Is there a need for a *Mathematics Intervention* program in Grades 3 and 4?

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The standard of numeracy skills of young Australians continues to be of concern to mathematics educators, teachers, parents and politicians. This paper discusses whether a mathematics intervention program developed for Grade 1 warrants extension into Grades 3 and 4. The focus is on 1997 data that show that although Grade 3 and 4 students had made significant progress since Grade 1, there is still need for additional assistance in Grades 3 and 4.

Introduction

Considerable concern is currently being expressed by mathematics educators, teachers, parents and politicians, about the standard of the literacy and numeracy skills of young Australians. Recently Australian Commonwealth, State and Territory education ministers agreed that "every child leaving primary school should be numerate, and be able to read, write and spell at an appropriate level" (Masters & Forster, 1997). They further agreed that "every child commencing school from 1998 will achieve a minimum acceptable literacy and numeracy standard within four years" (McLean, 1997, p.1). In this context numeracy was defined as "the effective use of mathematics to meet the general demands of life at school and home, in paid work, and for participation in community and civic life" (McLean, 1997, p. 2).

A National Plan (Kemp, 1997) has been developed to ensure that these goals are met. The plan requires that education authorities provide support for teachers in their task of identifying children who are not achieving adequate literacy and numeracy skills and in providing early intervention strategies for these students. *Mathematics Intervention* is one such program that was designed, implemented and monitored by classroom teachers in the hope that their intervention strategies would assist children achieve adequate numeracy skills (Pearn & Hunting, 1995; Pearn & Merrifield, 1996).

Mathematics Intervention

Mathematics Intervention was developed as a collaborative project involving the principal and staff of a state primary school in the metropolitan area of Melbourne and mathematics educators from a nearby university (Pearn & Hunting, 1995; Pearn & Merrifield, 1996). The program, first implemented in 1993, aims to identify, then assist, children in Grade 1 "at risk" of not coping with the mathematics curriculum, as suggested in the National Statement on Mathematics for Australian Schools (Australian Education Council, 1991).

In November 1997 testing was extended into Grades 3 and 4. At that time 57 students were tested. Of these, 32 had been tested when they were in Grade 1. There were two reasons for the 1997 testing program. The first was to validate the effectiveness of the Grade 1 *Mathematics Intervention* program currently operating at the school. The second was to determine whether there was a need to establish a similar program for Grade 3 and Grade 4 students. This paper focuses on the second of these aims, that is, to determine whether there is a need to establish a *Mathematics Intervention* program for students in Grades 3 and 4. To investigate this need a procedure similar to that introduced when developing the Grade 1 *Mathematics Intervention* program was used. As it was anticipated that the Grade 3 and 4 *Mathematics Intervention* program would be an extension of the same underlying mathematics research the rationale and scope of the Grade 1 *Mathematics Intervention* program is documented.

Previous research

Since its introduction in 1993, the Grade 1 *Mathematics Intervention* program has featured elements of both Reading Recovery (Clay, 1987) and Mathematics Recovery (Wright, 1991; 1996). *Mathematics Intervention* offers students the chance to experience success in mathematics by developing the basic concepts of number upon which they build their understanding of mathematics. Students are withdrawn from their classes and work in small groups with a specialist teacher to assist with the development of their mathematical skills and strategies.

The theoretical framework underpinning the Grade 1 *Mathematics Intervention* program is based on recent research about children's early arithmetical learning (Steffe, von Glasersfeld, Richards and Cobb, 1983; 1988; Wright, 1991; 1996) and about the types of strategies used by children to demonstrate their mathematical knowledge (Gray & Tall, 1994). In particular, the program aims to assist students' progression through the counting stages as developed by Steffe et al. (1983, 1988) and which are summarised below. It was planned that the Grade 3 and 4 *Mathematics Intervention* program would similarly use this same theoretical framework.

Counting stages: Research into *Mathematics Intervention* has confirmed the research by Steffe et al. (1983; 1988) that stated children move through the counting stages documented below.

