Which jar gives the better chance? Children's decision making strategies.

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In task-based interviews 48 Kindergarten to Year 6 children were asked to choose between two jars containing different mixes of red and yellow toy bears, with the aim of giving themselves a better chance at drawing out a red bear. The children applied a variety of strategies, ranging from idiosyncratic reasons to proportional reasoning. These strategies are examined in relation to the ratio pairs presented in each jar and are compared to other strategies reported in the literature.

Probability can be defined as being "..composed of two subconcepts: chance and proportion." (Falk, Falk & Levin, 1980; p183). It is necessary to be aware of uncertainty in a situation so that proportional reasoning can be appropriately applied. Some studies have established that young children, with under-developed proportional thinking, are able to apply non-numerical or estimation strategies to make appropriate probability judgements (Spinillo, 1995; Falk, Falk & Levin, 1980). Fischbein, Pampu & Manzat (1970) advocate that, with appropriate instruction, young children can learn to develop their intuitive strategies into proportional reasoning. Their research revealed that 9 and 10 year olds were particularly responsive to the instruction that involved the use of discrete objects to model ratios and to form subgroups to assist in the comparison of ratios. However, other researchers argue that proportional reasoning is slow to develop and is not consistently used until formal operational thinking is well established (Piaget, 1975; Green 1983, Lovell & Butterworth, 1966), although there is disagreement about the age at which this occurs. Perhaps the type of research most useful to mathematics educators is the type that provides some insight into the strategies that children use in probabilistic decision making as they gradually develop proportional reasoning.

The classic 'two-urn-choice task', introduced in Piaget's studies has often formed the basis of research tasks for exploring proportional thinking. In such tasks, the child is shown two containers holding different mixes of two colours of small objects. The target, or 'favourable colour' is specified, and the child is asked to choose the container that would provide the better chance of drawing out the target colour without looking. According to Piaget (1975), applying proportional reasoning to such a task would require one of two approaches: constructing a ratio for each container that compares the number of each colour; or constructing a probability of the specified outcome occurring with each container. The first approach involves examining the part-part relationship within each sample space. The second approach involves part-whole thinking. Also of importance is the conservation of ratio and proportion, which enables us to work with equivalent fractions and recognise the equality of two ratios (Falk, Falk & Levin, 1980; Lovell & Butterworth, 1966).

There are, however, other appropriate strategies that require the understanding of proportions, but do not require precise calculations. Behr, Harel, Post and Lesh (1992) refer to research investigating children's qualitative reasoning when dealing with order relations between fractions and between ratios. The problems they presented to children were based on the equation $a/b = c$. Information about the relationship between two of these variables, without allocating specific numerical values, was provided to the children, who were required to determine the nature of the effect this relationship would have on the third variable. For example, if a remains the same and b decreases, then the value of c would increase. The problems also required comparison between two sets of relationships; $a_1/b_1 = c_1$ and $a_2/b_2 = c_2$, not unlike the two-urn-choice' task referred to previously. For example, if a_1 is less than a_2 and $b_1 = b_2$, which is greater c_1 or c_2. The authors refer to the orange juice mixture tasks used in Noelting's (1980) research, and to Siegler's (1976) masses on a balance beam tasks, to illustrate the differences between two main categories of task type; invariance of ratio and invariance of product. These categories are each further split into two subcategories; find-product-order and find-factor-order, and find-rate-order and find-rate-quantity.

These categories and subcategories are relevant to two-urn-choice studies in several ways. The children are asked to make a comparison between two sets where a is the number of target-colour objects and b is the number of other-colour objects. Some children choose to focus on the relationship between a_1 and a_2 , or on the relationship between b_1 and b_2 or to determine c_1 and c_2 . Even though the children are presented with specific numbers for each variable, it is possible to produce a 'correct choice' and valid explanation by making qualitative judgements to determine the direction of a relationship rather than calculate precise relationships. In some tasks, depending on the colour-colour ratios, additive or subtractive strategies may be all that is required for a successful choice, while other tasks may demand multiplicative thinking to support an appropriate choice.

