Towards a psychology of knowing-to

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While considerable attention has been focused on forms of teacher knowledge (eg. Shulman, Elbaz, Petersen, Carpenter *et al*) as essential components of expert and effective teaching, but knowing-that, knowing-how, and knowing-why are insufficient to enable a teacher to act upon that knowledge. What is critical is that teachers know-to act in the moment-by-moment unfolding of a mathematics lesson. We distinguish different types of knowing, and following Gattegno, relate these to levels of awareness. Our methods use descriptions and illustrations such as a fine-grained study of one teacher involved with innovative materials for teaching algebra to 13 year-olds to help the reader experience distinctions which we have found fruitful. Furthermore, the situation is self-referent, for what we want children to learn is to to know-to act themselves when formulating or solving problems. Drawing on the study, on our experience, and on the experience of others, we move towards a psychology of how knowing-to arises and can be supported.

Background

As a teacher, I know (theoretically, in that I have used them) many many tactics for use when teaching mathematics, but in the moment I do not always know-to use them: they do not always come to mind. As a student of mathematics, I know many many techniques, heuristics, and processes, (again theoretically in that I have experienced them and even written about them), but in the moment when working on a problem I do not always know-to use them: they do not come to mind.

Whitehead (1923 p7) described the central problem of all education as 'the problem of keeping knowledge alive, of preventing it from becoming inert' (p7). He forcefully asserts that 'this evil path represented by a book or set of lectures which will practically enable a student to learn by heart all the questions likely to be asked' (p7) 'culminates in a uniform examination [which] is so deadly' (p8). He wants theory to be applied within the curriculum, but points out that how this is done will depend on the 'the genius of the teacher, the intellectual tyope of the pupils, their prospects in life, the opportunities offered by the immediate surroundings of the school, and allied factors' (p7-8). The notion of inert knowledge has been taken up and examined by a multitude of authors seeking both explanation for it and strategies to over come it (see Renkl *et al.* 1996 for a survey). We are trying to probe what it is like from the inside when requisite knowledge does come to mind, and means for assisting that process.

Despite passing tests and exams and so indicating the presence of knowledge (however short term), it is very possible to get into a situation and not to have relevant 'knowledge' come to mind. For example, Schoenfeld (1988) describes in detail how students who were known to know all requisite facts did not (Schoenfeld says 'were unable to') employ them to achieve a geometric construction.

Shulman (1987) found that a teacher with a broad subject knowledge was the most 'conceptual' in his teaching, and a teacher with little subject knowledge was the most 'rule-based'. This fits with experience, for when faced with a problematic situation, people will move to where they are confident, controlling uncertainty by specialising, restricting, and falling back on confidence-inspiring strategies and entities (Mason 1993).

But broad subject knowledge in itself is insufficient to make an effective teacher. Dewey (1902) distinguished between the expert immersed in advancing a subject, and a teacher who is concerned with how 'the subject matter may become a part of experience' (p22). He called this, the *psychologising* of the subject matter:

> 'to see it is to psychologise it ... It is the failure to keep in mind the double aspect of subject-matter which causes the curriculum and the child to be set over against each

other . . . The subject-matter, just as it is for the scientist, has no direct relationship to the child's experience. ' (p22)

The *transposition didactique* (Chevellard 1985, Kang & Kilpatrick 1992) has become a well used label for the transformation of expert awareness (subject matter as it stands to the specialist) into instruction in behaviour (as it is construed by the student). Dewey, and every committed teacher and author, constantly struggles against this transformation.

Awareness

Gattegno (1987) used the term *awareness* in order to approach Dewey's concern and the *transposition didactique*. For him, an awareness need not be conscious, because it may have been integrated into functioning. He referred to awarenesses which enable people to act, and to awareness of those awarenesses, which creates a discipline. We go further, for it is awareness of discipline, which is necessary in order to teach. In order to keep the levels and forms of awareness to which Gattegno draws attention clear, it will be convenient to speak of *awareness-in-action* to refer to awarenesses which make it possible to perform actions, but which we need not be aware of explicitly in order to carry out those actions. The term draws metonymically on Vergnaud's notion of *theorem-in-action* (Vergnaud 1981, Binns & Mason 1994). Thus,

> You can count without being aware of one-to-one correspondence, you can add, subtract, multiply, and divide, without being explicitly aware of your awarenesses-in-action of numerals, place-value, routines, the role of order, etc. which make that arithmetic possible.
>
> You can form and detect patterns and locate formulae that generalise specific cases without being explicitly aware of your awarenesses-in-action of same and different, relatedness, induction, stressing and ignoring.
>
> You can combine fractions according to rules, without being aware of how fractions relate to decimals and to integers, and how they generalise number, without being aware of the slide between operator and object, without relating them to a number-line.