1. *Perceptual*. Students are limited to counting those items they can perceive.
2. *Figurative*. Students count from one when solving addition problems with screened collections. They appear to visualise the items and all movements are important. (Often typified by the hand waving over hidden objects.) If required to add two collections, one of six items and one of three items, the student must first count the six items to understand the meaning of "six", then count the three items, then count the whole collection of six and three.
3. *Initial number sequence*. Students can now count on to solve addition and missing addend problems with screened collections. They no longer count from one but begin from the appropriate number. If adding collections, one of six items and one of three items, students commence the count at six and then count on: six, seven, eight, nine.
4. *Implicitly nested number sequence*. Students are able to focus on the collection of unit items as one thing, as well as the abstract unit items. They can count-on and count-down, choosing the most appropriate to solve problems. They generally count down to solve subtraction problems.
5. *Explicitly nested number sequence*. Students are simultaneously aware of two number sequences and can disembed smaller composite units from the composite unit that contains it, and then compare them. They understand that addition and subtraction are inverse operations.

As well as assisting students to progress through the counting stages the *Mathematics Intervention* program focuses on the strategies used by students when solving mathematical tasks. In particular, the focus is on work by Gray and Tall (1994).

Strategy choice: Assessment for the Grade 1 *Mathematics Intervention* program has confirmed research studies by Gray and Tall (1994) that have shown that young children who are successful with mathematics use different types of strategies from those who are struggling with mathematics. Students struggling with mathematics are usually procedural thinkers, dependent on the procedure of counting and limited to the "count-all" and "count-back" procedures. In summary, Gray and Tall (1994) defined procedural thinking as being demonstrated when:

> ... the numbers are used only as concrete entities to be manipulated through a counting process. The emphasis on the procedure reduces the focus on the relationship between input and output, often leading to idiosyncratic extensions of the counting procedure that may not generalize (p. 132).

For example, when asked to count back from a given number, students have been heard to count up to each number before responding with the number required. This procedure is highly unlikely to generalise into a backwards counting sequence.

While some students tested in Grade 1 were dependent on rules and procedures, other students gave instantaneous answers. For example, when students who gave an instant correct response to tasks were asked "How did you do that" they gave several different strategies they could have used and then checked that their solutions were correct. According to Gray and Tall (1994) this use of known facts and procedures to solve problems, along with the demonstration of a combination of conceptual thinking and procedural thinking, indicates that these children are proceptual thinkers. Gray and Tall (1994) defined proceptual thinking as:

> ... the flexible facility to ... enable(s) a symbol to be maintained in short-term memory in a compact form for mental manipulation or to trigger a sequence of actions in time to carry out a mental process. It includes both concepts to know and processes to do (pp. 124-125).

Another feature of the Grade 1 *Mathematics Intervention* program requires teachers to use a one on one clinical interview to assess the extent of the child's mathematical knowledge. This allows the teacher to observe and interpret the child's actions as he/she works on a set task. Leder (1990) advocates encouraging students to talk about their mathematical strategies to obtain information on children's own mathematical understanding and knowledge.

> Encouraging students -- particularly those deemed to have difficulties -- to talk about mathematics and listening carefully to what is being said provides invaluable information about students' learning. It is a strategy that can be used readily by classroom teachers to probe and monitor their students' learning. The data obtained can serve if necessary, as a rich data base for subsequent error analysis (p.26).

Grade 1 Initial Assessment

In 1997, children who had been tested in 1994 and 1995 when in Grade 1, were now in Grade 4 and Grade 3 respectively. As we are going to look at the progress of these children later in the paper, Table 1 shows the results of the testing not only from 1997, but also from 1994 and 1995. The results of the small group of 32 students who were tested both in Grade 1 and in 1997 are given in bold in Table 1. The results in the brackets are those of the whole year level.

Table 1: Results from Level AA tests (Grade 1): 1994, 1995 and 1997 (in percentages).

Year	ones	back 20-1	back 10-1	twos	fives	tens	6 -12	before /after	14 beads	pattern	numeral	6+3=	10= +2
1994													
n=18	**94**	**50**	**89**	**44**	**44**	**61**	**61**	---	**83**	--	**67**	---	**72**
(n =48)	*(98)	(69)	(77)	(44)	(48)	(67)	(69)		(88)		(71)		(71)
1995													
n=14	**100**	**64**	**93**	**71**	**50**	**57**	**86**	**86**	**100**	**100**	**43**	**93**	**64**
(n=26)	(96)	(65)	(96)	(50)	(50)	(54)	(88)	(92)	(100)	(96)	(46)	(85)	(69)
1997													
(n=62)	100	73	97	53	47	66	63	84	90	94	37	79	56

* Figures in brackets are the results of testing all students at this level and in this year.