Method

The particular piece of research reported in this paper is part of a larger study using task-based interviews to investigate children's understanding of basic probability concepts prior to formal instruction. The main purpose of the task reported here was to examine the levels, and types, of proportional reasoning being applied by the children. The total sample was drawn from two different NSW schools, one a country school (sample J) and the other from the western fringe of the Sydney metropolitan area (sample S). K-6 teachers were asked to provide four 'average' ability children from each grade who would be reasonably comfortable in a one-to-one interview situation within the school. Some of these children may have experienced some lessons dealing with basic probability concepts, but because this is not yet part of the NSW Primary Mathematics Syllabus, they will not have received any systematic instruction. Table 1 shows some details about the sample.

Age Group	Male	Female	Total
5/6 yrs	7	7	14
7/8 yrs	4	8	12
9/10 yrs	9	3	12
11/12 yrs	5	5	10
Total	25	23	48

Table 1: Games 1 to 4 Samples J and S

Age Group	Male	Female	Total
5/6 yrs	5	4	9
7/8 yrs	3	3	6
9/10 yrs	4	2	6
11/12 yrs	2	2	4
Total	14	11	25

Table 2: Games 5 to 7 Sample S

All the children played a 'warm-up' game to familiarise them with materials and procedures. The task took the form of a game, where the goal was to choose the jar of small plastic red and yellow bears that would give a better chance of picking out red bears without looking. The contents for each of the two jars were first lined up in front of the jars, in a line of red and a line of yellow. The question asked was: "Will one of these jars give a better chance for picking out red bears, or do they both give the same chance?" An explanation of the reason for the choice was sought. The bears were then placed into the jar/s chosen and five draws, with replacement, were made by the child. A reward of 'smarties' (tiny chocolate & candy) was given to the child according to the number of red bears drawn.

The main interview set, common to the children from both schools, contained four such games. The children from the second school (25 children) were given an additional three games which involved different types of colour mixes. The actual ratios of red and yellow used are presented in the column headings of Table 9. Each interview was tape recorded and transcribed into a data base.

Results

The choices made by each child in each game were determined to be either correct or incorrect based on the calculated probability for each jar. A reason was deemed to be 'correct' if it used the information provided by the contents of the jar (sample space) appropriately. Tables 3 and 4 contain these results in the form of percentages for each age group for the four main games, and for the additional three games. Tables 5 and 6 provide similar details for each separate game. In the overall results there is an obvious progression with age and a clear difference between the number of correct choices and 'correct' reasons. However, Games 1 and 3 produced some surprising results in the 5/6 year old age group, where these children performed better than older children. A possible explanation for this emerges when the details of the actual tasks and the choice strategies used by children are examined.

Age Group	Correct Choices	Correct Reasons
5/6 yrs	62.5	21.4
7/8 yrs	67.4	20.0
9/10 yrs	83.3	33.3
11/12 yrs	85.0	58.3

Table 3 : Percentages correct. Games 1 to 4

Age Group	Correct Choices	Correct Reasons
5/6 yrs	70.4	14.8
7/8 yrs	66.7	22.2
9/10 yrs	88.9	16.7
11/12 yrs	100	16.7

Table 4: Percentages correct. Games 5 to 7

	Game 1		Game 2		Game 3		Game 4	
	Choice	Reason	Choice	Reason	Choice	Reason	Choice	Reason
5/6 yrs	92.9	78.6	21.4	0	92.9	0	35.7	0
7/8 yrs	83.3	58.3	41.7	16.7	63.6	0	81.8	0
9/10yrs	83.3	83.3	58.3	33.3	75.0	16.7	91.7	16.7
11/12yr	100	100	70.0	66.7	70.0	44.4	100	22.2
TOTAL	87.5	76.6	45.8	25.5	76.6	12.8	72.9	8.5

Table 5: Percentage Correct each Game. Games 1 to 4.

	Game 5 Choice	Reason	Game 6 Choice	Reason	Game 7 Choice	Reason
5/6 yrs	88.9	0	44.4	0	77.8	44.4
7/8 yrs	66.7	0	66.4	0	66.7	66.7
9/10yrs	83.3	0	83.3	0	100	75.0
11/12yr	100	0	100	0	100	75.0
TOTAL	84.0	0	76.0	0	84.0	56.0

Table 6: Percentage Correct each Game. Games 5 to 7.

Closer examination of the reasons given by the children for their choices revealed five distinct categories, with a further six subcategories. There was some indication that a larger sample of children may have provided opportunity for creation of a few more subcategories. These categories are explained below.