Awarenesses-in-action include the natural powers we all possess for making sense of the world, manifested in specific ways when mathematics is the content. The behaviours to which awarenesses-in-action contribute can to some extent be trained without explicit reference to awareness, for it is not necessary to be explicitly aware as long as situations remain routine and do not require innovation, novel interpretation, or creativity. Thus, short term success can be achieved in getting children through assessment hurdles, or teachers through in-service events on reform topics such as investigations, co-operative learning, structured lesson plans, constructivism, and whole-class teaching. But education is about more than training of behaviour in routine actions on symbols on paper, and teaching is about more than carrying out sequences of instruction. Teaching is about inducting children into disciplined forms of thinking and perceiving, and these emerge when awarenesses-in-action are made explicit and formalised.

Gattegno defined a discipline as the study arising when people become aware of awarenesses-in-action (Tahta 1988 quoting Gattegno): "Sciences are born when someone states that what occupies his mind *is*, and, because of that, is part of reality and worth being considered by others."

Mathematics arises as a discipline when we become aware of the awarenesses-in-action that constitute counting, ordering, classifying, and relating, and start to formalise these in the languages of algebra and geometry. Gattegno suggested that algebra emerges when we become aware of our awarenesses-in-action involved in the dynamics of relationships, and geometry through awareness of the awarenesses-in-action involved in the dynamics of our minds (of which mental imagery is a major part). For example, when we attend to how we order, how we classify (the notion of equivalence, of same and different), how we count (through one-to-one correspondence), and formalise these in

general statements we produce algebra, and simultaneously, the rules of algebra. When we attend to our powers of mental imagery to conceive mentally of points, lines, curves, surfaces, and movements and inter-relations between them, when we formalise them into assertions of what *must*, *might*, and *mayn't* happen (Mason 1991), and justify these assertions, we develop the discipline of geometry.

What makes teaching a discipline? In his comprehensive review of teacher knowledge, Shulman (1986) reminds us that the original meaning of higher degrees (masters and doctorate) was recognition of the ability to teach the subject, not just prosecute it as an apprentice (bachelor). He quotes Wheelwright's Aristotle:

> We regard master-craftsmen as superior not merely because they have a grasp of theory and *know* the reasons for acting as they do. Broadly speaking what distinguishes the man who knows from the ignorant man is the ability to teach, and this is why we hold that art and not experience has the character of genuine knowledge (episteme) – namely that artists can teach and others (i.e. those who have not acquired an art by study but have merely picked up some skill empirically) cannot (Wheelwright, 1951, p69).

We are going further and suggesting that to teach requires more than knowing reasons. It requires knowing-to act, prompted by awareness of awareness of awarenesses-in-action, in order to provoke students into themselves knowing-to act.

Knowing

The noun *knowledge* has an unfortunate finality and rigidity associated with it, implying a finished product separated from time and situation. By contrast, the gerund *knowing* carries a sense of dynamic change situated in the moment.

Forms of Knowing

Changing prepositions often provides insightful contrasts. We find it useful to distinguish between

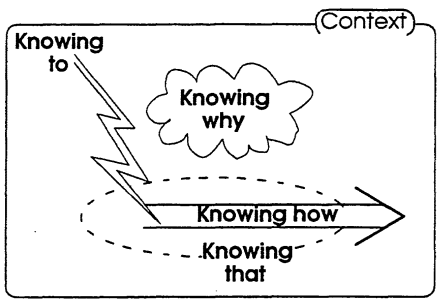
Knowing-that:

the sum of the interior angles of a triangle is 180 degrees, ...;

fractions and decimals are names for numbers;

doing-and-undoing is a major theme in mathematics generally and in algebra in particular;

stressing and consequent ignoring is the process behind generalisation;



