The Potential of Computer Manipulatives for Overcoming Place Value Misconceptions

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The difficulty in teaching and learning place value concepts is well documented. One suggestion to overcome this is to use computer manipulatives to help students "bridge the gap" between symbols and concrete materials. A teaching experiment investigated Year 3 students' place value learning using either conventional or computer-generated place value blocks. Observations of one lower-achievement student using computer software indicate that well-designed mathematics software offers advantages to teachers for overcoming students' misconceptions not possible with conventional materials.

Background

The importance of place value in the primary mathematics curriculum and the difficulty teachers experience in developing place value concepts in their students are well documented. As Resnick (1983) stated,

> The initial introduction of the decimal system and the positional notation system based on it is, by common agreement of educators, the most difficult and important instructional task in mathematics in the early school years. (p. 126)

Ross (1990) likewise noted that "children find place value difficult to learn and teachers find it difficult to teach" (p. 13). One reason for this mutual difficulty is that, as Skemp (1982) pointed out, mathematics' "[conceptual structures] are purely mental objects: invisible, inaudible, and not easily accessible even to their possessor" (p. 281). Thus teachers are often not aware of what their students are thinking of with regard to mathematical objects. To answer questions about students' mental representations, researchers investigating place value understanding generally use observations of participants' behaviour to posit "various cognitive structures and processes believed to produce the behavior" (Putnam, Lampert & Peterson, 1990, p. 65). When observed behaviour reveals misconceptions about place value concepts, models of faulty internal structures can be proposed.

Misconceptions of Multidigit Numbers

The place value literature "is replete with studies identifying children's difficulties learning place-value concepts" (Jones & Thornton, 1993, p. 12). Lack of space precludes a detailed description here of the misconceptions shown by students; as Sinclair and Scheuer (1993) noted, misconceptions of place value concepts held by children are "very diverse" (p. 200). One misconception in particular, called the "face value" construct (Ross, 1989) has been reported by many researchers, and is considered in some detail here.

The "face value" construct: Though it is efficient to compute answers to multidigit questions *as if* each digit was a single unit, many children apparently believe that each digit *actually represents* only its face value. One set of tasks used by researchers to identify students using this construct is "digit correspondence" tasks (Miura & Okamoto, 1989; Ross, 1990). Participants in a number of studies were asked to show what was represented by each digit in a two-digit number showing the number of objects in a collection. Students operating from the face value construct would indicate unit objects as representing each digit, rather than tens and ones (i.e., a student might represent 43 as a set of 4 ones and a set of 3 ones).

This faulty conceptual understanding of multidigit numbers has been observed by researchers who were investigating a variety of mathematical abilities. These included counting (Cobb & Wheatley, 1988), representations of two-digit numbers (Miura & Okamoto, 1989; Ross, 1989, 1990), comparison of pairs of two- and three-digit numbers (Sinclair & Scheuer, 1993), handling two- and three-digit numbers in novel problem-solving exercises (Bednarz & Janvier, 1982), and completing written algorithms (Fuson & Briars, 1990; Kamii & Lewis, 1991). The presence of this construct points to a critical misconception of multidigit numbers, but one that may be difficult to detect (Ross, 1990). It was found to be present in the thinking of one participant in this study, reported below.

Teaching Place Value Concepts

There is widespread support for the idea that the key to the development of higher level conceptual structures for numbers is teaching students to make connections between numbers and their referents (Fuson & Briars, 1990; Hiebert & Carpenter, 1992). Traditionally these connections have been made via concrete materials: MABs have been used for over thirty years now (Dienes, 1960) as the predominant place value materials in many countries, including the USA, the UK, New Zealand and Australia. Yet despite the popularity of MABs among teachers and academics, research shows learning effects from their use are equivocal (Hunting & Lamon, 1995). Hart (1989) summed up an idea that a number of authors seem to share:

> Many of us have believed that in order to teach formal mathematics one should build up to the formalization by using materials, and that the child will then better understand the process. I now believe that the gap between the two types of experience is too large, and that we should investigate ways of bridging that gap by providing a third transitional form. (p. 142)

Several authors have recommended the use of computer manipulatives for bridging this gap between concrete materials and symbols (Champagne & Rogalska-Saz, 1984; Clements & McMillen, 1996; Thompson, 1992). However, there is little research that investigated precisely what the advantages or pitfalls of using computer manipulatives for place value teaching may be, and how they compare with standard MABs. The present study is planned to address this gap in the literature. A software application (Price, 1997) designed to help students develop accurate models for multidigit numbers was used in this study with one cohort of students, and MABs with another.