Reasoning Categories

1. Idiosyncratic

a) *No reference to the number* of bears in either jar: For example: Samantha (5 yrs 2 mths), "It's the jar that won before."

b) *Reference to number* is made but does not refer to any comparison of colours, contradicts the choice or makes no sense to the researcher. Sally (5yrs 6mths), " The same. There's 1R and 2R there."

2. Comparison of Favourable Events.

The children refer to the sample spaces but are only really concerned with comparing the number of favourable colours (Red) in each jar. The jar with more Reds is chosen. For example, John (9yrs 1mth) "This has got 3 red, that's got 1 red. It's a better chance having the red."

3. Comparison of Unfavourable Events

The children refer to the sample spaces but are only really concerned with comparing the number of unfavourable colours (Yellow) in each jar. The jar with fewer Yellows is chosen. For example, Sinclair (8yrs 11mths), "There's less yellow than the other one."

4. Subtractive Comparison

The difference between the number of Reds (favourable) and Yellows (unfavourable) in each jar is determined and compared. The jar with the smaller difference is chosen. Sometimes the child clearly states the numerical differences in their explanations, and sometimes the comparison is less specific. For example, Jason (9yrs 11mths), "They're the same because that one's got 3 more yellow (than red) and that one's got 3 more yellow (than red)."; and Sven (9yrs 4mths), "You have more red than the yellow (in the 2nd jar). In this one (1st jar) you have nearly exactly the same (amount of red and yellow)."

5. Proportional Comparison

To fit into this category a response had to indicate that an attempt had been made to define the relationship between the number of Red and Yellow objects in each jar, and that this relationship formed the basis for comparison of the two sample spaces. Within this category the range of responses can be further sorted into subcategories. Although the order in which the subcategories are listed suggest a gradual increase in the level of sophistication of reasoning, this may not necessarily be the case for all responses. This is because there is some evidence that the nature of certain ratios used in the interview prompted certain strategies.

a) Multiplication (including doubling by addition): In some cases fractions were not constructed but multiplication was used in the reasoning. For example; Justin (9yrs 6mths), "2 red against 4 yellow (in 1st jar), and this one (2nd jar) it's got double the red (of the 1st jar) and double the yellow."

b) Approximation: Some children used approximations to express the comparison they had made. Once either the balance or the direction of the imbalance had been determined, sufficient information was available for making an appropriate decision without determining a specific numerical relationship. For example, Jane (11yrs 5mths), "There's 1 red and 3 yellow (in 1st jar). There's more yellow on this side (in 2nd jar) - because there's only 2 red and there's 8 yellow it's not even a half, it's probably only a quarter." (Note: This strategy illustrates the qualitative reasoning reported by Behr, Harel, Post and Lesh (1992)).

c) Component Ratios: A small number of children used component ratios, or subgrouping, to define the relationship between the numbers of colours in each sample space. In the following example a child is trying to explain that one jar containing 2 red and 4 yellow objects presents the same probability for drawing a red as the second jar containing 4 red and 8 yellow, because the objects in the second jar can be physically divided into two groups of 2 red and 4 yellow; Julie (12yrs 3mths), "2 fours go into to 8 and that's 4 ..um..it's the same amount, 2 there, 2 there. (In the 1st jar) those 2 (red) match (with 2 yellow) and there's 2 again (total of 4 yellow). (In the 2nd jar) 4 (red which she separates into pairs) match 4 (yellow) and there's 4 (yellow) again." (Note: This is the strategy used in Fischbein, Pampu & Manzat's (1970) instructional study).

d) Fractions, actually part-part ratios: Some children constructed precise fractions from the number or red and yellow objects in each container, then compared the fractions and chose the 'larger' fraction. No child constructed a part-whole probability fraction. For example, Jack (11yrs 8mths), "'Cause that's half and that's only quarter."

Strategy Category	5/6 yrs	7/8 yrs	9/10 yrs	11/12 yr
1. Idiosyncratic	24.1	24.2	7.6	4.2
2. Comparing favourable	53.0	32.3	18.2	14.6
3. Comparing unfavourable	6.0	27.4	40.9	8.3
4. Subtractive	16.8	25.8	42.4	41.7
5. Proportional	0	1.6	18.2	25.0

Table 7: Percentage of children in each age group using each strategy. Games 1 to 7.

