Affective factors and the improvement of algebraic problem solving

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Problem solving is an important activity in algebra, as all mathematics, and research has shown that affective factors can be crucial to mathematical progress. This research study considered 105 15 year old students, and investigated the relationship between their affective domain and algebraic problem solving ability. Analysing students performing at different ability levels, our test and interview data have enabled us to describe a preliminary model of the interaction between self-concept, self-perception, enjoyment, interest, motivation, anxiety and problem solving ability, describing how they may be influencing students' learning.

Background

Problem solving is important in mathematics, and since the National Council of Teachers of Mathematics (NCTM, 1989, p. 23) announced that "Problem solving should be the central focus of the mathematics curriculum", the role of affective factors in mathematical problem solving has been the subject of a number of research studies (e.g. Macleod, 1992; Lester & Kroll, 1990; Mendler, 1989). Results indicate that the affective domain plays a crucial role in mathematics achievement, influencing the individual's ability to solve problems. Some research studies (Mendler, 1989; McLeod, 1992) have concentrated on how the affective variables influence mathematical problem solving, while others have considered the role of affective variables such as self-concept, interest, anxiety, self-perception, motivation, enjoyment and usefulness and the relationship between them (e.g. Marsh, 1989; Fennema, 1989; Mitchell, 1993; Skaalvik & Rankin, 1995). Marshal (1989) noted that repeated episodes of errors while solving problems may lead to a sense of frustration, a distrust of the child's own skills, resulting in strengthening or weakening of the nodes formed among the problem situations. The nodes thus formed may play an important role in the conscious control of both cognitive and affective information processing, which, according to Mendler (1984) allows one's meta-cognitive process to direct efforts to find a solution to a problem. The transition from arithmetic to algebra is a difficult passage for many students, and, as McLeod (1989, p. 33) emphasises "evidence gathered from secondary school teachers suggests that some students who have got on well in mathematics in the past will suddenly lose confidence when they have to address a more difficult problem than they are accustomed to". Thus knowledge of the relationship between affective factors and achievement in algebra problem solving would be very helpful for educators seeking to assist students' learning.

This study was based on the hypothesis that a relationship between mathematical problem solving achievement and the affective domain exists, and can be described, and we have previously reported (Kota, 1997; Kota & Thomas, 1997) some gender-based differences in the relationship between affective factors and algebraic problem solving performance during adolescence. More recent analysis of all our data, in which we studied separately students performing at two different levels of ability, has led us to believe that we may have identified how some of the factors we measured might be influencing problem-solving ability. We present the evidence in this paper along with a preliminary model describing the role of certain affective factors in algebraic problem solving.

Method

The data described and analysed in this paper were collected from 105 form 4 students (15 years old) drawn from 7 secondary schools, both single-sex and co-educational, in the Auckland region of New Zealand. One form of average ability students from each of forms 3 and 4 in the schools was randomly selected to form the subject group and the students were taught by their normal specialist mathematics teachers. All the schools were following the recent New Zealand curriculum published by the Ministry of Education, and in particular, its algebra section.

There were two different types of instruments used in this research. Firstly, there was a group of tests designed to measure the seven affective factors, self-concept, self perception, anxiety, interest, enjoyment, usefulness of mathematics and motivation, using self-descriptive Likert scales. Secondly, an instrument, comprising five word problems constructed from three pilot algebra tests, was designed to measure algebraic problem solving ability. Full details of the tests used can be found in Kota & Thomas (1997).

In the first half of 1996, before the algebra syllabus was taught for the year, students from both forms 3 and 4 in the schools were given both questionnaires, on affective factors and algebra, during their regular mathematics classes. They were also given the same two tests at the end of the school year, in November 1996. Only 105 of the form 4 students completed all of the tests and so only the results of these students are analysed here. Ten of the students, representative of the groups identified, were later interviewed for about fifteen minutes, and asked to relate their feelings regarding mathematics and their work in the subject. These interviews, conducted by the first named researcher, were tape recorded and later transcribed.

Results

The algebra test comprised five questions, each assessed on a scale of 1 to 5, enabling a finer measure of the students' progress, both in terms of understanding and correctness, to be obtained, and giving a range of possible scores from 5-25. Using the median score of 15 we divided the students into two distinct groups, those scoring below 15 were designated lower achievers (LA), and the others, higher achievers (HA), on that test. To keep track of which group each student was in on each test we added I for the earlier and II for the later test. Thus, LAII indicates those students below the median in test II. Figure 1 shows that, of the 42 students in LAI, 22 were also in LAII, while 20 students improved, and were in the higher achieving group, HAII. Similarly, of the 63 students in HAI, 54 were also in HAII while 9 were in LAII. We sought to examine, and identify, any changes in the affective factors of the students over the period between the tests and to try to relate this to their algebra performance, outlining any patterns discovered.

