Ability Grouping: Some Implications for Building Mathematical Understanding

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In response to an interview question on features of classroom organisation that support the development of mathematical understanding, all four teachers interviewed referred to the benefits of "ability grouping". This was seen as a means of structuring learning conditions so that teachers could attend to children with similar levels of understanding. Subsequently, achievement-grouped classes were videotaped. Analysis of the data showed that interactions within this mode of organisation demonstrated the same problematic features as whole-class teaching. It is concluded that if achievement grouping is to be used, other strategies need to be used to make mathematics pedagogy inclusive.

Introduction

In a research project entitled *Teachers' constructions of their roles in building mathematical understanding*, four teachers from one school were videotaped teaching about six lessons each. Throughout this observation period, interviews and less informal discussions with the teachers were recorded. The results of this stage of the project will be used to inform the construction of a survey instrument researching the notion of mathematical understanding, to be used with a wider population of primary mathematics teachers.

During the interviews held with the teachers in the first few weeks of their observation periods, each was asked to "... talk about the different ways (they) organise the classroom that seem to facilitate the development of children's mathematical understanding". This paper focuses on what was the first claim made by three of the teachers and at a later of the interview with the fourth teacher. In response to this question, all four teachers claimed that the use of ability groups allows teachers to focus on children's understanding because the children in such groups have "common levels of understanding" (Teacher R, October 1997).

This research report presents some of the teachers' claims and comments about ability grouping, a summary of the results of this aspect of the videotape analysis, and some data to represent a fairly typical pattern of interaction in these groups. It concludes with a statement about the implications for teachers of this piece of research.

Ability grouping

How classroom participants act is largely a product of their theories about learning and teaching, about the subject matter involved, and about the roles individuals should play within the social contexts of schooling. Teachers' ideas about ability grouping in mathematics classes are a product of their personal histories as students and as professionals working in an environment where underlying assumptions about pedagogy frequently remain unchallenged. The notion of *ability* is rarely portrayed as problematic, and the teachers' right to set up various forms of ability groups is often actively encouraged by administrators and parents, and rarely questioned. Nevertheless, most administrators, teachers and even students are aware of some of the potentially undesirable effects of ability grouping, and in practical contexts they are aware of the tensions created. For instance, the conflict between the educational goals of excellence and equity is a significant problem in mathematics teaching, and the tensions surrounding ability grouping are one of the manifestations of this larger conflict. Amongst other teaching facets, Sullivan (1996, p. 1) identified a tension that he called "educating for the best <—> educating for everyone", just one of the many excellence–equity dilemmas that make decisions about curriculum planning and classroom organisation problematic.

The literature on ability grouping is extensive, and much of this research has been carried out in mathematics education because mathematics is used as a mechanism for streaming students into particular courses and occupations, and hence it is the most commonly ability-streamed subject in primary and secondary schools (Loveless, 1994). Since the 1960s, there has been a continual stream of evidence to show that within-class grouping has little effect on achievement in mathematics, and given the likelihood of loss of self-esteem in slower groups the argument is generally made that ability grouping and/or streaming should not be used. Recent studies taking this form include those of Bode (1996) and Fuligni (1995). Similarly, some studies have shown that the use of fixed ability groups led to lower achievement than a whole-class model (for example, Mason & Good, 1993). Some researchers have criticised ability grouping in mathematics classes because it has been shown to increase the gap between students through exposure to limited models, or because it encourages differences in teachers' expectations thus leading to tailoring of instruction (see for instance Cahan, 1996).

However, the implications of research finding in this area are not unambiguous. If one is to take an "educating for the best" position, it is clear that there are some potential advantages to ability grouping, and for this reason the right of schools to stream students at secondary levels is rarely questioned. In fact, some extensive studies, such as that by Brewer (1995), have shown that long-term streaming leads to increased performance for uppertrack groups—although the research in this area rarely gathers data on, or even questions, the social implications of such practices. Brewer noted the social pressure on schools to accept the idea of grouping by ability, collecting evidence that in schools where this style of grouping is common, the students, teachers and parents generally fear that achievement would be reduced if streaming were to be abandoned.