This study investigates the use of two different forms of representations for multidigit numbers, computer-generated and standard place value blocks. The aims of the research are: (a) to identify the conceptual structures for numbers held by the participants; and (b) to look closely at the interactions between individual students' conceptual structures and their use of either conventional MABs or an innovative software application. Specifically, we are interested in whether either material will *support* or *confirm the validity* of accurate models of multidigit numbers, and on the other hand, whether they will *challenge* or *contradict* students' misconceptions.

Research Design

Research Questions

Two broad questions are being addressed in this research:

a) Are there differences in children's development of conceptual structures for multidigit numbers when using two different representational formats (place value blocks and computer software)?

b) What differences emerge as children of different ability groups learn place value concepts using two different representational forms and associated processes?

Assumptions

A number of assumptions underlie this research. First, whilst deliberately avoiding the troublesome question of what numbers actually are, and where they exist, it is assumed that most people will by their actions treat numbers as entities, and so for them numbers do exist in some form (Sfard, 1991; Sfard & Thompson, 1994). Second, the conceptions that a person holds for numbers form a system that has a structural form; the system incorporates rules by which the conceptions may be manipulated (Ohlsson, Ernst & Rees, 1992; Resnick, 1983). Third, there is some relationship between internal and external representations of numbers (Hiebert & Carpenter, 1992; Putnam et al., 1990). Fourth, the nature of internal representations of numbers may be deduced from a participant's responses to particular mathematical tasks (Resnick, 1983, 1987).

Method

The basic methodology used for this study was a teaching experiment (Cobb & Steffe, 1983). Ten daily teaching sessions were used, with the author taking the role of teacher for all participants. A pretest and posttest developed for this study were used to analyse specific features of place value knowledge held by each student. Participants worked either in pairs or singly, and used either conventional MABs or a computer model of MABs. Both types of material were used as a teaching tool with an experienced teacher; it is not assumed that the materials themselves will be effective in improving children's ideas of numbers. As Hunting and Lamon (1995) pointed out, the mathematical structure of multidigit numbers is not contained in any material. Learning can only take place when the "[learning] experience is meaningful to pupils and . . . they are actively engaged in thinking about it" (Baroody, 1989, p. 5).

Software design: The software application has been designed to provide up to five different representations of any whole number up to 999 that may be switched on or off by the user. The software will represent a number as: (a) a standard arrangement of hundreds, tens and ones blocks; (b) a written symbol; (c) a numeral expander that may be opened or closed at each place; (d) an audio recording of the spoken number name; or (e) a variety of non-canonical arrangements of blocks (i.e., with more than nine in a place). Though there are some features of MABs that are *not* present in the software, educational software offers a number of advantages over conventional concrete materials that cannot be disregarded (Clements & McMillen, 1996). In particular, Clements and McMillen noted:

> Actual base-ten blocks can be so clumsy and the manipulations so disconnected one from the other that students may see only the

> trees—manipulations of many pieces—and miss the forest—place value ideas. The computer blocks can be more manageable and "clean". (p. 272)

It is hypothesised that by making multiple representations of a number available to a student, the software will aid children in the development of useful, accurate conceptual structures for multidigit numbers.

Participants

Participants in the study reported here were selected from the population of Year 3 students (aged 7-8 years) at a primary school in a small rural town north of Brisbane. Participants were selected at random from two pools: students of either high or low mathematical achievement, based on the previous year's Year Two Diagnostic Net, a state-wide test used in Queensland to identify students at risk in the areas of literacy and numeracy. Students in pairs were matched for gender and ability, on the assumptions that at this age children will prefer to work with same-gender partners, and that maximum learning would be possible if both children were of similar mathematical ability. Results for only one child are reported here, to illustrate how the specific features of the software enabled her to develop a more accurate understanding of place value.

Case Study: Nina

One participant in the study reported here demonstrated a number of key points already alluded to in this paper. Nina (not her real name) was identified by the Year Two Net as being in need of remedial help in mathematics and language.

The pretest demonstrated that Nina had severe misunderstandings of numbers, and showed great confusion about the meanings applied to symbols and concrete representations of numbers, and about the magnitude of numbers. It was evident from several of her responses that Nina was operating according to a face value concept of multidigit numbers. For example, when asked to show a two-digit number (43) with MABs, she showed 4 tens and 3 ones without hesitation. However, when asked to "show 43 another way", she exchanged the numbers of blocks, and showed 4 ones and 3 tens. When shown 7 hundred blocks and 3 one blocks, she simply counted the blocks as individual entities, and stated that the represented number was 10.

