The effective use of calculators in the teaching of numeracy

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Current debates about numeracy in the UK focus on the availability of calculators but although standards of numeracy are poor, they have not changed significantly since pre-calculator days. It is the way numeracy is taught which is the problem, with or without calculators. Effective use of calculators should be associated with the concurrent development of students' number sense and mental methods. Calculators can enhance the teaching and learning of mathematics if used as a focus for discussion and reflection on learning.

Definitions of numeracy vary from the "mathematical literacy" of the Crowther report (1959) to the more mundane emphasis on basic arithmetical computation implied by current UK government proposals (Reynolds, 1998). To be numerate suggests both an ability to cope with the practical mathematical demands of everyday life and also an "at-homeness" with number (Cockcroft, 1982) which requires an understanding of the mathematical potential of situations. Numeracy thus demands not only a familiarity with number facts, associated processes and an appreciation of the interconnections between them but also an awareness of the power of number and a predilection to seek numerical explanations to situations.

The Mathematical Association (1992) suggested that a numerate person has "the ability to solve simple everyday problems involving number, by using effectively the knowledge and skills that they possess" and that this effective use should include being able to choose and to devise appropriate strategies for calculations (ibid, p71). Numeracy here requires both mathematical knowledge and skills and, in addition, an awareness of this knowledge base so that effective choices can be made. The choice of an effective strategy for a problem is dependent not only on the knowledge which has been learned but also on one's awareness of that knowledge and the realisation that its use would be appropriate. To devise a strategy requires confidence, an at-homeness maybe, and a view of mathematics as a subject in which students can create their own methods.

To be numerate then is not merely to have secure knowledge of numerical facts and processes, numeracy also includes the capability and disposition to construct personal approaches to the solution of problems which are appropriate to the context and are based on knowledge of individual strengths and weaknesses. To be numerate is to be able to mathematize situations using techniques and processes which are confidently known to generate a secure answer. Numeracy therefore involves an interaction between mathematical facts, mathematical processes, metacognitive self knowledge and affective aspects of mind including self confidence and a disposition to construct personal methods.

The role of the calculator with respect to numeracy is contentious. Girling (1977) equated numeracy with the ability to use a calculator sensibly but the Numeracy Task Force suggests that "numerate primary pupils should: ... recognise when it is not appropriate to use a calculator" (Reynolds, 1998, p7). Girling's suggestion implies that the use of a calculator is but one strategy among many, to be chosen or not as considered appropriate for the task. However, the assertion that there are occasions when it is not appropriate to use a calculator implies that in some way calculators can impede mathematical development and that to labour over pages of arithmetical calculations is intrinsically good - an aspect of mathematical character building!

Mathematics teaching in the UK seems poised to return to the technological dark ages. The use of calculators is to be "discouraged as far as possible" until the start of secondary education (Reynolds 1998, p 25) and availability heavily restricted in examinations . At the same time working parties have been established and targets set to improve students' standards of numeracy. The assumption underlying such a stance is that the use of calculators leads inevitably to a lack of mental fluency and a decline in basic arithmetic skills. However, the research evidence suggests that this is not the case (eg: Shuard et al, 1991; Hembree & Dessart, 1992; Groves, 1994; Beaton, 1996).

The impact of calculators on mathematical development

A recent research study (Jones & Tanner, 1997) explored the effects of calculator use on the basic arithmetic skills of Welsh secondary students. A questionnaire sent to all secondary schools in Wales probed the ways in which calculators were used in mathematics. 126 questionnaires were returned (63%) and were analysed to examine the mode of calculator use by Year 7 (12 year old) students. Three groupings emerged:

1 - *Available*	Calculators were allowed in the majority of lessons and their use was not discouraged.
2 - *Discouraged*	Calculators were restricted to certain topics, and their use was discouraged.
3 - *Restricted*	Calculators were restricted to certain topics, and their use was not discouraged.

The distribution of schools across the groups (1-3) was in the ratio 4 : 4 : 3. A stratified sample of eleven schools was then taken to reflect this distribution. From the national league tables of examination success at age 16 the median and interquartile points were determined. Schools close to these points were then randomly selected to reflect the proportions of the groups.