Shorter-term and more flexible ability grouping is also seen to bring benefits to all children (Numeracy Task Force, 1998). For instance, if group size is adjusted according to the apparent need for individual attention, teachers can spend more time working with lower-achieving individuals, and they are not so likely to have teaching that is directed at their peers go "over their heads". Similarly, the better students can be challenged and coached together, benefiting from higher-level interaction with their peers and the teacher; and middle groups need not either be daunted by work that is challenging for children with higher levels of understanding or be held back while lower achievers have their needs met. The Numeracy Task Force report, published only recently in England recognised such potential advantages when it recommends that:

> One purpose of group work is to allow a manageable degree of differentiation around a common theme. Groups can be organised by attainment, and while the main body of the class works on a set task, a more challenging task can be set for the most able, with a simplified task for those who would benefit from this. (Numeracy Task Force, 1998, p. 20)

However, as with all of the key dilemmas in mathematics education, there are no simple solutions to the conflict between teaching for excellence and teaching for equity. There is a necessity to investigate what happens within ability groups, and to gather data on whether assumptions such as "common levels of understanding" can be supported.

Methodology

The teachers in the school that provided the context for my research work as pairs in planning, teaching and assessing mathematics. Four teachers (two Year 2, and two Year 6) were the focus of this project. The general aims of the research were to identify (a) what the teachers claim to do to build their pupils' mathematical understandings, as well as what they actually do; (b) the different types of understanding required of primary-school mathematics students; (c) some personal, institutional and socio-cultural factors that support or constrain teachers' work in developing mathematical understanding; and (d) how success (or lack of success) in this field is recognised.

About six lessons taught by each teacher, and some jointly-taught lessons for each pair, were videotaped. Casual discussions were held with teachers after most of the lessons, and more structured interviews were held midway through each observation period as well as at a later date when some of the data had been analysed. The discussions and interviews were audiotaped.

To date, the videotape analysis has aimed to identify four distinct components of the interaction, although it is recognised that these factors worked interactively. The first area of focus has been what the teachers did to develop their pupils' understandings of particular concepts being taught. Here, the teacher actions studied have included components such as patterns of explanation, types of questions and responses, the use of metaphors and examples, and demonstration with gestures, diagrams and aids, etc. A second level of analysis has focused on how some child exhibited what seemed to be a growth of understanding, with a particular emphasis on "breakthrough" episodes, as well as different types of understanding exhibited (e.g. understanding of explanations c.f. understanding of mathematical concepts; instrumental versus relational understanding [Skemp, 1976]). Teachers' and children's body language, oral exchanges, use of concrete materials, written work, children's explanations, their responses to the teachers' questions, and other similar indicators of the children's understandings have been noted. Third, broader aspects such as modes of classroom organisation and the dynamics of the interaction that seem to impinge on learning of mathematics concepts, including social interactions that seem to support or inhibit the development of understanding have been examined; with particular focus on the experiences of individual children. Factors noted here have included prior understandings that different children demonstrate early in the lessons, the ways in which they attended to or ignore the teaching and learning activities, the amounts and types of individual attention and feedback each child received, and so on. The fourth category of focus has consisted of phenomena taken from the literature on what teachers can do to develop understanding, such as the "folding back" technique expounded by Martin, Pirie and Kieren (1995) that developed from a "dynamic theory of mathematical understanding" (Pirie and Kieren, 1991), metacognitive experiences (e.g. Greeno & Riley, 1987), relationships between understanding and mathematical processing (e.g. Arnold, 1997), and qualities of understanding (e.g. Skemp), as well as constructivist and socio-cultural perspectives on the development of understanding.

The focus on ability group in this section of the research project was not part of the original research design. It was identified as only one of many potential areas of engagement related to the third focus above, when each of the four teachers mentioned in the first formal interview that this mode of classroom organisation makes it easier for teachers to build their pupils' mathematical understanding. Subsequently, the tapes of the four lessons (three Year 2 and one Year 6) where children had worked in ability groups were re-analysed in the light of this claim.

In each of the four achievement-grouped lessons, the Number area of the curriculum was central. Two of the Year 2 lessons and the Year 6 lesson were on place value, and the remaining lesson taught with ability groups was a Year 2 lesson on problem solving using multiplication. In all cases, the groups taught by the classroom teacher were small (5-6 pupils in each of two more able and two less able groups being videotaped), while the rest of the class worked in groups under the direction of other people—other teachers, parents, or child monitors—in the same room, other classrooms, withdrawal areas, or outside. I did not choose the groups or children to film, because I was "following" each of the classroom teachers in turn.

