Looking Back Towards the Future: A Case Study in Mathematics

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This paper focuses on the affective and cognitive behaviours of a small group of grade 7 students who worked for eight lessons on an extended mathematical task in the classroom. Data sources included videotapes, field notes, students' reactions, self-report affective measures, and interviews. By drawing on these data sets, inferences were made about cognitive and affective behaviours influencing learning and the students' longer term involvement with mathematics five years later, in grade 12.

Introduction

The main aims of the present study were to describe students' affective and cognitive behaviours as they worked, over an extended period of time, in a small group learning setting on a substantial and realistic mathematics task and thus to infer their likely long term commitment to mathematics. Our observations of students' behaviours and interactions served as important data sources, as did the students' responses on different affective measures and when interviewed. Possible gender differences were also of interest. Specifically, we aimed to answer the following question: are students' cognitive and affective behaviours in class predictive of their longer term involvement in mathematics?

Previous Research

Links between affect and cognition in the learning of mathematics have been examined in several studies. Various authors in McLeod and Adams (1989) described affective reactions experienced by students during mathematics problem-solving. Data gathering techniques included interviewing students about their problem-solving strategies, eliciting affective information as part of that process, and focusing on affect during interviews. Boekaerts, Seegers, and Vermeer (1995) used faces with very sad to very happy expressions to trace students' feelings as they worked on mathematical problems. Finding ways to infer attitudes from behaviours has been a challenge to researchers (McLeod, 1989).

In extensive reviews of gender issues in mathematics learning (Leder, 1992; Leder, Forgasz, & Solar, 1996) it was reported that, internationally, participation rates in elective and specialised mathematics courses persistently reveal gender differences favouring males. Mixed results were found for student achievement. Explanatory models to account for these observed gender differences share a number of common features:

> the emphasis on the social environment, the influence of other significant people in that environment, students' reactions to the cultural and more immediate context in which learning takes place, the cultural and personal values placed on that learning and the inclusion of learner-related affective, as well as cognitive, variables (Leder, 1992, p.609)

Included among the affective variables are: self-concept of ability, confidence, attributional style, expectations for success, sex-role congruency, perceived difficulty of maths, learned helplessness, societal influences and teacher attitudes. Cognitive variables include: general ability, spatial skills, mastery orientation, engagement in high level cognitive tasks, willingness to work independently. Several of these variables were incorporated in the research design of the present study.

Research Methods

Our research problem called for a combination of quantitative and qualitative research techniques. Observational methods, using videotaped records and a descriptive system similar to that used by Clements and Nastasi (1988) and interviews were supplemented with self-report data.

The mathematics task and setting

The group task was set by the teacher as part of the regular mathematics program. It involved a study of the feasibility of building a new tuckshop in the school. With anticipated growth in enrolments and pressure already being felt on existing facilities, an additional cafeteria was seen as an option worth considering. To allow the students to put forward their views was consistent with the philosophy of the school.

The teacher organized the 28 students into six small groups of four or five. Each group was required to investigate the feasibility of building the new tuckshop and to prepare a report supporting the case presented. Students had covered the topics of simple data analysis, presentation and interpretation (tallies, bar charts, pictographs etc.) and percentages prior to the period of monitored lessons. The problem set thus allowed them to use skills learnt and revised earlier and to work on a substantial mathematical task over a number of lessons. The students were also encouraged to draw on skills learnt in other subjects. For example, the survey ultimately used by the observed group was compiled during a 'computer' lesson (the same teacher taught both subjects).

The groups worked on the mathematics project for eight lessons, each of 45 minutes duration. Seven lessons were videotaped. One was missed due to a timetable clash (a second class in the school was also being monitored by the researchers).

The sample

The sample comprised one of the small groups consisting of five students: three females (Carol[1] [C], Cheryl [Ch], and Jenny [J]) and two males (Brian [B] and Mark [M]). The male teacher (DM) was asked to nominate articulate students for inclusion in the targeted group. Each was described by him as being very good or excellent at mathematics. For one lesson each week, a female teacher (BL) assisted DM. Due to absences and attendance at special lessons such as music, the target group did not have its full complement at all lessons. This is not unexpected in the reality of classrooms and the changed composition of the small group provided additional insights into the interpersonal dynamics of group members and the interplay of cognition and affect.

Procedures

Data gathering involved videotaping each lesson, subsequently transcribing the tapes, keeping field notes, monitoring students' reactions to the work done each day and their contributions to it, administering self-report instruments tapping the students' attitudes and beliefs about mathematics and themselves as learners of the subject, and interviewing the students. The data sources and their use are described below.

Data sources and modes of analysis

Lesson transcripts: Focussing on the targeted group, lessons were videotaped and subsequently transcribed. The students' behaviours and conversations were captured.