A test of basic arithmetic skills was developed which included questions similar to those used in studies which pre-dated the widespread use of calculators in schools (APU: Foxman, 1985; CSMS: Hart, 1981). Calculators were not allowed in the test. A short questionnaire at the start of the paper probed students' attitudes towards calculators. The test was given to all students (n>1500) in the selected schools at the start of Year 8.

Comparison of basic skills: There were three sections to the test: number, decimals and fractions. Overall the response rates for the number and decimals sections were comparable with the previous studies but facilities were worse in the fractions section. Data from the questionnaire suggest that, in line with the National Curriculum for England and Wales, fractions are no longer taught extensively at this age.

Impact of mode of calculator use on test scores: If calculators do have a detrimental effect on arithmetic skills then students from group 2 might have been expected to out-perform those from group 1, with group 3 in an intermediate position. Alternatively, if calculators have a beneficial effect then group 1 should outperform the others. Contrary to all such conjectures, no significant differences were found between the groups. The extent to which the use of calculators was restricted or discouraged had no significant impact on the students' basic skills (F probability = .82)

Analysis of test score by mode of calculator use (ANOVA)

Group	No. of students	Mean	S.D.	S.E.
1	562	27.77	16.67	.70
2	483	28.40	15.32	.70
3	500	28.02	16.23	.73
Total	1545	28.05	16.11	.41

The groupings were derived from the description by the head of department of the mode of calculator use in year 7. However it is possible that variations occurred between individual classes. To explore this from the students' perspective, responses to statements on the students' attitude questionnaire were analysed. Test scores were compared according to the extent to which the students considered that they used calculators in mathematics. The students who claimed to use calculators outperformed those who did not.

T-test on test score for modes of calculator use identified by students
Statement: I use a calculator in most of my maths lessons

Variable	No of cases	Mean	SD	SE
Agree	380	31.55	17.21	.88
Disagree	922	26.68	15.60	.51
Mean Difference = 4.87		2-tailed significance = .000		

Thus, although the mode of calculator use as identified by the head of department was associated with no significant difference in the students' test scores, the students who reported that they used calculators performed better than students who did not. Two possible explanations are offered. It could be that only the more able students are allowed to use calculators in their mathematics lessons or alternatively, use of calculators has a beneficial impact on students' arithmetical skills. The direction of causality can not be inferred from the data. A similar correlation based on analysis of data from the Third International Mathematics and Science Survey is reported in SCAA (1997 p13).

It is also worth noting that only a modest percentage of students reported a free or frequent use of calculators (23% and 41%). This supports the data from the heads of departments and contradicts the impression of uncontrolled access given by the media.

Impact of calculator use on students' misconceptions: The use of calculators might have been expected to produce an improvement in students' understanding of "reversals", - the non-commutativity of subtraction and division, but mode of calculator use identified by the head of department made no significant difference to the students' responses. This suggests that the potential offered by calculators to enhance students' understanding of mathematical structure and symbolisation was not always being used to best effect.

Mental arithmetic: Only half of the heads of departments reported mental arithmetic practice to be a regular feature in their year 7 classes and this frequency of practice declined sharply after year 8. There was no significant difference in the frequency of practice by the mode of calculator use. This suggests that the increased use of calculators is not associated with a parallel increase in emphasis on students' mental strategies.

The mode of calculator use identified by the heads of department had no significant impact on students' basic skills although those students who reported free and frequent use of calculators did significantly better in the test. The survey identified the availability of calculators but did not indicate how they were being used. It would appear then that the issue is not whether to allow the use of calculators but how best to use them in combination with other strategies to improve the teaching and learning of mathematics.

The effective use of calculators

Similar conclusions have been reached by other studies. A meta-review of research into the effects of calculator use (Hembree and Dessart, 1992) concluded that, from the weight of evidence available, calculators enhanced the learning and performance of arithmetical concepts and skills, problem solving, and the attitudes of students. They suggested that future research should focus on ways to implement and integrate the calculator into the mathematics curriculum. Similarly in the UK it would appear that guidance is needed on how best to use calculators:

> Only a very small proportion of schools submitted detailed policies that offered teachers clear guidance about the principles, purposes and practice of calculators use ..." (SCAA, 1997 p10).