The school context in relation to ability grouping in mathematics

Ability grouping is used throughout the school, but it is school policy that students not be "tracked" in longer-term streams. Typically, ability groups are used only once a week, and the teaching focuses on a particular concept. The groups are temporary, lasting from one to three weeks; and are created by the teachers on the basis of diagnostic tests and/or their perceptions of individual students' knowledge and skills as the pupils complete regular seatwork. The groups do not have names—a typical instruction to form new groups is "F,R,L,C, ... and R; get a bead frame and work with Mrs Smith, please". If the same groups continue for several mathematics lessons, the teacher will call up, "Those people who did bead frames with Mrs Smith", and so on. Thus groups are extremely flexible, with, for instance, no achievement-grouped lessons in one week, several over the next few weeks, etc. Groups dissolve as the class moves on to another topic or the concept is explored further in a whole-class situation.

The teachers' ideas about ability grouping

The following three quotations from transcribed discussions are typical of the reasons given for the use of achievement grouping:

Teacher B: (After place value lesson 1, Year 6) There are some bright kids here who really do not get extended in ordinary lessons, and some slower ones who get left behind. This works well at both ends. But not all the time—it would do more harm that good if we used it all the time. (Question: Why?) Two things really. One is that the slower kids would not progress, because they learn a lot from the others and they would never be challenged by listening to higher-level ideas. It would not be good for them socially either. Kids this age do not need to be labelled "smart" or "dumb" by being stuck in a particular group. There is already enough of that at this age.

Teacher W: (After place value lesson 2, Year 2) Kids like M and S get left behind in a grade situation. I just can't spend enough time with them. They are really weak. They need lots of time on really basic stuff. Even counting—you saw M yesterday. I really don't think she learns anything most of the time because she has not got the underlying concepts. In a small group at least I can give her time and really get down to her level.

Teacher W: (After problem solving lesson 1, Year 2) I wanted to do problem solving in groups because that lot never gets extended. They really fly, and learn a lot from each other when they are together—like L did today. She is so bright, she needs some challenging work. Nearly all of what we do as a grade, she already knows. She works so fast—too fast because she is so keen to finish that she makes mistakes. She needs work like this to get her thinking and to teach her that maths is about nutting out real problems, not just flying through routine work and getting it all right.

In summary, the teachers studied used ability grouping with the interests of the children in mind, and saw it as a means of catering for individual levels of knowledge and skill—and hence as a way of creating opportunities for enhanced development of mathematical understanding. Given this context, I was interested to analyse whether the achievement grouping did seem to serve these purposes for all, or at least some, children.

Results of video analysis

Analysis of the videos of interactions in the achievement-grouped classes showed that many of the "excellence <—> equity" issues present in whole class teaching also arose in these smaller groups. Space does not allow a representative sample of interactions to be displayed so I present one snippet of transcript as a context for the discussion to follow. This snippet was selected because it portrays many of the features noted in each of the four achievement-grouped lessons filmed.

Lesson: Year 2, Place value 1. The weakest of five groups was working with the teacher. The five children had been making groups of ten with counters and had been asked to record "one group of ten and two ones" on the whiteboard. Two of them had written it in words, but the teacher was aiming at the "number sentence" type of representation, or at least "12" so that she can point to the ten and the ones.

