed (see Frost *et al*, 1994). This may have an important bearing on the search to understand the differences in the cognitive domain of the students, especially with regard to the often stated underachievement of girls. Delineation of such *'extra-cognitive auxiliary inputs'* would illuminate the mechanics of cognitive functioning. Variables from both cognitive and affective domains have been seen to influence the problem solving ability (Boekaeris et al, 1995) as well as intra-domain interactions (Skaalvik and Rankin, 1995). Ethington &Wolfle (1984) concluded from their longitudinal study that there is a complex interaction between gender, selected other variables and mathematics achievement. These other variables include cognitive and affective variables. We have previously attempted to describe problem solving as the resultant of the interaction between these sets of variables (Thomas and Kota, 1996) and the variables in each set. Maker (1982, p. 30-31) stated that 'there is a cognitive component in every affective objective and affective component in every cognitive objective'. McLeod (1988, p. 134) sees this relation with problem solving, ' if the students obtain a solution to a problem, they typically express feelings of satisfaction , even joy' and these are seen to be important for problem solving performance. The National Council of Teachers of Mathematics (NCTM, 1989) and National Research Council (NRC, 1989) have encouraged mathematics educators to incorporate affective factors and cognitive factors such as algorithmic procedures and problem solving (Schoenfeld, 1985), in their mathematics teaching and learning. To do this efficiently and effectively, knowledge of the relationship between affective factors and problem solving ability and the changes in this relationship with age and academic level, would be very helpful for mathematics educators. This knowledge could help to narrow the gap between the genders in enrolment and achievement in mathematical sciences in their further education.

Affective factors

A body of research records the analysis of the relationship between affective factors and achievement in mathematics. Enemark and Wise (1981, p. 22) demonstrated that '...the attitudinal variables are a significant indicator of mathematics achievement'. Reyes (1984) examined a numbers of affective variables influencing the learning of mathematics. These variables included self-concept, anxiety and perceived usefulness and others considered enjoyment of mathematics, self-perception, motivation and interest (Skaalvik & Rankin, 1995; Mitchell, 1993; Watson, 1983). Kloosterman (1991) and Minato & Yanase (1984) tried to examine the relationship between mathematical attitudes and achievement which Anderson (1981) inferred through mathematics participation.

Other researchers have tried to establish a relationship between these affective factors and various other components such as class-room environment, participation and achievement and problem solving in the process of mathematics learning and teaching. McLeod (1992) proposed a framework in which beliefs, attitudes and emotions reflect the range of affective reactions involved in mathematics learning and teaching. Forgasz and Leder (1996) established a link between classroom environment and affective factors and stated those would provide vital clues about the factors shaping student beliefs which exhibit gender differences at high school.

Affective factors and problem solving

After the announcement of the agenda by National Council of Teachers of Mathematics (NCTM, 1980), 'problem solving should be the main focus of school mathematics'; additional research has gone into finding the role of affective factors in mathematics problem solving. Lester and Kroll (1989, p. 57) stated that 'the affective domain...is an important contributor to problem solving behaviour' and 'student performance in solving a problem may be very much influenced by affective factors, sometimes to the point of dominating the students thinking and actions.' Some researchers established a positive influence of these variables while others noted the negative affects of lack of some of these factors (see Wong, 1992, and Davis, 1973).

Affective factors, problem solving and gender

Evidence is accumulating that establishes gender as an important influence on affective factors as well as performance in mathematics. Hyde *et al* (1990) performed a meta-analysis on the magnitude of gender differences in the mathematics performance. They noted problem solving as one of the areas where gender differences exist and reported that junior high school years are important to notice favourable or unfavourable changes in attitudes towards mathematics (see also Aiken 1986 & Callehan, 1971). Aiken (1976) suggested a separate analysis by sex in prediction studies involving affective factors. From the meta-analysis by Xin Ma & Kishore (1997), Frost *et al* (1994) and Hyde *et al* (1990), the following points emerge about the relationship between gender, mathematics attitudes and achievement of students: a) junior high school appears to be the more suitable age group since they are not too young to lack the ability of expression nor too old to have fixed attitudes precluding interventions, b) grouping on the basis of differential abilities seem more appropriate, c) area specific tests such as algebra, geometry and the like may be more revealing, & d) high stake tests like problem solving could be better representatives of mathematical ability. The inference from these observations is that studies incorporating the above ideas would provide a clear view of the various relationships in the quest for bringing appropriate changes in line with directives of learned bodies (National Council of Teachers of Mathematics, 1989 and National Research Council, 1989).

