Identifying Separate and Connected Knowing in Mathematics Education

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This paper describes the development of a questionnaire to identify Separate and Connected Knowers (Belenky, Goldberger, Clinchy and Tarule (1986) in mathematics. Justification for the selection of items is provided from the work of Belenky et al (1986), Buerk (1985), Becker (1995, 1996), Koch (1996), and Erchick (1996). The questionnaire was administered to 67 university students as part of a larger study. Interviews within the main study showed that the questionnaire had served its purpose; it had accurately identified a group of Separate Knowers in mathematics.

Introduction

The purpose of the main study referred to within in this paper was to investigate the influence of moral orientation (Gilligan, 1982) on participation in traditional mathematics education. However, in this paper I report only on the methodology used to generate the sample for the main study interviews. The influence of moral orientation on participation in mathematics education has been discussed in a preliminary way elsewhere (Ocean, 1997).

Over the last decade, Belenky, Goldberger, Clincy and Tarule's description of Women's Ways of Knowing has been adapted to mathematics education by Buerk (1985); Becker (1995, 1996); Morrow and Morrow (1995); Koch, (1996); Erchick (1996), Morrow (1996), and Boaler (1997a, 1997b). These authors describe the Separate Knowing approach to mathematics as rule-based, memory-reliant, individualistic and competitive. In contrast, a Connected Knowing approach to mathematics is concept-based and creative, and places the emphasis on co-operation rather than competition. Such an approach has been found to produce greater success for girls (Boaler, 1997a) and women (Rogers, 1990). Erchick expresses one of the concerns of those who write in the area of Connected Knowing in mathematics: "If mathematics is (or is perceived to be) a formal system that threatens connectedness, a language that expresses power-over and control, a masculine space, a space that disallows subjectivism, perhaps most women really would choose not to participate in it" (Erchick, 1996, p.120). That is, if mathematics is presented only as Separate Knowing (sometimes called "traditional" mathematics education in this paper), the attraction and retention of female students will be low.

The questionnaire was designed to identify individuals who had had an extreme Separate Knowing experience of mathematics education at school. It identified nine such individuals from a sample of 67 third- and fourth-year university students enrolled in engineering and education courses. Subsequent interviews with six of these students confirmed the questionnaire's usefulness as an identifier of a Separate Knowing school mathematics education. No claims are made here for any wider application of the questionnaire than the generation of a sample for later interviews. However it is suggested that the questionnaire, because it did serve its purpose, may be useful as a basis for further work in this area.

Women's Ways of Knowing

Women's Ways of Knowing was developed into a theory of intellectual development by Belenky, Goldberger, Clinchy and Tarule (1986). Prompted by Perry's (1970) widely-accepted model of intellectual development, which used only interviews with men to validate the model, they asked what else women might have to say about their own intellectual development. They conducted interviews over an extended period of time with 135 women of diverse ages, races, and circumstances.As a result, Belenky et al posited five "stages of knowing"; silent knowing, received knowing, subjective knowing, procedural knowing, and constructed knowing. These stages are not necessarily developmental; that is, women do not necessarily pass through them in this order, or through all of them.

The five stages of knowing were expanded with reference to mathematics education by Erchick (1996). Erchick's work will now be referred to in detail to describe each stage. This will be followed by Table 1, which draws on the work of Belenky et al (1986), Becker (1995, 1996), Koch (1996), Buerk (1985), and Erchick (1996) to further illustrate each of the stages.

Women's Ways of Knowing in Mathematics

In interviewing women who were primary school teachers about their experiences of mathematics, Erchick repeatedly heard talk of the *silent* way of *knowing*. These women asked no questions, and few asked teachers for help. They felt voiceless, and stayed silent regarding mathematics until well into adulthood. Erchick suggests that the silent learner may fit rather well into the mathematics classroom, especially in the primary school years, and "may even be rewarded for their silence with praise for being a good student" (Erchick, 1996, p. 112).

In contrast, the *received knower* depends on words; she learns by listening. But she listens to the voice of an authority - to her, there is only one right answer that the teacher will dispense. The formal system of mathematics, with its "one way to do things" should feel comfortably familiar to women in this way of knowing (Erchick, 1996, p. 112). This type of knower was evident in Erchick's (1994) interviews with primary school teachers, with one woman, for example, describing her expectation that simply attending all classes, listening, and taking notes would be sufficient to produce learning (Erchick, 1996, p. 113).