Speaker	Dialogue (edited)	My comments
Claire	I've thought of it. (takes the pen; starts to write "Ten and two ...")	
Teacher	Mmh. ... No, without the words. Can you write it with the numbers. Just without the words, but using the numbers. ... No, I don't want you to draw the picture, I want you to write the number for me, of what that is. It's (said with pauses between words) one group of ten; and two ones. How can you write that with just numbers? (Stephen raises hand quickly)	Sees that Claire is still thinking in terms of words, so stops hinting and gives specific directions, using repetition for emphasis.
Stephen	I know!	Claire had written 10 + 2 earlier, and had drawn the correct MAB representation, but she does not understand the idea of "one group of 10 and 2 more". The problem seems to be the "groups" idea/word.
Janelle	Eleven. (Claire has drawn a circle with two little lines in it.)	
Claire	It's one group with two in it.	
Teacher	One group with two in it. No, it's one group of *ten*. If I do this on the board ...(teacher draws a multiplication sign) ... what does that mean, that cross. What does that mean? (Claire does not respond.) What does that mean Janelle?	
Janelle	It means ... (pauses).	Teacher presents what she had expected (use of x symbol, leading to lesson aim of 1 x 10 + 2 x 1 representation)
Teacher	What's that sign there? What's that mean?	Janelle does not understand the meaning of x ...
Janelle	A cross.	
Teacher	It's a cross, but what does it mean?	
Claire	I know.	... and guesses. It also seems she does not know the meaning of the equals symbol.
Janelle	Equals?	
Teacher	No, it doesn't mean equals.	
Dianne	Plus?	Janelle and Dianne demonstrate they are not ready do understand what the lesson is aimed at teaching (representing tens and ones as symbols).
Teacher	It does not mean plus ... (teacher looks at Claire).	
Claire	Times?	
Teacher	Times. What else does it mean? (no response) Groups of! Or lots of!	
Stephen	I can do it.	
Teacher	You can do it? Oh, Stephen, you are working so well today. (Teacher gives Stephen the pen).	Claire knows name of symbol, but perhaps not its meaning.
Claire	I know another way.	
Teacher	What?	Teacher probably assumed Stephen has taken her hints.
Claire	Four, plus four, plus four.	Claire is confident with plus. Did she link times with repeated addition?
Teacher	Four plus four, plus four. Good girl! That's a great way of doing it.	
Dianne	I know another way.	
Teacher	You know another way of doing it? What?	
Dianne	You get ... you do a circle and you put ten in it.	
Teacher	Yes. Exactly what Stephen has just done. (Stephen has drawn a	
Dianne	circle with ten little lines in it.) You've got one group of ten. Good	Stephen's representation is not what the teacher expected.
Teacher	boy. This is actually what Stephen has done and this is what Janelle has done, and Claire. You've all done one (writes figure 1, long pause; writes x).	
Stephen	Times.	Stephen knows x means "times" but does he think of it as "groups of"?
Teacher	Put them down (points at Diane who was playing with counters) ... one ... leave them, look ... (points to x) group of (pauses) ten (writes ten, pauses) and (writes +).	Teacher presents what she was aiming at, but lack of understanding from Dianne is obvious. Stephen is the only one attending.
Claire	Plus.	
Teacher	Two (writes 2) ... groups of (writes x) of (writes 1)...	
Dianne	(incorrectly pre-empting what teacher is about to write) Ten.	Diane watches teacher writing but has not followed any of the preceding exchange.
Teacher	One!	
Dianne	One.	

In the next section of the lesson (about fourteen minutes later), the teacher focused on Stephen, who seemed to be closest to understanding her aim—to teach symbolic recording of tens and ones. He remembered "times" and could represent a group of ten. The following interactions took place after the group had explored twelve then fourteen, and after the teacher had asked all of the children to make "one group of ten and three ones".