Throughout the ten teaching sessions Nina continued to demonstrate evidence of the face value construct. When considering two-digit numbers, she frequently used the method of adding the face values of the number to decide its value. For example, she stated that 39 was "twelve altogether", and that 72 was "nine". The following excerpt from a teaching session illustrates some of the confusion she had in understanding symbols for two-digit numbers:

I - Nina, can you tell me what these two numbers [36 & 63 written on paper] are?

S - 66 *[puts hand on mouth]* Oh, 36, 63. They're both the same, because the 3, the 3 is at the back, and the 3 is at the front, and [points] the 6 and the 6, and the 3 and the 3.

I - Well, Nina, I was going to ask you which one is bigger?

S - Oh, which one is bigger? 36 is bigger than 63.

I - How do you know 36 is bigger?

S - [indistinct; holds up both hands with fingers outstretched] because . . . [waves hands in the air] I don't know.

Though Nina frequently changed her mind in answering questions, her fundamental (faulty) conceptions were very robust, and would stand up to sustained challenges from the interviewer. One of her beliefs that seemed to reinforce her misunderstandings about tens and ones was her claim that numbers are different in New Zealand! She evidently started pre-school in New Zealand, and had deduced from her experiences on either side of the Tasman that numbers were different in the two countries.

The interviewer used the idea that misconceptions may be overcome through challenges presented by a teacher (Ginsburg, 1981). Ginsburg used the technique of testing a participant's "strength of belief" by challenging the child when he or she seems uncertain or hesitant in the response to a question. The two techniques recommended by Ginsburg to test strength of belief were "counter-suggestion" and "repetition of the problem or introduction of highly similar problems" (p. 10). Both these techniques were used to challenge Nina's beliefs about two-digit numbers. The computer software was used to augment these techniques, by presenting a number of representations of numbers that countered the participant's assertions. For example, when asked to compare written symbols for 48 and 73, Nina was certain that 48 was larger, saying:

S - That's got 7 and 8 is more than 7, and 4's more than 3 so that *[48]* is the highest number and that*[73]*'s the smallest.

Though the interviewer repeatedly attempted to show her that the computer-generated block representation showed that 48 was smaller, she did not accept it. She persisted in her belief that 48 was larger even when an on-screen numeral expander was used to show that 73 was 7 tens and 3 ones, or 73 ones (Figure 1).