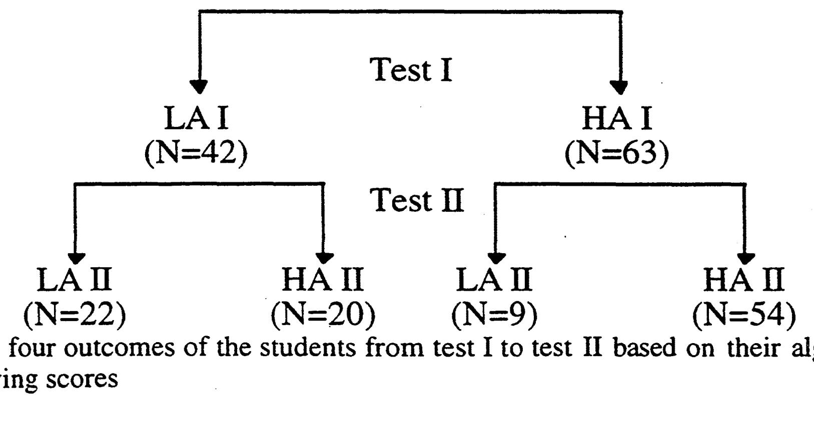

Figure 1: The four outcomes of the students from test I to test II based on their algebra problem solving scores

To accomplish this we used the possible outcomes to compose four groups of students, designated as LA I to LA II, LA I to HA II, HA I to HA II and HA I to LA II.

Table 1 shows a summary of the test results for the students in HA I to LA II (N=9), that is those who did significantly worse in the second algebra test. Although the number in this group is small, and it is difficult to attach weight to the results, the significant decline in their algebra score is matched by a significant decline in 3 of the factors, and weak evidence of significantly lower self-concept and interest, with the remaining factors, usefulness and anxiety, also apparently lower.

Table 1: The affective factor and algebra results for the HA I to LA II group (N=9)

	Test I		Test II		t	p
	Mean	SD	Mean	SD		
SC	95.1	12.03	89.6	11.44	1.92	<0.1
Int	69.7	9.45	63.7	11.2	1.48	=0.16
Anx	27.2	4.24	26.7	4.42	0.35	n.s.
SP	22.9	1.81	19.8	3.70	1.98	<0.05
Use	52.0	6.76	50.0	6.44	1.12	n.s.
Mot	50.9	10.39	44.9	9.57	2.31	<0.05
Enj	41.6	8.46	37.2	6.59	2.10	<0.05
Alg	19.0	1.73	11.6	3.21	5.25	<0.001

SC=Self-concept, Int=Interest, Anx=Anxiety, SP=Self-Perception, Use=Usefulness of Mathematics, Mot=Motivation, Enj=Enjoyment, Alg=Algebra Problem Solving

We considered that the above group, which did significantly worse in the algebra test II, would be most closely related to the LA I to LA II group, containing students whose algebra score remained below the median in both tests. Table 2 shows that, for this group too, interest, usefulness and enjoyment declined significantly, with some evidence that anxiety did too. We had also expected to see self-concept, significantly down for this group, but it is important to note that their self-concept, measured at 80.9 in the first test, was already significantly below all the other groups.

Table 2: The affective factor and algebra results for the LA I to LA II group (N=22)

	Test I		Test II		t	p
	Mean	SD	Mean	SD		
SC	80.9	12.06	79.0	12.14	0.71	n.s.
Int	65.9	12.79	59.6	8.77	2.89	<0.01
Anx	24.4	3.69	23.2	3.63 .	1.57	=0.13
SP	19.3	3.45	18.1	3.22	1.31	n.s.
Use	47.2	4.33	43.8	7.33	2.89	<0.01
Mot	41.6	10.84	40.8	10.72	0.51	n.s.
Enj	35.1	8.22	31.9	7.66	3.36	<0.005
Alg	11.6	2.66	11.4	3.06	0.22	n.s.

These two sets of results indicate the possibility that the level of some of these factors might be linked to performance in the algebra test, with both a relatively low, or declining, algebra score, accompanied by a corresponding pattern in the affective factor levels. Any causal link between the two was not apparent at this stage.

If this was the case then, we hypothesised, we might expect the other two groups of students to display a different pattern of results, without this decline.