With the *subjective knower*, it is not only the authority who knows. With this perspective, women begin to become their own authorities. Belenky et al (1986) found that women move into this perspective as a result of having encountered a failed male authority. They still depend on authorities, from and with whom they learn a truth that they know and share, but these are usually now female authorities, more like themselves. For the subjective knower, truth is a private and individual matter for everyone. Abstraction, logic and analysis, which might produce common truths, are distrusted (Erchick, 1996, p. 114).

Procedural Knowing consists of two kinds of knowing, Separate Knowing and Connected Knowing. These are not regarded as gender-specific, but they may be gender-related - the preferred styles of men and women respectively (Belenky et al, 1986). For the connected knower, understanding is predominant- for the separate knower, knowledge is predominant. The model of Separate Knowing closely reflects the individualist and competitive values of traditional mathematics education, while Connected Knowing is more representative of a style that places emphasis on co-operation, discussion, and group work. Erchick (1996) observes that the connected knower finds both expertise and constructive criticism within the group, and so finds it helpful to maintain a group connection. The separate knower, on the other hand, is involved with others more as competitors than collaborators. She learns to distance the self from the object of knowledge. Certainty, mastery and control are features of separate knowing.

The key to *constructed knowledge* lies in the integration of thought, feeling, and experience (Erchick, 1996). The constructed knower can call on both connected and separate knowing when appropriate to create or recall the mathematics that she needs.

Table 1 provides a summary of these Ways of Knowing in mathematics, with a description of how the knower might behave and an example of what the knower might say in the mathematics classroom.

Table 1 Women's Ways of KnowingIn Mathematics Education
(drawn from Belenky et al (1986), Buerk (1985), Becker (1995, 1996), Koch (1996), Erchick (1996)).

Ways of Knowing	The Knower	The Knower's Voice
Silent Knowing In the first stage knowing is subliminal. It is not articulated, and the knower does not believe that she can learn from her own experience (Becker, 1995).	She does not speak, and does not expect reasons to be given. She does not believe that she would understand any explanation given (Erchick, 1996).	Silent knowers rarely speak. They may be rewarded for their silence with praise for being a good student (Erchick, 1996).
Received Knowing Received Knowers believe they are only able to receive and reproduce knowledge and not to produce knowledge on their own. Knowledge emanates from external authorities (Koch, 1996).	The received knower will feel confused if asked to do original work (Erchick, 1996). A received knower is not ready to give up her belief that mathematical knowledge comes from the teacher and that mathematics is a series of calculations (Koch, 1996).	"How could she learn if he wouldn't pass along the answers?" (Belenky et al, 1986, p.40)
Subjective Knowing Authority locates within the knower - knowledge derives from what "feels right". For a subjective knower, there is no place for the integration of contradictory evidence - it is ignored. There is absolute truth, but it exists only for the individual (whereas the silent and received knowers believe it to be universal) (Erchick, 1996).	Students might rely on other students who show them how to do the work. These students may be seen as authorities even though they might have failed the course previously. The experience of difficulty and effort may be the common bond between a subjectivist learner and the new authorities who would be people very like the student herself (Erchick, 1996).	"I know that base angles on an isosceles triangle are equal. Just look at them: they're equal" (Becker, 1995, p. 163).

Procedural Knowing consists of two kinds of knowing, Separate Knowing and Connected Knowing. These are not regarded as gender-specific, but they may be gender-related - the preferred styles of men and women respectively (Belenky et al, 1986). **Separate Knowing** Separate Knowing is based on the use of impersonal procedures to establish truths (Becker, 1995). The Separate Knowing approach gets right to the solution in a structured, algorithmic way, stripping away any context (Buerk, 1985). **Connected Knowing** Connected Knowers build on personal experiences. Context is important. A Connected Knower will want to know what circumstances led you to your perception, rather than what logical steps led to your conclusion (Becker, 1985). Connected knowing is complex, related, considers many things simultaneously.	**Separate Knowing** In problem solving, for example, a problem would be solved repeatedly by many examples solved exactly the same way. **Connected Knowing** In problem solving, for example, the same problem would be solved in different ways, rather than additional examples solved exactly the same way.	**Separate Knowing** Example : "the main thing that's nice about math is: that's it. You do what it says right there. I like that...one can read a Shakespeare play for the rest of one's life and not have any definite idea of what's going on" (Becker, 1996, p. 22). **Connected Knowing** Example : "(Mathematics) is beautiful. It ties in with so many things, it encompasses so many things" (Becker, 1996, p. 23)
Constructed Knowing When an individual has learned both separate and connected ways of knowing, and can use each appropriately, this is called Constructed Knowing. In this stage answers are dependent on the context in which questions are asked and on the frame of reference of the asker (Becker, 1995).	Constructed Knowers are able to use both Separate Knowing and Connected Knowing when required, in solving problems. "The solution of an equation is dependent on the domain being considered, and what can be proven is dependent on the axioms being assumed...here, the knower would use the creative aspect, induction, together with the rules of discourse, deduction, in order to know something" (Becker, 1995, p.168).	"Let's physically compare the angles" (Becker, 1995, p. 165). "Tell me why you think that base angles are equal" (Becker, 1995, p. 165).