Knowing-how:

to add two fractions, multiply two finite decimals, find the area of a triangle;

to stimulate productive discussion amongst pupils such as talking-in-pairs, say-what-you-see, card-sorting, conducting an interactive lesson in silence;

to provoke disturbance so there is something to conjecture and resolve;

Knowing-why:

only some numbers with infinite decimal names also have fraction names;

to divide one fraction by another you flip the second over and multiply;

discussion with peers can be an aid to learning mathematics;

a task has been structured in a particular way in a text-scheme;

Knowing-to (in the moment when needed, not just theoretically):

use some strategy or heuristic;

draw attention to some mathematical thinking process just provoked;

draw attention to some mathematical theme that has emerged;

divert from the text-scheme because of genuine class interest and readiness.

Knowing-why means having some 'story' to account for knowing-that or knowing-how, but the story does not have to be valid or true in someone else's theories.

We are not claiming hard and fast distinctions between these categories. Some know-how depends on knowing-that, knowing-to draws upon knowing-how, and knowing-why encompasses knowing-that and knowing-how. But it is often the case that people apparently know-that, yet not how to act on that knowledge; know-how but fail to recognise an opportunity to employ it; know-why something must be the case but do not use it.

It is not surprising that the first three of the *knowing* modes have been used before. For example, Ryle (1949) used them in the context of a discussion of mind, and Miller, Malone, & Kandi (1992) used similar terms.

Skemp (1979) distinguished *knowing that*, *knowing how*, and *knowing to*, though he described the latter in terms of *being able to* use a technique in a novel situation. That particular choice of language, which is common, is frequently misunderstood as 'able to in the sense of knowing-how, that is, having demonstrated out of context. By using the language of 'knowing-to in the meoment', we can make progress beyond the simple possession of abilities.

Biggs (1994) rehearses Ryle's distinctions and then directs attention to what he considers to be a different way of considering knowledge, in five hierarchical levels following Bruner's spiral curriculum and resonant of van Hiele's levels (1986):

Tacit: manifested through doing without conscious awareness or stories

Intuitive: directly percieved or felt

Declarative: description of how and why expressed in some symbol system that is publicly understandable

Theoretical: abstracted or generalised statements going beyond particular instances

Metatheoretical: knowledge about the process of abstraction and theory building

Biggs includes two additional forms, *Procedural* and *Conditional* which do not seem to fit into his hierarchy. Yet each of these seven forms finds their equivalent in combinations of the types of knowing we have delineated, when coupled with awareness. Our distinctions have the advantage of being based on what it is like as a knower, rather than being analysed from outside.

Jong & Ferguson-Hessler (1996) use a matrix of types against qualities in order to classify research on knowledge. They use categories such as *situational knowledge*, *conceptual knowledge*, *procedural knowledge*, and *strategic knowledge*, and qualities such as *level* (as in surface or deep), *connectedness, automatedness, modality* (as in verbal and pictorial), and *generality* (as against domain specific). Again, these categories are based on the same sorts of distinctions we are making, but whereas Jong & Ferguson-Hessler are trying to analyse other people's knowledge, we are trying to get at how it is that people know-to act in the moment.

Renkl *et al.* (1996) review much of the literature on inert knowledge as a common phenomenon. They locate three types of explanation for it: *metaprocess* (there is a disturbance to accessing what is needed), *structure deficit* (there are aspects missing in what is known), and *situatedness* (mismatch between current situation and previous situations). In the last category they note particularly Clancey (1993), and Greeno, Smith & Moore (1993) who see knowledge as relationally defined and not a property of the individual. Greeno *et al.* use the analogy of motion. Motion is not a property of the object because it depends upon frame of reference; motion is a relation between a frame of reference and an object. So too with knowing.

Proponents of situated cognition often write as if they have eliminated questions of behaviourists 'trasnfer', but fail to address the question of how the situatedness in which a skill is employed broadens and becomes more general, more abstract. The language of knowing-to provides an appropriate access to such questions.