Table 3: The affective factor and algebra results for the HA I to HA II group (N=54)

	Test I		Test II		t	p
	Mean	SD	Mean	SD		
SC	93.4	11.12	93.4	13.66	-0.67	n.s.
Int	67.2	12.05	66.4	11.89	0.61	n.s.
Anx	26.4	4.14	27.4	4.14	-1.99	<0.05
SP	20.7	3.12	21.3	3.46	-1.04	n.s.
Use	47.8	6.60	47.6	6.37	0.37	n.s.
Mot	44.1	10.1	45.9	10.04	-1.65	<0.1
Enj	36.7	8.18	37.0	7.70	-0.42	n.s.
Alg	21.9	2.99	22.0	2.84	-0.2	n.s.

What did an analysis of the results of those students who either significantly improved their performance, or maintained it at the higher level indicate? The analysis of the test data for the HA I to HA II group in table 3 confirms that, for these higher achieving students, in contrast to the previous two groups, not only was there no significant decline in any of the affective factors, but they are clearly very stable, except for anxiety and motivation; the former having *increased* significantly and weak evidence that the latter had too. It is the final group, LA I to HA II, those who improved significantly in their algebra score, from a mean of 12.25 to 19.55, which in many ways should be the key group. Since one would hope to improve student achievement in algebraic problem solving, these students, who had managed that very well, might hold the key. We see that, again, none of the affective factors shows a decline but all are remarkably stable in their values, with the exception of enjoyment, which fell significantly. This is a surprising, anomalous result which requires further investigation.

Table 4: The affective factor and algebra results for the LA I to HA II group (N=20)

	Test I		Test II		t	p
	Mean	SD	Mean	SD		
SC	85.1	13.17	84.1	15.16	0.49	n.s.
Int	64.2	10.05	63.1	11.2	0.72	n.s.
Anx	24.9	3.98	24.6	3.51	0.37	n.s.
SP	19.9	2.86	19.5	2.74	0.99	n.s.
Use	46.7	7.25	46.5	7.08	0.19	n.s.
Mot	45.3	9.81	44.5	10.01	0.77	n.s.
Enj	37.9	7.96	35.4	7.89	2.02	<0.05
Alg	12.3	2.17	19.6	2.30	-9.91	<0.0001

Comparing the factor levels of those students in the LA group at each test (not the same students), with those in the HA group (see table 5), reveals that, for these form 4 (year 10) students, those who are achieving the higher algebra standard have higher affective factors scores. Whilst initially the difference is significant only for self-concept, self-perception and anxiety, at the end of the year, when the HA group had increased in size from 63 to 74, it is significant for all the factors except usefulness and motivation.

Table 5: A comparison of the affective factors for the lower and higher achievers groups

	TEST I				TEST II			
	LA I Mean (N=42)	HA I Mean (N=63)	*t*	*p*	LA I Mean (N=31)	HA I Mean (N=74)	*t*	*p*
SC	82.9	93.6	4.47	<0.001	82.1	90.9	3.11	<0.005
Int	64.8	67.5	1.29	n.s.	60.8	65.5	2.23	<0.01
Anx	24.6	26.5	2.36	<0.01	24.2	26.7	2.81	<0.01
SP	19.6	21.1	2.37	<0.01	18.5	20.8	3.06	<0.005
Use	47.0	48.4	1.17	n.s.	45.6	47.3	1.06	n.s.
Mot	43.3	45.1	0.83	n.s.	42.0	45.5	1.61	n.s.
Enj	36.4	37.4	0.60	n.s.	33.5	36.6	1.9	<0.1

Clearly, the measurement of affect is very difficult, with beliefs, feelings and emotions constantly changing, and what we have sampled is merely a snapshot of this shifting landscape. However, on the basis of the above data, we hypothesise that maintaining high levels of the affective factors, and in particular self-concept, self-perception, anxiety, interest and enjoyment, is strongly associated in form 4 students (age 15 years) with higher performance in algebra problem solving, as measured by our instrument, and alternately declining levels of these factors are related to a lower algebra problem solving performance. We were very interested in examining the relationship between algebra performance and these affective factors further, particularly for evidence of a causal link, and for this we used the data from the interviews with the students.

Modelling a possible causal relationship

The interviews with the students were very revealing about their perception of the links between affective factors and their performance in mathematics lessons and we were able to add to those measured in our tests, two other factors which seemed important for modelling the causal relationship, one external and one affective. First we will consider some excerpts from the interviews with the HA I to LA II students and examine how they describe the role of their feelings in mathematics.