Of interest is the tendency for 5 to 8 year olds to apply strategies 1 and 2, and the strong similarity of the figures for the 9/10 yrs and 11/12 yrs groups, except for Strategy 3. There is quite a clear increase in the use of proportional reasoning with age. It is also worth noting that, with the exception of 5/6 year olds and Proportional Reasoning, children of all age groups used the full range of strategies. However, very few children applied the same strategy to all games.

Strategy Category	1	2	3	4	5	6	7
1. Idiosyncratic	8.7	23.9	10.9	19.6	12	24	12
2. Comparing favourable	84.8	17.4	50	13	32	8	12
3. Comparing unfavourable	0	10.9	6.5	34.8	0	4	4
4. Subtractive	6.5	17.4	17.4	23.9	56	64	72
5. Proportional	0	30.4	15.2	8.7	0	0	0

Table 8: Percentage of children using each strategy in each game.

The top of Table 9 shows the ratio of red and yellow bears in each jar, with the first vertical pair referring to the first jar and the second vertical pair referring to the second jar. The remainder of the table indicates whether application of a particular strategy would result in the correct jar selection. It can be seen that although Proportional thinking is appropriate for all games, there are less demanding strategies that lead to correct choices in all games except Game 2.

The results in Table 9 suggest that certain tasks are more likely to stimulate particular strategies than others. For example, Game 2 produced more proportional reasoning than any other, while Games 1, 5, 6 & 7 produced none. The relationship between the type of ratios used in each game and the type of strategies used warrants further examination.

	Game 1 1R 3R 4Y 4Y	Game 2 2R 4R 4Y 8Y	Game 3 1R 3R 4Y 6Y	Game 4 1R 2R 3Y 8Y	Game 5 3R 5R 2Y 3Y	Game 6 4R 6R 2Y 5Y	Game 7 3R 3R 5Y 7Y
1. Idiosyncratic a) No number	Not approp.	Not approp.	Not approp.	Not approp.	Not approp.	Not approp.	Not approp.
b) Number	Not approp.	Not approp.	Not approp.	Not approp.	Not approp.	Not approp.	Not approp.
2. Comparing favourable	Correct choice	Wrong choice	Correct choice	Wrong choice	Correct choice	Wrong choice	Not approp.
3. Comparing unfavourable	Wrong choice	Wrong choice	Wrong choice	Correct choice	Wrong choice	Correct choice	Correct choice
4. Subtractive	Wrong choice	Wrong choice	Wrong choice	Correct choice	Wrong choice	Correct choice	Correct choice
5. Proportional a) Multiplication	Correct choice	Correct choice	Correct choice	Correct choice	Not approp.	Not approp.	Correct choice
b) Estimation	Correct choice	Correct choice	Correct choice	Correct choice	Correct choice	Correct choice	Correct choice
c) Component ratios	Correct choice	Correct choice	Correct choice	Correct choice	Correct but hard	Correct but hard	Correct but hard
d) Ratio construction	Correct choice	Correct choice	Correct choice	Correct choice	Correct but hard	Correct but hard	Correct but hard

Table 9: The potential of each strategy in producing a correct choice in each game.

Discussion

The main focus of this paper is to report on the use of various strategies employed by the children in problems involving pairs of ratios. There have been a number of studies conducted using simular problems that have also revealed a variety of response categories These include Chapman, 1975; Fischbein, Pampu & Manzat, 1970; Green, 1983; Lamon, 1993; Siegler, 1986; Truran, 1992. The categories reported by these researchers range from being too narrow to include all the categories found in this study (Siegler, 1986; Fischbein et al.,1970), to being too finely divided to be of great practical use (Truran, 1992). The strategies listed by Lamon (1993) perhaps supply the best compromise between these extremes. There are, however, sufficient overlaps across all the reported categories to lend support to the idea that there is a set of consistently identifiable proportional reasoning categories. Table 10 provides a summary of the overlaps between some of these studies, particularly in reference to the categories found in this study. An expansion of this table to include the examination of several other significant studies should provide sufficient information to propose a stable set of categories that could become a very useful resource for formulating specific research tasks and informing educators on beneficial instructional approaches.