Of the 13 tasks used in the initial Grade 1 interviews in 1994 there is good overlap in the results for both the small group and the whole cohort of students for all but two tasks: counting backwards from 20 and counting backwards from 10. The small group achieved better results for counting backwards from 10 but were less successful in counting back from 20 than the whole cohort.

In 1995, the small group's results overlapped with the whole cohort results in all but three of the tasks: they achieved better results as a group on counting by twos and the first counting stage task but were less successful on determining the number before and /or after a given number.

Table 1 highlights that the results for the small group are representative of the Grade 1 results for the whole group. Most Grade 1 children, over the three years given, were successful in counting forwards by ones to 20 and backwards by ones from ten, counted patterns of dots and counted out exactly 14 beads. They were less successful identifying the numbers between the numbers six and twelve or determining numbers "before" or "after" a given number. Children included in *Mathematics Intervention* were those who displayed difficulties with most tasks and were at or below Stage 1 and used procedural strategies such as "count all".

Extending *Mathematics Intervention* into Grades 3 and 4

The 1997 Grades 3 and 4 interview tasks were designed by two of the teacher-clinicians involved in designing the original set of tasks for the Grade 1 clinical interview. It was decided to maintain the focus on number as initiated in the Grade 1 *Mathematics Intervention* program although we acknowledge the importance of a breadth of mathematical activities. However, as the Curriculum and Standards Framework [CSF] states:

> As a student acquires an appreciation of different levels of understanding of number, intersections occur with other mathematical studies in ways which give number a central unifying role. Work in the number strand links with work in all other strands ... Later work in all strands requires that they understand and work confidently with all kinds of numbers (Board of Studies, 1995, p. 42).

To determine each child's development in terms of the counting stages, and the types of strategies they used in solving mathematical tasks, the interview included tasks testing knowledge of verbal counting sequences, whole numbers, oral responses to addition and subtraction facts and word problems. Whole number tasks were given orally by the teacher while cards showing the task were also presented simultaneously. For example, for the subtraction task *52 -17* the child was asked "Can you tell me what 52 take away 17 is?" and at the same time a card was presented on which 52 *-17* was written horizontally. As with the Grade 1 interviews the Grade 3 and 4 tasks were designed so children were given every opportunity to demonstrate the various strategies they used to solve mathematical tasks. This interview was trialed in November 1997 and is now under review.

Grade 3 and 4 results from clinical interviews

Most Grade 3 and 4 students were able to count by ones, fives and tens but several students had difficulty counting forwards by twos starting at 15 and backwards by tens starting at 120 although most students could successfully count backwards by tens from 100. It appeared that they had learnt some sequences by rote but had no strategies for counting from a specific number.

Most students were successful in identifying the number 'one more than', 'ten more than', and '100 less than' 342. They found the missing addend and subtraction tasks much more difficult due to an apparent reliance on rules and procedures. For example, six students from the same class all responded that 52 - 17 was 45 because "you take the small number from the large one". Difficulties experienced with subtraction are reflected in Table 2 where results from computational tasks are given. Although most students were able to identify the word problem as subtraction, 15 of the group of 30 were unable to complete the computation successfully due to a reliance on faulty or "buggy" algorithms.

Table 2: Results of 1997 Grade 3 and 4 computational skill tasks (in percentages).

Year Level	addition facts	subtraction facts	addition problem	subtraction problem	multiplication problem	division problem
Grade 3 n = 13 (24)*	**86** (71)*	**50** (54)	**86** (88)	**50** (42)	**57** (63)	**64** (63)
Grade 4 n = 17 (33)	**72** (70)	**61** (61)	**89** (91)	**56** (52)	**56** (55)	**72** (79)
Total n = 30 (57)	**78** (70)	**59** (58)	**86** (89)	**53** (47)	**56** (58)	**69** (72)

* Figures in brackets are the results of Grades 3 &4 interviews of 57 students.