Laura	I've got it, Mrs W…	Laura has made a row of thirteen counters. Perhaps she doubts this is correct, as Stephen and Claire have put out two groups each (10 + 4). Perhaps she does not understand what the teacher wants her to say in repose to "What have you made?"
Teacher	Good. Show me. Can you show me where your group of ten is? (child points to a long row of 13 counters) Where? 'cause they're all joined up. Can you show me which is your group of ten? (Laura points to first ten, indicating a small gap that follows them) Oh, I see, that's your group of ten and those three ones are separate. They're in another group by themselves. What have you made? (Laura does not respond, after waiting for a few seconds, the teacher attends to Stephen who has fourteen counters out.)	
Laura	I know what I made.	Laura has thought of an appropriate response.
Teacher	(to Stephen) Two groups of one …	
Laura	I know what I made. Mrs W…, Mrs W…	
Teacher	(to Stephen) No, I said to make a group of three leftovers, so you only need three.	
Laura	Mrs W, I know what it is.	Was this just a slip? Perhaps not—he never understood the idea of "groups of one".
Stephen	Three groups of three.	
Teacher	It's not three groups of three.	
Laura	I know what it is.	
Teacher	One … That's one group of one … one group of one, two groups of one … if you put all those three together, that would make a group, and what would that be? How many groups have you got now?	Stephen does not seem to understand that understand that three counters can be called one group of three. (Their recent work with groups has been with MAB blocks, where the only groups are of ten.)
Stephen	Three.	
Teacher	You haven't got three groups. You've got … ? If I had them like that (spreads them out) , now you've got three groups.	
Laura	I know what it equals (loud, moving forward on knees, hand up).	
Teacher	(to Stephen) And how many are in each group? How many's in this group? How many's in this group? How many's in this group?	
Laura	I know what it is (softly, sitting back on heels).	
Teacher	(to Stephen) That would be three groups of one, but if I bunch them all up together and make a little group, like this, in my hand, you're got one group of … how many? (no response) … three. One group of three.	Teacher answers her own question. Does he understand?
Laura	Mrs W…, I know what it is (bending forward).	
Stephen	I get it.	
Teacher	You get it! Oh, good on you. How many's this? (ten counters)	Teacher expresses relief, but what does he "get"? Teacher has changed the focus of his attention from the three to the ten.
Stephen	Ten.	
Teacher	If I separate them all, like this, have you still got one group of ten?	
Stephen	No	
Teacher	No, what have you got? (long pause, no response; pushes the ten counters together again) How many?	Stephen is really struggling with the "groups" language.
Stephen	One group of one. Two groups of (hesitantly) two …	
Teacher	Two groups of? They're still ones.	Models teachers' counting of ones (probably what he "got" previously); but not the expected answer because he is splitting up the group of ten.
Stephen	One. Three groups of one, four groups of one, (continues[inaudible])..	
Laura	Mrs W…, I know what it is now.	
Teacher	Nine groups of one, ten …	Teacher emphasises the "group" idea that is needed to understand place value. Stephen understood this earlier,
Laura	Mrs W…, I know what it is.	
Teacher	Yeah, ten groups of one. If you put them all together, in a little	

	bunch, in a group, (emphasises each word) one group of ten.(turns to Laura) Show me your one group of ten and your three left over. What is it?	but does he now that he has started seeing teen a "groups of one"?
Laura	Thirteen.	Laura's demand is attended to. She has not realised the teacher is seeking a "groups of ten" response.
Teacher	Thirteen! Good. (to Dianne) Did you know it was thirteen?	
Dianne	Thirteen.	

At the end of the lesson, Stephen appeared to understand, although close analysis of the transcript and video actually gives little evidence of this. While other pupils were floundering, unable to grasp the "groups of ten" concept, Stephen certainly gave the impression that he had grasped what had been explained, and the teacher gave him some positive feedback.

Teacher	Two. So you've got ... one group of ten ... and you've got two left over. You've got one group of ten, right? And you've got two left over. How many have you got all together?
Janelle	Um.
Teacher	Count them all together. You've got?
Claire	&(simultaneously) Twelve. Twelve.
Laura	(teacher points to three children, each of whom says "Twelve"; turns to Janelle; points to Janelle).
Janelle	(Not looking at counters, copying the girls ho called out) Twelve.
Teacher	Twelve! Can you count your up and make sure? (Janelle starts counting.)
Stephen	That was easy.
Teacher	Oh, I'll have to think of a harder one for you next time?
Janelle	It's twelve.
Teacher	Very good. Okay. (looks directly at Stephen and smiles) Very good. (gives directions for packing up)

Of course, this teacher may be accused of allowing a boy to dominate her attention, and that criticism would be justified for this latter part of this lesson. However, in other lessons taught by the same teacher this was not the case, and boys were left out of the teaching while a girl who seemed to be taking a real interest in the new concept became the focus of attention. What seems essential to note is that these snippets are representative of a broader problem (not a necessarily a gender one), as follows.

Analysis of the four tapes of ability-grouped lessons, followed by a focused re-visiting of the other tapes, showed that in lesson after lesson, teachers responded to and came to concentrate their attention on, the child or group of children who seemed closest to meeting the teachers' aims. This was apparent for significant sections of lessons taught by all four teachers, with a variety of maths topics, and using any pattern of grouping or class organisation, and whether the teachers were attending to lower- or higher-ability or mixed groups of pupils.

Typically, the pupils who attracted attention came to the learning activity with what seemed to be the highest level of background knowledge, appeared to listen well, and gave indications (body language, responses, etc.) that they were understanding the instruction process and/or content—even if this were not the case. These children tended to give the teacher positive feedback with smiles and indications of a willingness to "have a go", and in contrast with the others were not distracted by the wider classroom environment or by the presence of manipulatives to play with. In short, they appeared engaged, and this engaged teachers.