. . . Interviewer: Has there been any change in how you feel over this year?

R1: I used to enjoy maths.... but it got harder now.

. . . Interviewer: When do you enjoy it?

R1: When I like it I understand it more and enjoy.

. . . Interviewer: Does any thing prevent you from learning maths?

R1: . . . I am not sure. . . no not much, but I used to enjoy maths. . .

. . . Interviewer: When do you feel confident?

R1: I get confidence through the teacher, get ideas, concepts on different ways to approach things. . . I think just not having the security of a good teacher, the teacher really affects me and how they teach . . . It affects my ability to work and my ability to understand. .

Interviewer: How does your confidence affect your work?

R1: Umm. . . it just makes me a bit better, yeah more sure of myself

So R1 feels a lowering of her enjoyment, as her test score predicted, and she thinks that this has affected her understanding. The first of the other factors, an extrinsic one, mentioned by a number of the students, was the role of the teacher. R1 clearly thinks that the teacher gives her more confidence, which in turn improves her understanding and hence the quality of her work. Student C1, also links the lowering of her performance to her feelings and to the teacher:

Interviewer: Has there been a change over the year?

C1: Yep from 3rd form to 4th form yep there had. . umm. . my maths dropped

Interviewer: What is the change in your feelings towards maths?

C1: I think it is just that I was not comfortable with the teacher I did not like the teacher so when I was in 3rd form it was more interesting, I think I used to enjoy more. In third form it is just working it was easier and in fourth form you got to involve more and is hard

C1 knows she is not doing as well as previously and, like R1, relates this to being 'comfortable' with the teacher. She also gives a lowering of interest, enjoyment and effort as relevant factors. Many students' comments described the concept of greater study effort as being promoted by feelings such as enjoyment and anxiety. Here, for example, R2 links confidence and enjoyment to focus in the classroom and states that lack of confidence causes her not to 'work as much':

. . . R2: Feel confident, concentrate more, do better

Interviewer: What are the feelings that help you?

R2: Confidence, my enjoyment, just my focus in the classroom, umm. . .

. . . Interviewer: Is there any specific feeling that helps you to do that?

R2: When I am confident, my confidence. . . I just feel not confident umm. . .that I can not answer the question or like that and do not do work as much when I do not know how to do

She also recognises her falling standard of work, linking it to the teacher, loss of interest and to her lack of enjoyment:

Interviewer: How do you feel about your ability to do maths?

R2: About average, umm... I was average at first and I dropped

Interviewer: What could be the reason?

R2: Probably the teacher, not concentrated, lost interest

Interviewer: What is the change you find from beginning to the end of the year?

R2: Umm. . . I used to enjoy it better, I do not feel so now

R2 feels a lowering of enjoyment and self-confidence, confirmed by her tests, and these like many others are associated with her teacher. It may be 'obvious', but teachers clearly play an important rôle in improving students' beliefs, attitudes and feelings (e.g. Forgasz & Leder, 1996), which in turn influence their ability to work at, and understand, mathematics. The importance of the teacher is confirmed by the comments of M1 and V, who moved from LA I to HAII. They maintain that their feelings have influenced their ability to work and understand, mentioning in particular, improvements in confidence, enjoyment and interest as helping them achieve.

Interviewer: Has there been any change over this year?

M1: Maths is much better now. . . when I had Mrs [A] it was good, her teaching was pretty good. . . My maths is getting better because of Mrs [A]. . . I am much better in my algebra

. . . Interviewer: Does that enjoyment do anything for you?

M1: It gives me. . . more of a . . . umph. . . to do algebra

Interviewer: Are you confident of your ability?

V: Umm...halfway yeah I am not that confident I guess I have improved from last year

Interviewer: What do you think is the relationship between your feelings?

V: Umm...I am interested it helps me to improve what I want to improve

The need for concentrated and focused effort again comes through in V's comments and, interestingly, as we found from the tests, the idea that anxiety can be a positive influence, causing one to work harder:

Interviewer: What do you think has helped you to improve your performance?

V: Umm,.... just put my mind into I guess and I concentrate

. . . Interviewer: Do you ever feel worried in maths class?

V: Umm... yes I do. Sometimes I do not get what other people do - that sort of puts me down

. . . Interviewer: Does it make you work better?

V: Yes

Students C2 and M2, who were in the higher achieving band on both tests, and showed improvement between the tests, similarly linked their performance to greater confidence, understanding, increased effort and enjoyment of the mathematics.

Interviewer: Has there been a change over this year?