Field notes: Field notes were kept. Notes were made on general observations, critical incidents out of camera view, and comments by teachers to the observer.

Students' perceptions of the lessons: At the end of each lesson, students completed "Today's Maths Lesson" sheets (based on Clarke, 1992). There were seven items (e.g., "Circle the face which shows how you felt about your *understanding* of today's maths lesson" and "Explain briefly why you felt this way").

Interviews: The five targeted students were interviewed individually some time after the monitored period. They were asked questions which encouraged them to reflect on the "Tuckshop" project, on the year's mathematics activities, and on their futures.

Self-report measures: Several Likert and Semantic Differential scales, as well as open-ended items, were administered to each student. The data are summarised in Table 1. The first six instruments were completed before the observational period; the remainder towards the end of the school year.

Results and Discussion

The researchers' perspectives

General comments: The group worked well with minimal prompting from the teacher. DM spent less time than average with the monitored group. The highest number of visits for any one lesson, eight, was recorded during Lesson 7 and totalled about eight minutes.

Table 1. Summary of self-report instruments

Instrument	Type and number of items	Items
1. *About you*	Semantic differential, 25	What kind of person are you? e.g., messy/neat; boring/interesting
2. *About you and maths I*	Open-ended items, 2	Do you like maths? Explain
3. *About you and maths II*	lines, 5: ├──────┼──────┤ Excellent Average Weak	Mark with a cross: • How good are you at maths? • How good do you think your teacher thinks you are?
4. *About maths lessons I*	Open-ended (drawing optional)	Write a description of the kind of maths lesson you usually have in grade 7
5. *Maths lesson activities*	28 activities: Frequency (1:never, 4:always) Enjoyment(1:dislike, 3:like)	• Teacher explains to whole class about a topic • Working alone from a textbook • Working in groups to discuss math problems
6. *About grading in maths*	Open-ended items, 5	How does your maths teacher find out what you know and have learnt?
7. *More about maths*	Likert scale, 25 items (Nicholls et al., 1990)	I like doing maths problems which make me think hard
8. *How good are you?*	Similar to *About you and maths II*	

The task was meaningful to the students as they had a personal stake in the eventual outcome. This appeared to provide some motivation for a genuine attempt at it. The level of mathematics to which some members of the group were engaged was unlikely to be so enthusiastically embraced by grade 7 students in a more traditional setting. The following episode from Lesson 6 reveals the level of complexity of a calculation performed by Brian and Mark using a calculator, their persistence with it and in interpreting the outcome, and their level of collaboration.

Lesson 6: Time - approx. 9 minutes into the lesson

M: 2630 divided by 2500
B: (inaudible comment)
M: Start again. 2, 6, 3, 0...
B: Yeh
M: Divided by..
B: Yep
M: 2500
B: 1.052
M: (incomprehensible comment)
B: No, it's not. The more students you have, the less it's going to be...
M: ...(unclear) How come we get .2 cents?
B: .052 cents.
M: That'd be about right
B: So that's a dollar and half a cent, um...

Individuals' levels of cognitive involvement: For the duration of the project, Mark, Brian and Jenny were very involved in mathematical tasks and discussions. Mark and Brian spent most of their time actively engaged in mathematical activities both at high and low cognitive levels. They plotted graphs, used calculators, consulted, and co-operated well. Jenny's cognitive involvement was mainly in discussing mathematical issues related to directions for further action, particularly in DM's presence. Her contributions were not always constructive and the cognitive level of many of her comments was questionable. Her remarks or assertions were often made without supporting mathematical reasoning. Throughout the monitored period, Mark and Brian spent most of their time engaged in higher order mathematical activities, Jenny and Cheryl took charge of the presentation of the project, while Carol was least involved in the group activities.

Overview of affective behaviours: Self report data after each lesson indicated that the girls were more confident than the boys that they had understood the work associated with the project. Since the girls' mathematical involvement was generally limited to discussions about what needed to be done, rather than actually undertaking any of the analyses or calculations, they may not have fully appreciated the complexity or levels of challenge and difficulty associated with the tasks completed by the boys. For example, at the beginning of Lesson 6, the following exchange took place:

Lesson 6: Time 0.10 - Brian gets up and leaves the table

J: (to Mark) Have you guys got all the facts and figures, sort of, or are you working on them? And have you got them or...

M: What do you mean by facts and figures?

J: Oh, just the sort of figures you guys were working on last week.

M: (picking up and looking at sheets) No, we've figured out that everybody spends...

J: We need the sort of profits and things like that

Ch: We need the costs and the profits

M: On average, everybody spends $1.17 every week...

J: Mmm

The teacher interacted differently with the students. For example, only some received positive feedback. Consistency in these interaction patterns may have longer term implications for individuals' perceptions of teacher support and of themselves as learners of mathematics.