The present study overcomes the majority of the disadvantages of the earlier research studies as implied in the above cited meta-analytical studies. When trying to establish a relationship between problem solving, affective factors and gender differences, a study based on students differential abilities, has not to the best of my knowledge been undertaken. Here, I have analysed the initial data of our semi-longitudinal study to note the gender differences in this relationship on the basis of ability grouping using correlational analysis. The results presented show that the extent of relationship between the problem solving ability and affective factors could be one of important reasons affecting students cognition and developing gender differences in algebraic problem solving abilities.

Method

Subjects

The data for the present study was collected from 345 form 3 and 4 students of ages about between 13 - 15 years, from 8 secondary schools in the Auckland region of New Zealand. Both single-sex and co-educational schools were represented in the study. Each school reported that they were following the 1992 New Zealand curriculum published by the Ministry of Education. One class of average ability students from each school was randomly selected to form the subject group. The students were taught by specialised mathematics teachers.

Instruments

Each individual student in the study was given two questionnaires during regular mathematics classes in the first half of the academic year 1996, under the supervision of the mathematics teacher, before the algebra syllabus was taught for the year. To ensure the uniformity of administration the teachers were provided with clear instructions and directions. These questionnaires were repeated at the end of the academic year. The first measured seven affective factors using self-descriptive questions in a 5-point Likert format. The second measured algebraic problem solving ability. The details of the tests used are:

Self-concept: The mathematical self-concept scale of 27 items developed by Gourgey (1982) was used to measure mathematical self-concept. The internal consistency reliability of the scale is 0.96 and Gourgey stated that the analysis of the scale provided support for its validity and reliability.

Interest: The scale developed by Mitchell (1993) and used in his subsequent research was used to measure the level of interest in mathematics. The internal consistency coefficient for the independent subscales used ranges from 0.77 to 0.93.

Anxiety: The anxiety component of the Skaalvik and Rankin (1995) scale was used to measure the students' mathematical anxiety. There are 8 items in this scale and the Cronbach alpha for this scale was reported as 0.90 for a similar subject group.

Self-perceptions: The self-perception constructs were measured by using Skaalvik and Rankin's (1995) three-item scale for each of self-perceived ability and self-perceived aptitude. The internal consistency reliability coefficients are 0.8 for 12/13 years and 0.83 for 15/16 years.

Usefulness of mathematics: The usefulness of mathematics scale of Fennema & Sherman (1976) served to measure the individual student attribute about the usefulness of mathematics.

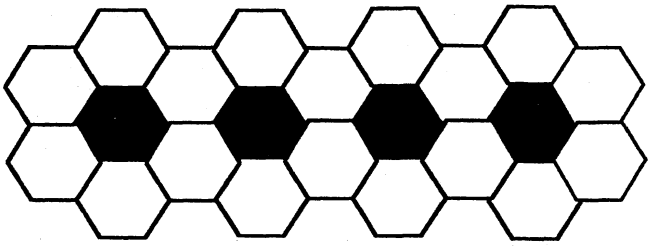
Mathematics Intrinsic motivation: Of the two types of motivation, intrinsic and extrinsic, we considered that the intrinsic factor was likely to be more consistent and reliable since it originates from the subject. This factor was measured using the English translation of the instrument developed by Skaalvik and Rankin (1995).

Enjoyment of Mathematics: Students enjoyment levels were measured using the instrument from Aiken's attitude scale as modified by Watson (1983). The internal consistency reliability coefficient was found by Aiken to be 0.95, using secondary school students.

Problem Solving ability: An algebra test consisting of five basic word problems was constructed from the problems used in the three algebra tests which we had piloted (Thomas and Kota, 1996). The problems were constructed based on the objectives for levels 2 to 5 of the New Zealand mathematics curriculum guidelines, published by the Ministry of Education. It was intended that factors like word order, situation, language and difficulty in carrying out numerical operations would not be obstacles in the process of solving the problems successfully. The type of problems used in the test were:

1. The number of girls in a school is 41 less than the number of boys. The total number of students in the school is 1539. How many girls are there in the school?
2. To hire the Pizza House for birthday party costs a basic rate of $70.00 plus $3.50 per person. If the total bill is $346.50, how many people attended the party?
3. Tickets for the school play cost $2 for children and $4 for adults. 500 tickets were sold for $1,640. How many children's tickets were sold and how many adult tickets were sold?
4. The Auckland city council wishes to create flower beds, surrounding them with hexagonal paving slabs according to the pattern shown below.

Complete the table below to find the number of paving slabs needed for 8 and 100 flower beds. Write an algebraic equation to find the number of paving slabs, N, needed to surround F flower beds.