In our Welsh study, no variation was found between the different modes of calculator use and the practice of mental arithmetic. In contrast, a common feature of the research and development projects which allowed unrestricted use of calculators at primary level has been the emphasis on the development of students' mental strategies and "number sense" alongside the availability of the calculator. The Calculator Aware Number (CAN) project (Shuard et al, 1991) required the children to explore "how numbers work", to develop their mathematical language and their confidence in talking about numbers. The project also emphasised the importance of mental calculation and the children were encouraged to share their methods with others. Through this CAN found that the children developed a wide range of strategies for computations without a calculator. Instead of replacing mathematical thinking, the calculator gradually came to be used as a source of ideas and starting points. Classroom talk emerged as a crucial feature through which children shared, extended and elaborated their ideas.

Similarly, the Calculators in Primary Mathematics (CPM) project (Groves, 1994) aimed to develop a learning environment which supported the development of "number sense". Teachers considered that they made more extensive use of discussion and the sharing of children's ideas. The project children did not become reliant on calculators, rather they outperformed others on a range of estimation and computation tasks.

There is little evidence to suggest that calculators are detrimental to mathematical development. What does emerge from the research is that the role of the teacher is crucial. Where teachers had training and support in ways to use calculators through their involvement in research projects they placed a greater emphasis on the development of students' own strategies and mental methods through the encouragement of classroom discussion. But why should this enhance learning?

Articulation

There are four major reasons why numeracy might be developed through articulation and discussion of students' own mental methods. They are associated with constructing knowledge and testing its viability; the development of corporate meaning; the development of reflective awareness or metacognitive knowledge; and the development of a disposition to construct.

The processes involved in the construction of knowledge suggest that opportunities should be presented for students to articulate their tentative constructions, to test them for viability against corporate knowledge. However, articulation does more than provide an opportunity for the student to test for viability against corporate meaning, it also contributes to the generation of corporate meaning by providing a further opportunity for construal to other members of the group. During the social process of negotiation, individual subjects arbitrate between rival construals of corporate meaning, using a continual process of representation and reflected thought (Clarke, 1994). The re-presentation and articulation of mathematical thoughts and explanations can thus be a significant factor in self generated learning as re-presentations may be combined in hypothetical situations to generate thought experiments such as: "What would happen if...?". Thought experiments can be a powerful tool for learning and have been seen to be within the capabilities of school students (Brown & Walter, 1992).

Articulation is also significant in the objectification of explanation. The act of verbalisation is associated with bringing the subconscious into the conscious and hence the development of reflective awareness and conscious control (Prawat, 1989). This is characteristic of the metacognitive knowledge required for higher levels of thinking.

> When students begin to consider the adequacy of an explanation for others rather than simply for themselves, the explanation itself becomes the explicit object of discourse (Yackel & Cobb, 1995, p269).

To control a mental function, Vygotsky (1962) claims that a student must be conscious of it, but suggests that unconscious self regulation should precede conscious self regulation, presumably appearing first on the social level between people (interpsychological) and then inside the child (intrapsychological) in an unconscious form (cf: Vygotsky, 1978). The shift to reflective awareness and deliberate control of cognition would then be achieved through a transition to "verbalised self observation" which denotes "a beginning process of generalisation of the inner forms of activity". This is a shift to a higher type of inner activity opening up new ways of seeing things and new possibilities for handling them (Vygotsky, 1962, p91).

> In perceiving some of our own acts in a generalising fashion, we isolate them from our total mental activity and are thus enabled to focus on this process as such and to enter into a new relationship with it. In this way, becoming conscious of our operations and viewing each as a process of a certain kind... leads to their master. (Vygotsky, 1962, p91-92)

When reflective discourse is encouraged within a classroom, teachers can be pro-active in encouraging construction, focusing the attention of students on significant aspects of the discourse for collective reflection. "What was previously done in action can become an explicit topic of conversation" and thus "participation in this type of discourse constitutes conditions for the possibility of mathematical learning" (Cobb et al, 1997). The social character of the discourse may be arranged to lend social status to "the disposition to meaning construction activities" which is a "habit of thought" that can be learned (Resnick, 1988).

The significance of articulation or verbalisation in making processes explicitly available as objects of thought is confirmed by research. Several studies have included articulation by students as significant aspects of their approach (eg: Gray, 1991; Wheatley, 1992; Cobb et al, 1992; Tanner & Jones, 1994; 1995; Galbraith, 1995).