Students' perspectives

Jenny's and Brian's *feelings about* and *understandings of* three representative mathematics lessons are summarised on Table 2.

Table 2. Jenny's and Brian's reactions (from "Today's maths lesson" sheets)

Student & lesson		*Feelings about lesson*: Point of lesson	Level of understanding
J	3	*Pleased*. Seeing whether we as a group could successfully establish a hypothesis based on our survey result.	Learnt to deduce facts from graphs. Easy to understand
	4	*Frustrating*. Sorting out what was messed up the previous lesson	Learnt nothing, but the work was easy
	8	*Pleased*. Finishing the project: frantic but fun. Learnt not to rush work	Work was easy: doesn't require much thought
B	3	*Pleased*. Graph work, how to cooperate	Understood what he had done
	4	*Neutral*. Graph work, working with figures	Not sure, more figures would have helped
	8	*Pleased*. Organisation and cooperation	Understood now they had finished. "It was a very good project which was difficult"

The different aims of the lessons identified by the students is intriguing. Jenny, for example, focussed more persistently on organizing and writing up the work. Brian included working with figures, with calculators, and graphing as important lesson goals. That all three girls were generally found to be more confident than the boys that they had understood the work was also noteworthy. Differences in the activities in which students engaged (see above) may account for their different perceptions.

Comparing the students' affective and cognitive reactions to the lessons with the affective and cognitive measures derived from the self-report data was informative. The profiles of the five students compiled from the self-report data are summarised in Table 3. The teacher also provided achievement levels for each student.

Table 3. Affective and cognitive profiles of the targeted students

Affective indicators	Cognitive indicators
Carol: Liked maths: important for future, useful, encouraged logic. HGM[1], CGM, TGM: 4.	TR: 4. Liked: talking to class about maths (not as part of a sma group), working in small groups. Pleased when: easy to get right answers, worked hard to solve problems, worked cooperatively.
Cheryl: liked maths: a challenge and important for real world. HGM, CGM, TGM: 5	TR: 5. Liked: talking about maths to friends and in small grou knowing more than others, challenging problems, working ha on them, alternative solutions. Pleased when: correct answers and no mistakes. Enjoyed lessons when: everyone understood worked co-operatively. Success from: understanding not just right answers, interest in learning, co-operation, persistence.
Jenny: liked maths: challenging, fairly simple, interesting, helped concentration, promoted logic, helpful later. HGM, CGM, TGM: 5.	TR: 5. Liked talking about maths in small groups. Pleased whe correct answers and no mistakes. Liked: challenging problems working hard to solve them, alternative solutions, answers tha made sense. Enjoyed lessons when: everyone understood, worked cooperatively. Success from: cooperation and persistence.
Brian: Liked maths: interesting, always different, never boring. HGM: 5, CGM, TGM: 3.	TR: 5. Did not like talking about maths. Liked: small groups tc discuss problems, hard work to solve problems, answers making sense, no mistakes. Success from: interest in learning, understanding more important than right answers.
Mark: Liked maths: challenge "to get it right", success meant "easier to find a good job". HGM, CGM: 5; TGM: 4.	TR: 4. Liked: talking about maths problems in small groups, challenging problems, solutions that made sense, alternative solutions, being only one to answer correctly. Enjoyed maths lessons when: everyone understood, co-operative. Success from: interest in learning, understanding not just right answers persistence, hard worked, setting work out neatly.

[1] HGM: self-rating of mathematics achievement, CGM: believed classmates rating, TGM: believed teacher rating, TR: teacher rating of achievement

Our predictions

Affective measures gathered from self-report data and from observed classroom behaviours did not always match. Greater consistency was evident for the boys than for the girls. Mark, for example, demonstrated his concern for both mathematical accuracy and neat presentation, and persisted at challenging tasks. Although there was some uncertainty about small group experiences evident on his self-report data, Brian indicated that he had enjoyed the group activity, displayed a preparedness to work hard and clearly tried to make sense of the mathematical solutions. As critical elements of the models postulating explanations for gender differences in participation rates in higher level mathematics courses, these observed behaviours and personal characteristics imply a likelihood for longer term involvement with mathematics.

On the other hand, there was little evidence that Cheryl discussed, persisted at or tried to find new ways to solve mathematical problems. While she showed task persistence, it was as reporter and recorder rather than as a user of mathematics.

Jenny's involvement in mathematical activities similarly did not go beyond discussion. She typically repeated strategies already discussed, did not initiate new approaches to solving the task at hand, and did not do any of the calculations required. At one stage Jenny argued for data selectivity. This behaviour seemed to belie her expressed concern for accuracy. The girls expended much effort on the task. They focussed, however, on the peripheral activities associated with reporting and presentation, rather than on mathematical activities. Jenny's leadership role, tendency to adapt others' "good" ideas masked her mathematically off task behaviours. She did, however, frequently engage in mathematical discussion, particularly in the presence of DM. Carol's behaviours were least consistent with her expressed beliefs. She was virtually uninvolved in the work at hand, simply observing or sent, by Jenny, to fulfil menial tasks. Her lack of involvement was exacerbated by the tendency for the others to work in same-sex pairs and marginalise her. DM was also implicated in that he did not encourage the others to include her nor did he pay much attention to her contributions when questioning the group on its progress. The signs did not appear to augur well for Carol's longer term commitment to mathematics.