Number of flower beds	1	2	3	4	8	100	F
Number of paving stones	6	10	14	18			

5. A wooden fence is made by placing 3 planks between 2 posts as shown in the figure. (A picture was given). Complete the table to find the number of planks needed for fences of 5 posts; 100 posts and S posts. Write an algebraic equation to find the number of planks L in a fence S posts long.

Number of posts	2	3	4	5	100	S
Number of planks	3	6	9			

Results

The total of 345 students, 216 girls and 129 boys, (boys and girls together, 177 form 3 and 168 form 4) participated in the study. The scores of these students are analysed using correlational analysis to look into the differences in the extent of the linear relationship between algebraic problem solving and affective factors. Whilst such an analysis does not permit definite inferences it is capable of giving some indication of the relationship. When we examined the difference in the mean scores on algebraic problem solving ability for female and male students, we found a significant difference. Girls performed better than boys in form 3 in algebra problem solving (for girls, mean = 13.702, for boys, mean = 10.171, t = 4.325, df=175, t critical = 3.98, $p<0.0001$). However, at form 4 there is no significant difference in algebra performance between girls and boys in (for girls, mean = 17.017, for boys, mean = 16.037, t = 1.073, df=166, t critical =1.974).

The student scores on each of the affective factors show a fairly high positive correlation with algebraic problem solving ability for form 3 girls (as described in table 1). But for form 4 girls, the correlation is positive and present only for self-concept and anxiety.

Table 1: Correlations between problem Solving Ability and Affective Factors in form 3 and 4 girls

Algebra Scores	*SC*	*Int*	*Anx*	*SP*	*Use*	*Mot*	*Enj*
Form 3 (n=101)	0.5133	0.3963	0.3853	0.398	0.2235	0.3893	0.3186
Form 4 (n=115)	0.2453	0.0120	0.162	0.0456	-0.014	-098	-0.130

Sc= Self-concept; Int = Interest ; Anx = Anxiety; Sp = Self- perception, Use = Usefulness; Mot = Motivation; Enj = Enjoyment

We also see that there are much higher positive correlations between each of the factors and problem solving performance for form 3 female students. It appears that there is a role played by age and academic level of the students in the relationship between affective factors and algebraic problem solving ability.

When the corresponding results were analysed for the 129 boys, 76 in form 3 and 53 in form 4, we obtained the figures shown in table 2. All the correlations for boys in form 4 are positive and are fairly high for self-concept, self-perception, enjoyment and interest. But none of the correlations is comparatively high for boys in form 3 except for anxiety. In fact self-perception and enjoyment show a slightly negative correlation. Here we see that a situation opposite of girls is true. As table 1 shows, the correlations between algebra performance and affective factors based on individual scores, are higher for girls in form 3 than in form 4. From these results it appears that the relationship between algebraic problem solving ability and affective factors is different for female and male students at secondary school level. There is an increase in the influence over this year for boys except for anxiety and decrease for girls.

Table 2: Correlations between problem solving ability and affective factors for form 3 and form 4 boys

Algebra Scores	*SC*	*Int*	*Anx*	*SP*	*Use*	*Mot*	*Enj*
Form 3 (n=76)	0.081	0.043	0.316	-0.010	0.130	0.0302	-0.006
Form 4 (n=53)	0.343	0.212	0.071	0.421	0.152	0.147	0.218

When we divide the students scores into low and high achievers, students whose algebra scores are between 5 and 15 and students whose algebra scores are between 16-25 (minimum and maximum scores obtainable in the algebra test are 5 and 25 respectively), we see some very interesting results which get masked when the result are taken as a whole for girls and boys.

Table 3: Correlations between problem solving ability and affective factors for high achievers and low achievers in Form 3 & 4 Girls

Algebra Achievement level		*SC*	*Int*	*Anx*	*SP*	*Use*	*Mot*	*Enj*
Form 3	low	0.298	0.178	0.155	0.228	0.029	0.120	0.001
	high	0.179	0.245	0.120	0.238	0.267	0.301	0.215
Form 4	low	0.313	0.163	0.400	0.320	0.350	0.271	0.259
	high	-0.105	-0.122	-0.104	-0.199	-0.307	-0.200	-0.200

When these correlations are examined for girls on the basis of ability grouping we obtain the figures shown in table 3. In form 3, the correlations are higher for the high achievers group in all the affective factors except self-concept and anxiety but in form 4, the correlations for low achievers group are higher in all the factors and in fact they are negative for high achievers group. Thus for the younger girls the scores in all the affective factors correlate positively with algebra problem solving performance for both high and low achiever groups, and are higher for the high achiever group for all the factors except self-concept and anxiety. But within just one year this effect has disappeared, and the correlations are positive only for the low achievers group and turned negative for high achievers group. It appears that higher levels in affective factors seems to decrease their scores in problem solving for high achievers group in form 4. The correlations have increased among the low achievers groups from form 3 to 4. Thus for girls as they grow in chronological age and academic level, the effect of affective factors seems to increase for low achievers but for high achievers it drops of markedly. One of the possibilities is that, they reached a *transient refractive affective state* thereby deviating from a positive view of mathematics and hence interest in further study.