These children were not necessarily the "stars" of the class—they varied with topics, grouping, etc. As in the lesson above, it generally took some time to identify them, and to begin to give them a disproportionate amount of teaching time and energy. Then the teachers worked hard with these students to, as socio-cultural theorists would say, pull them forward in the zone of proximal development (Vygotsky, 1962), possibly in an attempt to meet the teachers' aims for the activity rather than because of any conscious effort to develop one particular child's strengths.

After the lessons, whenever teachers were available for short discussions with me, a standard question I asked was "Which children seemed to understand (the concept the teacher had just articulated as his/her aim for the lesson or activity)?" In most cases, the teachers named children who had exhibited the behaviours outlined above. When asked how they knew that these children had understood, teachers actually mentioned behaviours easily identified in the videos (such as listening, responding, seeming confident) rather than getting work right or answering questions correctly. Thus the teachers' judgements about understanding were dependant on behaviours not necessarily linked with or correlating with the understanding of mathematical concepts.

Some implications of these findings

A study of only four teachers, and then only four ability-grouped lessons amongst the many that were videotaped, does not present enough breadth to make claims about this mode of classroom organisation. However, as limited as the research context and the data are, the findings of this part of the research project have important general implications for teachers.

First, it cannot be assumed that any group of children, no matter how small and no matter how they are grouped, have similar understandings of (a) instructions, or (b) mathematical concepts and processes, or (c) the individual and combined words and symbols we use to represent mathematical ideas. This brings into question the ability of teachers to plan lessons in full ahead of time unless very open and adaptable questions and conceptual contexts are employed.

Second, organising children into small groups based on what seems to be similar ability does not, in itself, allow teachers to develop the mathematical understandings of each child—inclusive practices, awareness of individual differences, and a spreading of any teacher's attention are still essential. There can be no assurance that skills mastered in one context (such as groups of tens with MAB) can be transferred into other contexts (such as groups of one with counters), or that every child will be able to make these links when they are explained and demonstrated by the teacher.

The next two implications are linked. Third, the aims that any teacher has for a lesson, or even one learning activity, are likely to be easier for some children to guess or grasp—perhaps because of their stronger procedural and/or content knowledge on entering the activity. Fourth, there is a danger that teachers will get more positive feedback from these children and hence will shape the lesson around their particular needs, subsequently giving only those individuals a feeling of achievement.

Fifth, there is a broader implication of this research—although I have not analysed any of the data with this point in mind—that certain types of pupils are most likely to exhibit the positive behaviours that attract teachers' attention. These include children who are overtly confident—a characteristic that is often linked with elements such as gender, sociocultural backgrounds, maturation, language ability, and self esteem arising from past successes (and the interaction of such factors). They also include children who are "good at maths". Such patterns of interaction would only serve to reify these advantages and to reinforce the disadvantage of others.

Conclusion

It has not been the aim of this paper to criticise the practice of ability grouping *per se*. In fact, it is likely that the pattern of usage observed has its advantages. For instance, it is possible that Stephen would have been likely to be left with less positive feelings after working with a more heterogenous group. Neither has it been my intention to find fault with the way that this form of grouping is used in a specific school or by particular teachers. It is clear that the school and teachers involved in the study use achievement groups temporarily and flexibly, with a knowledge of the potential effects of more fixed arrangements as well as with the interests of all students at heart. However, I do wish to raise the awareness of teachers and teacher educators in relation to the precarious nature of the assumption that commonly provides a rationale for this mode of classroom organisation: that there is a necessary connection between ability grouping and the development of better mathematical understanding.

A broader issue is that if the above pattern of interaction is present with small "like-ability" groups taught by experienced teachers who have good mathematical knowledge and excellent classroom organisation skills (that is, with the best-case scenario), then it is also likely to be present with other pedagogical contexts that are less controlled. It is clear that teachers need to be aware of the need to draw out and build on the mathematical understandings of children within different forms of classroom organisation, and to be trained in ways of doing this. At least, this study implies the need for teachers, teacher educators, and researchers to work together to develop more inclusive practices, no matter what form of grouping is used.

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