Developing The Questionnaire

In the pilot for the main study (not reported here - see Ocean, 1997), I had interviewed six adult women (friends and colleagues) to investigate the influence of moral orientation on participation in traditional mathematics education. I had obtained a sample by making requests of friends and colleagues who really disliked mathematics education and who I thought might provide rich data. This approach to generating a sample produced a particular problem, however, that an example will illustrate. One of the participants, Sue, turned out to be a silent/received knower. Sue viewed mathematics as entirely beyond her, and regarded that as entirely her fault. She was difficult to interview because she didn't see herself as participating in mathematics in any way - in effect, I got very little data from her at all. In the main study, I wanted to avoid this pitfall, and so developed the questionnaire to identify separate knowers and connected knowers. I thought they would (a) actually know more mathematics and (b) be more articulate about their own mathematics education than silent, received, or subjective knowers. I subsequently decided to interview only the Separate Knowers.

Methodology

Fifteen items chosen to identify Separate Knowing in mathematics education were taken from Schoenfeld's (1989) unrelated questionnaire *Explorations of Students' Mathematical Beliefs and Behavior.* The advantage of selecting items from an existing questionnaire are that some of the possible pitfalls of questionnaire design, such as ambiguous language, wording effects, and question order (Loewenthal, 1996) have already been addressed. Justification for the selection of items from Schoenfeld's questionnaire will be given shortly.

Schoenfeld's questionnaire contains 81 items. It surveys attributions of success and failure, perceptions of mathematics and of school practice, the nature of (geometric) proofs, reasoning and constructions, motivation, and personal and scholastic performance (Schoenfeld, 1989, p. 342). While the items were designed for senior high-school students, the language was suitable for adults. Minor changes were made to better fit some items to general mathematics, and to Australian English. For instance, the most extensive change was made to the item *(When I do a geometry proof), I'm finished if I can't remember the next step.* This was altered to *(When I solve maths problems), I'm often stuck if I can't remember the next step.*

Fifteen multiple-choice items amongst the eighty-one in the questionnaire corresponded principally to either Separate Knowing or to Connected Knowing. These fifteen items were sent to a reviewer who had completed a PhD on Separate and Connected Knowing two years previously. She was asked to rate each item. Inter-rater agreement for the fifteen items was 100%.

The questionnaire asked participants for their view of school mathematics. They were asked to disregard their experience of university mathematics, if they had any, when answering. Responses were asked for on a Likert-type scale, with participants being asked to strongly agree, agree, remain undecided, disagree, or strongly disagree with each item. While no more weight was given in the analysis to the stronger comment, five rather than three options provides more choice to the respondent. Space was left under each item for additional comment.

Items that were chosen for the questionnaire as representative of Separate or Connected Knowing reflect the descriptors found in Table 2 (Ways of Knowing in Mathematics). Recalling Erchick's (1996) comment that mathematics may be perceived as a formal, controlling, masculine space, with no room for subjectivism, Separate Knowing in mathematics involves facts, formulae, known procedures, reliance on memory, certainty (right vs wrong), lack of creativity, and no acceptance of alternative methods for solution.Mathematics is seen as an absolute and finite body of knowledge. Within Connected Knowing, mathematics involves creativity, conceptual understanding, complexity, context, inter-relatedness of ideas, and acceptance of alternative methods for solution.

It should be noted that items chosen to represent Separate Knowing may have been selected on the questionnaire by a silent, received or subjective knower. Alternatively, these knowers may have selected the 'undecided' option provided. Correspondingly, items that were chosen to reflect Connected Knowing may have been selected by a constructed knower. Results from the questionnaire should be read with this in mind. While it did not affect the usefulness of the questionnaire in my study (because a silent or received or subjective knower is still likely to have had a "traditional" mathematics education, and similarly a constructed knower will have been exposed to a connected knowing approach to mathematics), it may be an important point in any further application of the questionnaire.

Table 2 links each of the fifteen multiple-choice questionnaire items with the relevant descriptors of Separate Knowing and Connected Knowing.