Even though not all types of ratio pairs were included in this study it is obvious from the results that particular ratio pairs are likely to prompt particular reasoning strategies. Comparing the information in Tables 8 and 9 provides support for this idea. For example, in Game 1 it was only necessary to compare the favourable colours (red) in each jar to make a correct choice, and 84.8% of the children used this strategy. In contrast, Game 2 required some form of Proportional reasoning for a correct choice and 30.4% of the children used this strategy; the highest percentage of use across all games. Games 5, 6 and 7 involved ratios that were difficult to compare proportionally because they relied on one's ability to create equivalent fractions. Consequently, even children who had used Proportional reasoning in Game 2, relied on the less appropriate and less reliable Subtractive strategy.

In general, the outcomes of this study suggest that children as young as 5 years possess a repertoire of strategies to select from in reaction to the type of ratio pairs presented to them. These reactions may disguise an underlying developmental sequence to these strategies. Closer examination of the interview transcriptions may reveal response patterns in individual children, that may give further insight into the nature of these reactions. Several children possessed powerful intuitive or qualitative strategies that also warrant closer investigation. There were also hints of some interesting intuitive strategies in the Idiosyncratic category, particularly amongst the younger children who had some difficulty in verbalising their decision making processes. Again, closer examination of the raw data may reveal response patterns. These investigations, together with a completed synthesis of existing research on the emergence of proportional strategies (such as begun in Table 10), could provide the foundation for designing a revealing research project on children's reasoning in two-choice probability tasks.

References

Chapman, R. (1975). The development of children's understanding of proportions. *Child Development, 46,* 141-148.

Behr, M., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio and proportion. In D. Grouws (ed) *Handbook of research on mathematics teaching and learning,* (pp. 296-333). New York: Macmillan

Falk, R., Falk, R., & Levin, I. (1980). A potential for learning probability in young children. *Educational Studies in Mathematics, 11,* 181-204.

Fischbein, E., Pampu, I. and Manzat, I. (1970). Comparison of ratios and the chance concept in children. *Child Development,* 41, 377-389.

Green, D. (1983). From thumtacks to inference. *School Science and Mathematics, 83(7),* 541-551.

Lovell, K., & Butterworth, I. (1966). Abilities underlying the understanding of proportionality. *Mathematics Teaching, 37,* 5-9.

Lamon, S. (1993). Ratio and Proportion: Connecting Content and Children's Thinking. *Journal for Research in Mathematics Education 23*: 1, 41-61.

Noelting, G. (1980). The development of proportional reasong and the ratio concept, part 1 - Differentiation of stages. *Educational Studies in Mathematics Education, 11,* 217-254.

Piaget, J. & Inhelder, B. (1975) *The Origin of the Idea of Chance in Children.* London: Routledge & Kegan Paul. (Original work published 1951).

Siegler, R. (1976). Three aspects of cognitive development. *Cognitive Psychology, 8,* 481-520.

Siegler, R. (1978). *Children s Thinking.* Prentice Hall: Englewood Cliffs.

Spinillo, A. (1995). Can young children learn how to reason proportionally? An intervention study. In In L Meira & D. Carraher (Eds). *Proceedings of the Nineteenth International Conference for the Psychology of Mathematics Education.* Vol.3 (pp.192-199) Recife, Brazil.

Truran, J. (1992) *The Development of Children's Understanding of Probability and the Application of Research Findings to Classroom Practice.* Masters Thesis: University of Adelaide.

Table 10: Summary of commonalities between reported strategies.

Way 97	**Lamon 93**	**Truran 92** *	**Siegler 86**	**Green 83**	**Chapm**
	Avoiding	Strategy 1		Non-response	
Idiosyncratic a) No number	Visual or			Other, and "They are the same"	0 Rating No quan relations
b) Number	additive	Strategies 2 &5.		Comparison within bag	1 Rating Referenc number
Comparison of favourable		Strategies 7 & 8 Partly Strategy 17	Rule 1	Comparison between bags	2 Rating
Comparison of unfavourable	Preproportional reasoning	Strategies 9 & 10	Includes Rule 2	of one colour	Compari colour
Subtractive comparison		Strategies 14 & 15.	Rule 3	Difference comparison	
Proportional a) Multiplication	Qualitative proportional	Partly Strategy 17			3 Rating Both col
b) Estimation	reasoning. (Pattern Building?)	Strategies 12 & 13 & 16	Rule 4	Ratio comparison	and both containe
c) Component ratios	Quantitative proportional	Strategy 11.			compare
d) Construction of ratios	reasoning	Strategies 18 & 19			

*Several of Truran's categories have not been included because they did not correspond sufficiently with other identified categories.