Shulman (1987) identified a number of different components of teacher knowledge: subject matter content knowledge; pedagogical content knowledge, knowledge of other

related content, of curriculum, of learners, of educational aims, and general pedagogical knowledge. However he made no overt reference to self-awareness, to the dynamic of knowing which co-emerges with the situation, or to what it is like to be a learner (unlike Elbaz 1983). Rather, it is all knowledge about: knowing-that and knowing-how. Shulman located his categories through interviews with teachers about what they thought about and knew about, but, of course, these interviews did not take place in the moment of teaching. The research methodology naturally leads to knowing-that, with room for knowing-how and knowing-why to emerge, but is much less likely to highlight knowing-to, since that involves the internal psychological process of noticing an opportunity (Mason 1994). When teachers are able to re-enter the situation without elaboration or justification, some 'inflight-decisions' are captured. But when teachers elaborate and reason about what they 'must have felt', access to knowing-to is lost.

There is a parallel with teachers asking pupils reflection-intended metacognitive questions such as 'how did you do that?'. Asking questions does not necessarily draw attention to the fact of the question, does not necessarily expose such a question as useful, and the response does not necessarily reveal what it was actually like. Creating an inner teacher (Hyabashi & Shigematsu 1988, Mason & Davis 1989) through metacognitive intervention depends crucially on teacher awareness of their own awareness of their awarenesses-in-action.

Evidence

Our methods involve seeking confirmation and challenge in the experience of others. We aim to convince not by analysing data in an attempt to prove something, but rather by trying to provoke your awareness of the distinctions we ourselves find useful. In the short space available we offer, in addition to our exposition in the previous sections, two brief extracts from a study (Spence 1996) which suggest that the phenomenon we are addressing is recognisable.

Discussion in pairs

The following brief extract has been chosen to highlight both the effect of a knowing-to, and consequences of not knowing-to with respect to generating discussion.

While talkjing with the researcher about students' dependency, the teacher asked what could be done about it. The researcher suggested "I would be inclined at this point just to step back and let them struggle with it for a while rather than you doing all the struggle for them".

In the next lesson on pattern recognition and expression as a formula, the teacher was for the first time observed letting go of his usual firm direction and control of what was said and done by the students. Square brackets are researcher diary notes made during the lesson.

T1: What did you learn from p 25? . . . (little or no response)

T1: Share with someone what you learned from p 25. (buzz of conversation)

[This is unusual behaviour not previously seen. There were periods of children disagreeing with what another has said; of teacher writing at the board with a succession of organising questions.]

T1: Is that a formula for odd numbers? *S*: yes; *T1*: yes it is

[Later: kids are asking teacher "How did you know that?" and "Why does it do that." The atmosphere has switched from a pupil's earlier 'just tell me the answer and I'll be happy' to animated enquiry. The researcher was excited by what she had seen, but as the teacher approached her at the end of the lesson it was clear he was dissatisfied as he threw his hands in the air in a gesture of frustration.]

T1: "Help me. They're just not getting this"

R: "Some of them do. This is the first time they seem to care. They care enough to ask questions."

T1: "I knew I could trust you to make me feel better about this lesson."

The pre-lesson discussion seems to have supported a knowing-to get the children discussing in pairs in in that lesson which continued in subsequent lessons but was not used with other classes. The teacher's request for help can be interpreted as awareness of a missed opportunity, of not knowing-to do something, but also an unarticulated awareness that something was possible. We see the teacher as caught up in his agenda and in his perception and frustration that the pupils are not 'getting formulae' which is where his attention is focused. Yet the researcher could see a marked contrast in pupil involvement from normal practice. The teacher does not seem to be aware of the fact of his formula-focus, nor how it blocks children in working on locating and describing patterns. Without this awareness he is unlikely to be able to suspend it or let go of it as a goal and listen to what the pupils are saying and doing, using his awareness as a guide rather than as path.

Is Zero Even?

During work with sequences of integers (called number-strips because they are displayed in consecutive cells on a strip of paper), the issue arose in a lesson of teacher T2 as to whether 0 is to be considered even. Student S1 remarked that zero was not even as it was only a place holder. For S2, being a place-holder meant having no meaning. Students came up with a variety of justifications: 'It's got to be even because every number that's got a 0 on the end', '–1 and 1 are odd, so ...'. During the discussion, the teacher was seen for the first time to be departing from the material, opening up and sustainging discussion. S1 still insisted 0 was not a number. At one point the teacher says

> S6 triggered something in my mind. What happens when, I think you defined all the even numbers as 2 times the step number. OK. Which means that I also could take the number divided by 2 (writes on chalkboard) and get the step number can't I?
>
> S6: Yeah
>
> T2: So when you take 0 and divide it by 2, is there a remainder or is there not a remainder?
>
> S6: There's not a remainder.
>
> T2: So there's no remainder.
>
> S7: Yeah, but it's not even because ...
>
> T2: So what happens with a negative 2?
>
> S6: It's an even number.
>
> T2: OK. So did we agree that every other number is an even number?
>
> S7: Yeah, but ...
>
> T2: (laughs). Keep thinking about it. We'll play with it. Keep thinking. I want to get out of the way. I want you to go on now [to the next work] ...