When the individual student scores in algebra performance and affective factors are analysed on the basis of ability grouping for boys (shown in Table 4), all these correlations are higher for high achiever groups than low achiever groups in form 3 as well as in form 4. These correlations have increased for self-concept and interest from form 3 to 4.

Table 4: Correlation between problem solving ability and affective factors for low achievers and high achievers among form 3 and form 4 boys

Algebra Achievement level		*SC*	*Int*	*Anx*	*SP*	*Use*	*Mot*	*Enj*
Form 3	low	0.208	0.160	0.301	0.1866	0.256	0.2038	0.087
	high	0.388	0.235	0.380	0.425	0.399	0.375	0.456
Form 4	low	-0.284	-0.117	-0.127	0.001	0.092	-0.027	-0.053
	high	0.407	0.338	0.245	0.402	0.191	0.2633	0.426

Thus for boys, the effects of affective factors is maintained for the high achievers group but is decreased for the low achievers group within one year from 13 to 14 years.

For girls there is a positive linear relationship between affective factors and algebraic problem solving ability for the low achievers group in form 3 and form 4. For boys, this relationship is positive for the high achievers group in form 3 and form 4. An important result to be noted here is that this relationship between algebraic problem solving and affective factors is turning negative for high achievers group in girls and low achievers group in boys within just one year from form 3 to 4. That means for high achievers group in form 4 girls, algebra score and affective factor score move inversely to each other. Similarly, for boys the influence of the affective factors becomes negative for the low achievers group within one year, from age 13 to 14 years. It could be possible that high achieving girls have reached *a transient cognitive set point* so that their high levels in affective factors could be refracting their cognition away from mathematics. Literature on mathematics achievement and gender has noted that girls achievement is higher or equal to that of boys up to junior high school or early adolescence and by the end of high school boys achievement is higher than that of girls (see Frost *et al*, 1994). It could be possible that because of this *transient refractive affective state* girls compromise any advantage in mathematics over their male counterparts. This could be one of the reasons for female students to deviate from mathematical sciences preferring other subjects like social sciences and arts by the end of secondary school education.

Conclusions

Summarising the results we may say that affective factors have a positive effect on algebraic problem solving ability of girls and boys in form 3. For low achieving girls this effect increases with age from 13 to14 years. The positive effect is present in form 4 only for high achieving boys, increased for self-concept and interest. There is a clear cut gender difference in the effect of affective factors and changes in both the ability groups in the opposite directions for both the genders with age and academic level and in behaviour pattern of girls and boys.

The behaviour pattern seen is that, in boys the correlations between affective factors matched with their performance, ie, high achieving boys showing high correlations and low achieving boys showing low correlations but for girls they move in opposite directions, ie, low achieving girls having high correlation and high achieving girls having low correlations. This reflects the more mature out look of girls in that low scorers in problem solving ability project their ideas in the form of positive statements and those who are doing well or satisfactorily play down on their affective factors. In effect, high achieving girls have reached the *transient cognitive set point* arising from *transient refractive affective state* where their cognition gets refracted when they face a cognitive challenge in mathematics. A stage where the stimulus for the particular type of cognition disappears. This often occurs when cognition reaches a set point. In other words cognition ceases being reward by itself for those types of cognitive challenges where the affective factors failed to respond to stimulus. To sustain their achievement levels, *extra-cognitive inputs* like affective factors may be helpful to sustain their problem solving ability for high achieving girls and improve it for low achieving boys as they grow in chronological age and academic level. When the results of the second data set are analysed we may discern the magnitude and direction of changes in affective factors and algebra problem solving ability. It will be important to look for 1) the changes in the female and male students who are high in both problem solving and affective factors, 2) students who are high only in problem solving and only in affective factors, 3) student who are average on both, and 4) low in both problem solving and affective factors. It may be possible that the results noted in this study could then be seen with better clarity. Further research is needed to look into the other possible factors responsible for the changes occurring in the preferences of girls and boys after the adolescent period.

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