Table 2: The Links Between the Questionnaire items and Indicators of Separate Knowing and Connected Knowing in Mathematics.

Q1. Maths is mostly facts and procedures that have to be remembered. Indicators of Separate Knowing: Reliance on memory, facts, formulas and procedures.
Q2. Maths makes you think creatively. Indicator of Connected Knowing: Creativity.
Q3. When the teacher asks a question in maths, you have to remember the right answer to answer it correctly. Indicators of Separate Knowing: Reliance on memory, certainty.
Q4. When the teacher asks a question in maths, there are lots of possible answers you might give. Indicators of Connected Knowing: Opportunity for creative input, context, alternative paths to solution.
Q5. Good maths teachers show you the exact way to answer the mathematics questions you have to do. Indicators of Separate Knowing: Limited or no opportunity for creative input.
Q6. In maths, something is either right or it's wrong. Indicator of Separate Knowing: Certainty.
Q7.Good maths teachers show students several different ways to look at the same question. Indicator of Connected Knowing: Alternative paths to solution are possible, creativity.
Q8. Everything important about maths is already known. Indicators of Separate Knowing:Limited or no Opportunity for creative input, maths is absolute and finite.
Q9. In maths you can be creative and discover things by yourself. Indicator of Connected Knowing: Creativity.
Q10. Some people are good at maths and some just aren't. Indicator of Separate Knowing:Certainty.
Q11. Problems can be done correctly in only one way. Indicator of Separate Knowing:Certainty, maths is absolute and finite.
Q12. To solve maths problems you have to be taught the right procedure or you can't do anything. Indicator of Separate Knowing: Certainty, maths is absolute and finite.
Q13. The best way to do well in maths is to remember all the formulas. Indicators of Separate Knowing: Reliance on memory , facts, formulas and procedures.
Q14.When I solve maths problems, I can only prove something a mathematician has already shown to be true. Indicators of Separate Knowing: Limited or no opportunity for creative input, Certainty.
Q15. When I solve maths problems, I'm often stuck if I can't remember the next step. Indicators of Separate Knowing: Reliance on memory, facts, formulas and procedures.

Questionnaire Administration: The questionnaire was administered to 46 (fourth-year) graduate Diploma in Education students, and 21 (third-year) Electronic Engineering students, in class time. It took about twenty minutes to complete. All volunteers read an information sheet and gave their consent in writing. Refusal rates were 4/44 Diploma in Education (Primary), 8/14 Diploma in Education (Secondary), and 13/33 Engineering.

***Questionnaire** Analysis*: Each response was scored as either undecided or Separate Knowing (S) or Connected Knowing (C). Agreement with a statement originally chosen to represent Connected Knowing was given a C score, while disagreement with the statement was given an S score, and vice versa. Undecided responses were not counted in the total. As the number of undecided responses varied between students, a student's view of mathematics was labelled S or C if 75% or more of their responses fell into that category. Otherwise, their responses were assigned to the combined (S/C) category. This produced small groups of people at either extreme, which suited the questionnaire's purpose of identifying an extreme sample. Subsequently it was decided only to interview those with an extreme Separate Knowing mathematics education.Table 3 shows the distributions of the scores in total, and across courses.

Results and Discussion

Table 3: Distribution of Ways of Knowing by Course of Study

Course of Study	Separate Knowing	Connected Knowing	S+C Knowing
Engineering (n=21)	3.	3	15
Dip. Ed (Primary)(n=40)	6	11	23
Dip. Ed (Sec) (n=6)	0	1	5
Total (n=67)	9 (13%)	15 (22%)	43 (64%)

The questionnaire thus generated a pool of nine individuals who had a view of mathematics that was at the extreme end of the individualist, rule-based, competitive - "traditional" - approach to mathematics education. Six of these were later interviewed in depth in semi-structured interviews, primarily to seek their moral response to their experience of traditional mathematics education. However, the Separate Knowing categorisation of each student from the questionnaire was cross-checked in these interviews. This was done in two ways; by (a) asking questions in the interview about what mathematics was like for them at school, and listening for a Separate or Connected Knowing description of mathematics at school, and (b) by using a number of written descriptors of Separate and Connected Knowing as a prompt for a discussion of their view of mathematics. These descriptors were taken from the work of Buerk (1985); Becker (1995, 1996); Morrow and Morrow (1995); Koch, (1996); Erchick (1996), and Morrow (1996) . They were written on cards, and participants chose those that *most* and *least* represented mathematics to them. For Separate Knowing the descriptors included absolute, finite, rules, algorithms, just and fair. For Connected Knowing, the descriptors included connected, intuitive, contextual, tolerant, models and creative.