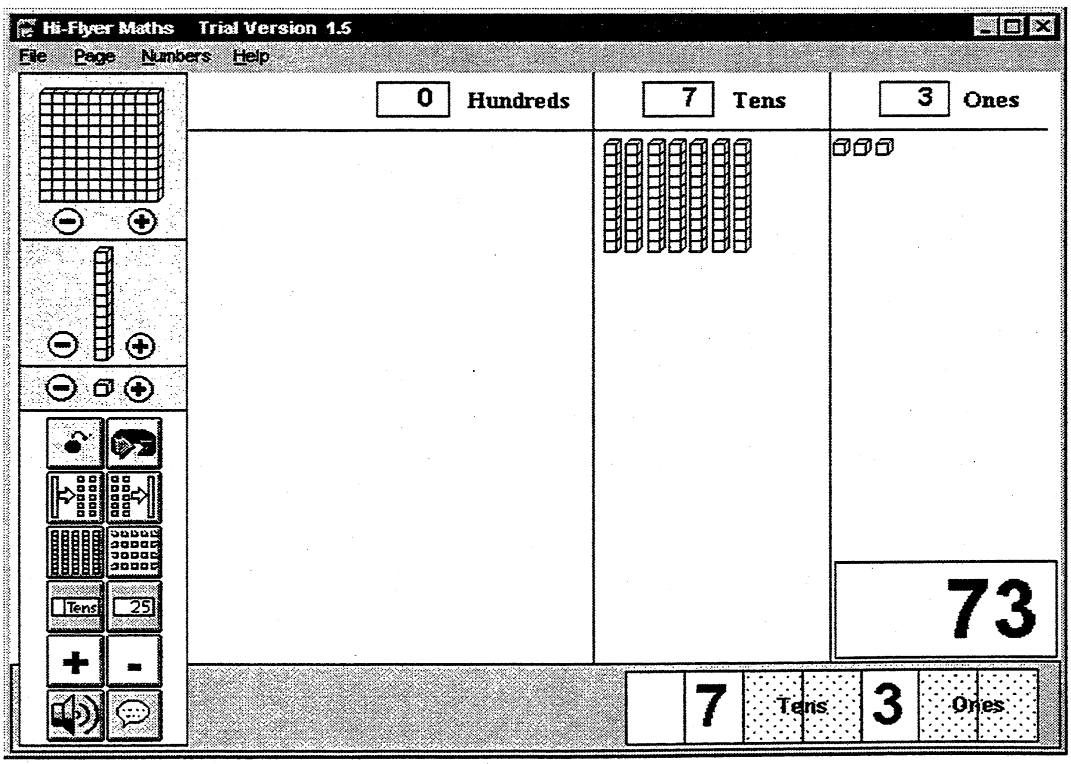

Figure 1 Screen shot of *Hi-flyer Maths* showing numeral expander and conventional block representation of number

Some success was achieved when another aspect of the software was accessed. The software includes a "show as ones" feature, which presents a number solely as ones blocks. When this feature was used, Nina was able to count the ones in 73 and see that it included more than 48 ones (Figure 2). The following conversation took place:

I - Stop. That one there is number 48 isn't it?

S - Yep. I said 48.

I - And all these over here are still to come. [points to remaining ones in 73]

S - Ahh...

I - Now remember when we looked at 48 you said 48 was smaller than 73.

S - Yes.

I - Have a look up here. If we count the first 4 rows and the next 8 here, that's 48. And we've still got all these here. [points again]

S - Oh, I think I'm wrong. I think that's higher. *[points to written 73]*

I - I think that's higher too. [smiles]

S - I was wrong.

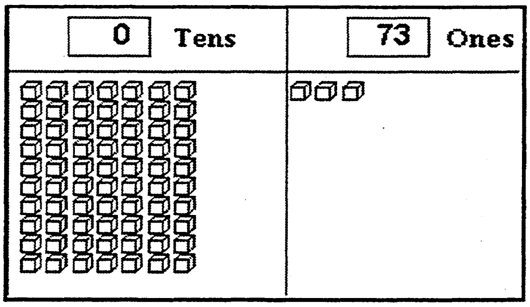

Figure 2 Section of screen showing "tens as ones" representation

It would be pleasing to report that this incident led to an immediate change in Nina's conceptual structures for two-digit numbers. However, the posttest revealed that many of her confusions remained. For example, after counting 36 pop sticks and asked to show which sticks were indicated by the two digits in the number 36, she put out a group of 3 and a group of 6 sticks, indicating that she was still operating according to the face value construct. On the other hand, there was some indication that she had a different understanding of two-digit numbers as a result of the teaching program. When asked in the posttest to compare 61 and 39 she stated that 61 was larger, because of the number of tens.

It seems reasonable to make the conjecture that for a child such as Nina there would need to be many more sessions where her misconceptions were challenged, and correct conceptions reinforced.

Discussion

In the situation described above the software enabled the interviewer to persist in presenting several counter-suggestions to Nina's assertions about the numbers 48 and 73, until she found that her belief could not be sustained in the face of contrary evidence on the computer screen. While that sort of process is possible with physical blocks, it is doubtful whether it would be as effective, for the reasons given by Clements and McMillen (1996) above.

The software offers a teacher great versatility in presenting several representations of a number simultaneously, as needed. In this instance the "show as ones" feature was accessed in order to challenge Nina's belief that tens were single entities, rather than collections of ten ones. In different session with a different child, another feature may be useful for illustrating an instructional point. The principal rationale behind the software design is that connections among numbers, symbols and other referents are of critical importance in learning place value concepts (Hiebert & Carpenter, 1992). By offering the teacher (and student) a range of available options in representing a number, the software provides a level of versatility in demonstrating connections that is not possible with conventional physical place value blocks.

Salomon (1979) noted that each form of technology offers a different set of "affordances" that set it apart from other technologies; we would extend this idea to individual software applications. This piece of software offered the particular advantage of presenting multiple representations for a number under consideration, which enabled the interviewer to lead the student to confront her own misconception. Without the specific features built into the software, or ones like them, it is likely that Nina's misconceptions may have been unaltered during the teaching session.

The study reported here is still in an early stage, and much data collection and analysis are still to be done. Nevertheless we believe that the evidence from these early observations indicates specific strengths that computer software can have for the teaching of the abstract concepts of mathematics, particularly in the area of place value.

Well-designed mathematical software enables a teacher to draw on resources that otherwise might not be available at all. We assert that software for the teaching of mathematics should be designed with these affordances in mind. The other factor of crucial importance in this endeavour is a clear model of how place value concepts are developed in children's minds; this research is hoped to offer new insights in this area.

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