Cobb et al (1992) describe a quasi-experiment involving second grade students following a course guided by a constructivist view of learning which facilitated mathematical dialogue between students and emphasised mathematical argumentation. The teacher's role included focusing group attention on and implicitly legitimizing aspects of students' explanations and redescribing them in more sophisticated terms. The teacher summarised parts which were thought to be shared and drew attention to critical points which were not yet understood.

Experimental classes and control classes gained equivalent scores in questions of an instrumental nature, whereas the experimental classes were significantly better in questions of a relational nature. It is claimed that the teaching approach facilitated the construction by students of "increasingly conceptually sophisticated operations". Furthermore it is claimed that students' "fundamental beliefs about mathematics and themselves as learners" were influenced and the development of "intellectual autonomy" was encouraged whereas the usual course fostered "intellectual heteronomy". It is suggested that, for the control students understanding meant following procedural instructions, whereas for the experimental students understanding had a "more extensive conceptual aspect" (Cobb et al 1992, p498-500). The development of numeracy in the sense of a capability and disposition to construct personal approaches to problems and individual mental methods is likely to require such an attitude.

Wheatley (1992) reports the success of third grade students following a course which demanded that students study problems together in small groups reaching consensus through negotiation before reporting back to the class in the "sharing time". In the sharing time, the class was obliged to try to understand explanations, perhaps requesting clarification and initiating negotiations chaired by the teacher. In this way students learn how to discuss within themselves and in the process of telling others how they think about a problem, they refine their thinking and deepen their understanding.

The discourse between students and teachers should involve a form of flexible scaffolding based on the selective emphasis of children's verbalisations and if

> ... the dialogue between the pupils and with the teacher involves listening and sense making by all participants then we expect the dialogue to include within it the intertwining of more and less sophisticated conceptions, formulations and explanations as each tries to make sense of the other. (Williams, 1997, p13).

The discourse should provide also opportunities for reflection. The teacher should manage the interplay of social norms and patterns of interaction to create opportunities for students to reason for themselves and "engage in reflective thinking or reflective abstraction". These two processes are exemplified by "conceptual reorganisations which occur while experiencing perturbations which are initiated by another student's reaction to their reasoning" and also motivated by a desire to participate in a discussion, students may reflect on and evaluate another's thinking and reasoning (Wood, 1996, 102-103).

The effectiveness of flexible scaffolding and reflective discourse for metacognition and mathematical development in the secondary school was demonstrated in The Mathematical Thinking Skills Project (Tanner, 1997). The most successful teachers utilised approaches based on a form of dynamic scaffolding. This required participation in a discourse in which differences of perspective were encouraged and group attention was focused by the teacher on useful constructions suggested by students.

When this sort of dynamic scaffolding was used in combination with techniques to develop a reflective discourse, for example in a group reporting and peer assessment session, accelerated mathematical development was observed. In reflective discourse students focus on the processes of mathematisation, abstraction and generalisation in the service of understanding structure, thereby encouraging the development of a mathematical disposition (Tanner, 1997).

Conclusion

To be numerate in the sense advocated in this paper requires students to develop a mathematical disposition. Students must be willing to construct individual solutions to problems based on their own self-knowledge, as well as having a secure understanding of number facts and processes. The development of numeracy is not determined by the extent to which calculators are used. Under certain conditions the use of calculators has been shown to enhance students' numeracy. However, allowing the use of calculators is not sufficient in itself and it certainly does not remove the need for good teaching. The research reviewed here indicates the importance of articulation, argumentation and dynamic scaffolding within a reflective discourse for the development of students' mathematical and numerical abilities. The metacognitive knowledge and skills developed during reflective discourse are necessary for the mathematisation of situations and the identification of strategies which are confidently known. When learned in this way, mathematics should also be perceived by students as a subject in which construction of their own methods is not only possible but required.

It is conjectured here that a crucial feature of effective numeracy teaching is likely to be the use of dynamic scaffolding within a reflective discourse. Calculators can be used as a stimulus to provoke mathematical argumentation within such a discourse. If calculators are made freely available to students to check answers obtained through the use of their own mental methods, they may develop the self confidence to articulate their strategies and challenge those of others in public debate. If calculators are used as a focus for such a reflective discourse, they might be expected to lead to the development of numeracy in the wider sense intended in this paper.

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