Gender differences were also evident in both cognitive and affective engagement. Differential task engagement was obvious and might partially explain the gender differences in affective outcomes. The girls' greater confidence in the understanding of the work provides an intriguing anomaly. Since they had not actively engaged in the mathematical aspects of the project, the small group experience may have engendered apparently functional beliefs about themselves as learners of mathematics. In future, when required to tackle similarly challenging mathematics problems, their lack of experience may expose the fragility of their expressed 'confidence'.

What actually happened

Five years after the initial data were collected, the targeted students' mathematical paths through school were explored. Brian had left the school to attend another. The other four students, now in grade 12, were interviewed. The students were also asked to complete an information sheet on which they indicated the mathematics subjects they had taken at school, achievement in these subjects, the extra curricular mathematics activities they had participated in, and some background biographical data including number of siblings, place in the family and parents' occupations. A few months later, when published in the daily press, the tertiary places offered to the students were determined.

Our predictions about longer term involvement in mathematics were largely born out for the four students we were able to trace in grade 12.Their mathematics choices followed gender-stereotypic lines. In grade 12, Mark was the only one taking *Specialist mathematics*, the most demanding option offered in Victoria. His parents, both mathematics teachers, had strongly supported his selection of subjects. His decision to enter the Australian Mathematics Competition (AMC) was rewarded each year (1991-1995) with a Distinction. In grade 7 he said that he hoped he would be a Victorian Certificate of Education [VCE] mathematics student, although he was not sure what career he would follow. At the grade 12 interview, he intimated that he wanted to be an engineer. He was, in fact, offered a place in an engineering degree. In grade 7, Carol was less certain of being a grade 12 mathematics student but said that her mother would like her to do so. She, too, was unsure of her future career direction. Carol also gained a distinction in the AMC in grade 7, but not in other years. She took a less demanding grade 12 mathematics study, *Mathematical Methods*, and gave this as her least favourite subject. Nevertheless, she hoped to get into a combined arts/science course and indicated that history was a particular passion. As early as grade 7, Cheryl's uncertainty about VCE mathematics was evident. She said that she hoped she would continue but she had heard that VCE students were under a lot of pressure and felt that if she did not enjoy the mathematics or experienced a teacher who "doesn't really help much", she might not persist. At that time, she was enthusiastic about a career in chemistry. Like many other students at the school, she also sat for the AMC and consistently received a Credit certificate. Her grade 12 studies also included

Mathematical Methods. She had chosen it because "I think you need maths ... (and) I think they should make it a compulsory subject". After considering many career options she decided she would like to do accountancy and was in fact offered a place in a commerce course. Pointers to Jenny's grade 12 choices were also evident in grade 7. We recognised that she was a motivated, articulate and confident student. Yet, in grade 7 her efforts in mathematics during the monitored period were directed to peripheral, non-mathematical tasks, although she frequently told the boys (and the other girls) what they should do and checked that they had done it. On the self-report data, she had indicated that she found mathematics "fairly simple". Aware that her grade for the activity was linked to the group's output, she may have consciously directed her efforts to the report and its presentation. She gained a Distinction in the AMC in grades 7 and 8, a Credit in grade 10 and had not entered the Competition since then. Jenny's grade 12 interview revealed that she was an *International Baccalaureate* [IB] student. She said she preferred the IB because it was examination-focussed, she liked its structure and emphasis on working independently, and she felt she performed best under these conditions. Her IB course included mathematics, but not the "high level" mathematics subject. Her father, an ambulance para-medic, was her role-model, resulting in her desire to practice medicine. As a result of her grade 12 efforts, she was offered a place in a medical course.

Final Words

Clearly, there are many dangers in attempting to predict, through a snapshot of some eight consecutive lessons spread over two weeks, students' long-term motivations and achievements. Nevertheless, multiple source data including observations of students' cognitive and affective behaviours during mathematics lessons, gathered and analysed when the students were in grade 7, seemed remarkably prophetic of their grade 12 mathematics subject choices and tertiary destinations. While we do not wish to ignore or minimise the many other factors, inside and outside the classroom and school context, which can influence subject and career choice, it appears that intensive observations of students can yield early indictors of eventual pathways. What can we, as mathematics educators do with this knowledge, to ensure that *all* students realise their full potential?

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Notes

1. Fictitious names have been used for the students and the teacher