C2: Umm.... No not really, I've had to work a bit harder this year

Interviewer? What change did you find from the beginning to the end of the year?

C2: Change in feelings on stuff. I think I enjoy maths now. Last year I didn't like it that much

. . . Interviewer: How do you feel when you are stuck on a problem?

C2: Umm... pretty much annoyed

Interviewer: Does this help you?

C2: Yeah I do more work and do more and more 'til I solve it

Interviewer: Has there been a change in how you feel over this year?

M2: A little, I understand it a little better. . . umm. . . it's a bit more fun

Interviewer: What do you think the relation between your confidence and your performance is?

M2: Well like, when I feel good I get better marks and my performance is better, but if I don't feel very well then it just doesn't work. . . I just find it more enjoyable and it just makes me want to do more because I like it so much

Conclusions

We believe that this study has provided some evidence linking the maintenance in adolescence (15 years of age) of high levels of self-concept, interest, enjoyment, anxiety, motivation and self-perception with a higher standard of performance in algebraic problem solving, but a falling off in these factors with a lowering of this performance. In addition, two further important factors which emerged from the interview comments were the students' level of effort and an external contribution to their understanding, primarily from their teacher. Students' comments clearly showed that they linked the teacher with their understanding and confidence, confirming results found by Forgasz & Leder (1996), who describe the influence of the teacher on student beliefs. Student understanding then, raises or lowers self-confidence (whilst we measured self-concept and self-perception, these are factors which find there expression in the more commonly understood term self-confidence), which, in turn, influences levels of interest, enjoyment and motivation, leading to greater or lesser effort and hence performance. The following comments from three of our interviewees sum up particularly well some of these relationships, emphasising the driving role of the affective factors.

V: Umm. . .well I feel really good about maths and that is I like doing it, so I do it, and I get the best results I can

Here V explains how her confidence and enjoyment affect her work rate, which she sees as influencing attainment.

M1: I feel happy and I enjoy myself at what I do. That gives me more encouragement to do some more because I understand it. . . now I understand it and feel more confident to do what I am doing

M1 too perceives enjoyment of mathematics as promoting greater effort and understanding as increasing confidence.

C1: If you don't feel like doing it, you won't do it and you won't do as well and you won't achieve as much, but if you know you can and you are feeling happy about what you are doing then you just carry on, keep on going

C1 describes how, for her, greater motivation and enjoyment lead to more effort, and hence better results, or doing well.

Analysing the interview data in this way has thus led us to propose a preliminary model of the relationship between these factors as seen in Figure 2.

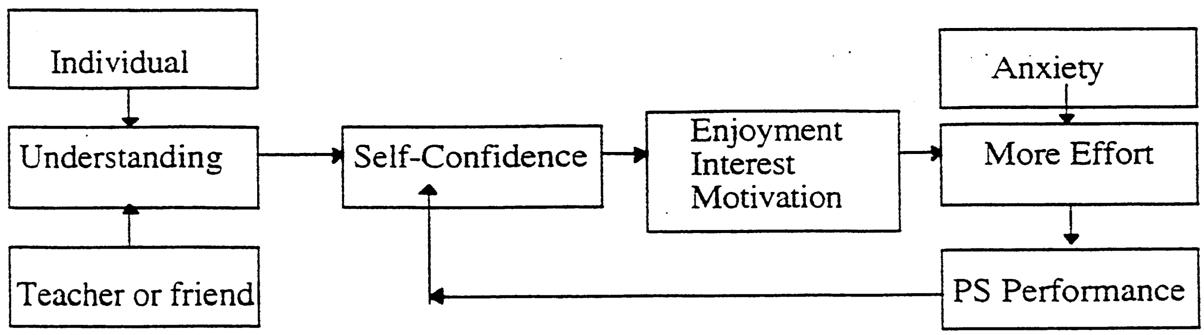

Figure 2: A Preliminary Model of the Relationship Between Some Affective Factors and Mathematics Problem Solving (PS) Performance

At first sight this description does not seem surprising, but there are several points which are worthy of note:

- the entry point for the cycle appears to be via understanding, and students look outside themselves for assistance with this, especially to their teacher
- self-confidence appears to play a major role and raising this should be a priority
- the students' perceived level of effort is a key influence; anxiety may raise this
- the process is cyclical, with improved work and achievement increasing confidence, but with the affective factors driving the cycle, not the performance

We have recently completed an analysis of corresponding data from 90 form 3 students (14 years old) to test the robustness of this preliminary model we have proposed. Very similar results are emerging which appear to support the findings presented here.

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