[The class is over. Children leave the room while the teacher is still talking and trying to explain. As I leave, the teacher asks me to come tomorrow with ideas about what he can do.]

> T2: I'm not doing too well here.

The teacher has unusually allowed the lesson to follow an unexpected direction, but remains dissatisfied. The researcher saw an opportunity for work on what makes a number a number, to draw upon historically different perceptions (to ancient Greeks, not only was zero not a number, neither was one or two), to engage with children in a perennial philosophical questions about the denotation of 'nothing' (Rotman 1987), and to develop the notion of definition in mathematics. With the requiste awareness it was an opportunity for checking whether something has required properties, and then either accepting it as satisfying the definition, or modifying the definition so as to exclude zero, and for finding that definitions are framed so as to make theorems easy to state rather than having objective existence, in true Lakatosian style (Lakatos 1976). In fact, no reference was made to this issue in the following lesson.

Text-scheme authors have to depend upon teachers seizing opportunities to develop aspects of mathematical thinking and mathematical themes as they arise, for it is not possible to guarantee that instructions written on a page will generate specific discussion, no matter how well the materials are trialled and honed. Successful teaching depends on teacher awareness in the moment, their knowing-that, knowing-how and knowing-to all in one instant; their awareness of mathematical content, themes and processes all coming to the surface at once.

It is much easier to see opportunities from the back of a classroom than when you are caught up in the midst of the lesson. That is why observing others can be as important as having someone observe you. Opportunity can only be seen for yourself if you have the requisite awareness. It is not sufficient to have been through a course which 'covers' such topics, not even enough to be able to answer direct questions about whether zero is a number from someone outside of the classroom. There has to be a match between the situation as it emerges and the ethos in which the teacher works.

Towards a psychology of knowing-to

Consistently with our methodology, we have interrogated our own experience to locate what it is that informs our knowing-to in the moment, as well as observing and talking to others, and getting them to try things out in theoir experience. We have found, as illustrated above, that *grasping* (being caught up in a dominant agenda) can block a freedom to go with the flow guided by mathematical awarenesses. But *going with the flow* without richly developed mathematical awarenesses is as likely to divert attention away from mathematical thinking as towards it. Whitehead and Shulman are correct in stressing that the teacher needs active content and pedagogic knowledge.

In the midst of the flow, something in the situation triggers recognition of a possibility, whether as student or as teacher. We have found that collecting brief-but-vivid accounts of situations which share the possibility to act in a certain way, combined with a label to act as a focus or hub for that act, enhances the possibility that that act will occur to us in the future. Furthermore by imagining ourselves in such a situation and actiung in the desired way, we can further enhance the poossibility of the act coming to mind (Mason 1996).

Knowing-to in the moment is not usually a process of carefully considered options. Rather, something comes to mind as a possibility at the same time that it starts to happen. This requires a sensitivity to the situation, and a rich collection of prepared actions. Labels such as *transposition didactique*, and many others, can serve as hubs in a network connecting situations and actions, allowing sensitivities and then actions to be triggered by situations.

Teachers taking Open University Inservice courses often report that the label-frameworks which form the core of the course have served to remind them to act differently, that is to know-to act. But this only happens when the teacher has been able to link past experience, future actions, and a label, in their own experience.

The principle lesson for us is that training behaviour is useful, but educating awareness is crucial, even if it requires time in the presence of a respected other. Certainly there is need for a community of enquiring, questioning colleagues. As individuals we are trapped in habits, and as the epigram says "Habit forming can be habit forming".

Teachers are traditionally isolated in their classrooms and trapped by fear of betraying their uncertainties in subject matter, pedagogy, and didactics, at all of Shulman's levels (*op cit.*). It is easier to automate and habituate as much as possible so as to reduce strain and energy drain. Unfortunately it also reduces the potential for teaching to stimulate learning.

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