This cross-check was an additional attempt to ensure that I was indeed listening to students who had a Separate Knowing mathematics education. It was not intended as a check on the validity of the questionnaire, as the numbers checked (6 out of 67) are too small. However, all six students did have a Separate Knowing approach to mathematics. None had a silent, received, subjective or constructed way of knowing. Because the questionnaire proved an accurate predictor of the participant's high school mathematics experience for the six individuals in this study, it very adequately served the purpose for which it had been designed. No claims are made here for its wider application. However, since it affirmed rather than contradicted data obtained from interviews some researchers may think it worthwhile as a starting point for further development of items to assess Separate and Connected Knowing in mathematics. For instance, following a re-examination of the "Mathematics as a Male Domain" scale (Forgasz, Leder, and Gardner (in press), these researchers are trialing some of the questions related to Separate and Connected Knowing drawn from this questionnaire (personal communication, Leder and Forgasz, 1998).

Conclusion

This paper describes the development and implementation of a questionnaire which was designed to identify Separate and Connected Knowing in mathematics, drawing on the work of Belenky, Goldberger, Clinchy and Tarule (1986), Becker (1995, 1996), Koch (1996), Buerk (1985), Erchick (1996) and Schoenfeld (1989). It proved effective in identifying a sample of students who had had a traditional Separate Knowing mathematics education. No claims are made for its wider application than the generation of a sample for in-depth interviews. However, it may be of use as a starting point in the development of further questionnaires to measure Separate and Connected Knowing in mathematics education.

References

Belenky, M., Clinchy, B., Goldberger, N., & Tarule, J. (1986). Women's ways of knowing: The development of self, voice, and mind. New York, Basic Books.

Becker, J. R. (1995). Women's ways of knowing in mathematics. In P. Rogers, & G. Kaiser (Eds). Equity in mathematics education: Influences of feminism and Culture (pp. 163-73). London: The Falmer Press.

Becker, J. (1996). Research on Gender and mathematics: One feminist perspective. Focus on Learning Problems in Mathematics, 18, 1,2&3, Center for Teaching/Learning of Mathematics, pp. 19-25.

Boaler, J. (1997a). Reclaiming school mathematics: The girls fight back. Gender and Education, 9 (3).

Boaler, J (1997b). When even the winners are the losers: Evaluating the experience of "top set" students. Journal of Curriculum Studies, 29 (2) 165-182.

Buerk, D. (1985). The voices of women making meaning in mathematics. Journal of Education, 167, 59-70.

Erchick, D. (1994). Mathematics teacher leader pilot final report: Columbus Public Schools. Columbus, OH.

Erchick, D. (1996). Women's voices and the experience of mathematics. Focus on Learning Problems in Mathematics, 18, 1,2&3, Center for Teaching/Learning of Mathematics, pp. 105-22.

Forgasz, H., Leder, G., and Gardner, P. (in press). The Fennema-Sherman "math as a male domain" scale re-examined. To be published in the Journal of Research in Mathematics Education.

Gilligan, C. (1982). In a different voice: psychological theory and women's development. Cambridge, MA: Harvard University Press.

Koch, L. C. (1996) The development of voice in the mathematics classroom, Focus on Learning Problems in Mathematics, 18: 164-75.

Loewenthal, K. M. (1996). Introduction to psychological tests and scales. UCL Press: London.

Morrow, C., & Morrow, J. (1995). Connecting women with mathematics. In Rogers, P., & Kaiser, G. (Eds). Equity in mathematics education: Influences of feminism and culture (pp. 13-26). London: The Falmer Press.

Morrow, C. M. (1996). Women and mathematics: avenues of connection, Focus on Learning Problems in Mathematics, 18, 1,2 & 3, pp.4-18.

Ocean, J. (1997). Is there both care and justice in the mathematics classroom? The Weaver,Vol 1, No. 1. http://www.latrobe.edu.au/www/graded/JOed1.html. Accessed March 1998.

Perry, W.G. (1970). Forms of intellectual development in the college years. New York: Holt, Rinehart & Winston.

Rogers, P. (1990). Thoughts on power and pedagogy. In L. Burton,(Ed). Gender and mathematics: An international experience (pp. 38-46). London: Cassell.

Schoenfeld, A. (1989). Explorations of students' mathematical beliefs and behavior. Journal for Research in Mathematics Education, 20 (4), pp. 338 -355.