To illustrate the additional information revealed in using a clinical interview rather than a paper and pencil test, Table 3 includes three responses given by students in response to the subtraction word problem. Students were given the problem which they were asked to read aloud. If they experienced difficulty in reading the problem it was read for them.

Table 3: Typical responses to the subtraction word problem.

Task	Lynda	Mike	Barry
Richard is 131 cms tall. Mary is 17 cms shorter than Richard. How tall is Mary?	Successfully counted back by ones from 131 keeping track on her fingers. Gave correct answer	Took 10 away from 31 then 7 away from 21. Gave correct answer.	Written algorithm. 131 - 17 126 Incorrect answer.

It can be seen that Lynda used a procedure of counting back by ones, Mike successfully used an 'invented' strategy while Barry used a 'buggy' or faulty procedure which he used consistently through the interview for all subtraction tasks.

A preliminary investigation of the Grade 3 and 4 results has highlighted extremes in the knowledge and strategies used. Those who appeared to be struggling with mathematics relied on their memory of rules and procedures. While two of these students had participated in the Grade 1 *Mathematics Intervention* program, there were many students who had not had the opportunity to do so because they had come into the school after Grade 1.

Grade 3 and 4 students who were successful with the mathematical tasks were flexible in their use of strategies (Gray and Tall, 1994) and used appropriate "invented strategies" (Carpenter et al., 1998) when tasks were presented without access to paper and pencil. Carpenter et al. (1998) noted that many students constructed their own procedures for adding and subtracting multi-digit numbers without using physical materials. They called these strategies "invented strategies". They found that students who use invented strategies developed a knowledge of base-ten number concepts earlier than students who relied more on standard algorithms. In their study they found that children made "relatively few conceptual errors in using invented strategies, whereas children exhibited a number of buggy procedures in using standard algorithms " (p. 19).

Scoring of interview results

In Figure 1 the two sets of interview results were compared for the small group of 32 students who were tested when in Grade 1 and again in 1997. Since only two students had participated in the Grade 1 *Mathematics Intervention* program the data were given for all 32 students who had participated in Grade 1 and 1997 testing programs. To compare these results, solutions to tasks were given a numerical score, based on the knowledge displayed and the types of strategies used. For example, for the counting task in the Grade 1 interview, when asked to count backwards by ones from 20, children would score 0 if no attempt was made to count backwards, a score of 1 if successful counting backwards from 10, and a score of 2 if successful counting backwards from 20. The maximum score for the Grade 1 test was 24 and for the Grade 3 and 4 test was 54. In order to compare these results only scores for tasks common to both Grade 1 interviews in 1994 and 1995 are included. All scores have been converted to percentages.

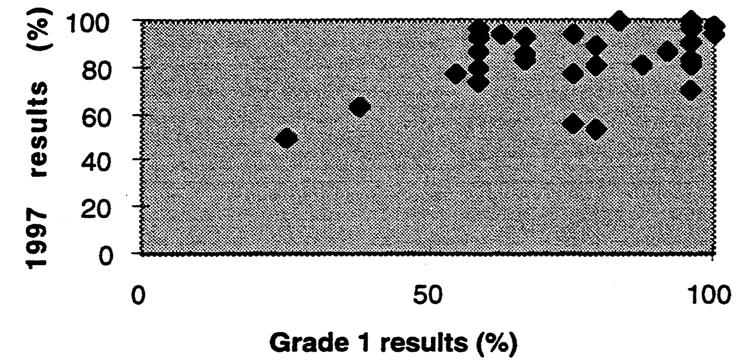

Figure 1: Comparison of results from 1994/1995 with those of 1997 (n=30).

The median for the Grade 1 results was 18.5 (77%) and the lower quartile score was 14 (58%) while the median for the 1997 results was 46.5 (86%) and lower quartile score was 42 (78%). From Figure 1 it can be noted that there are students whose results were:

- in the lower quartile in both Grade 1 and 1997,
- in the lower quartile of the 1997 results but not in the lower quartile in Grade 1,
- not in the lower quartile in Grade 1 or in 1997 but who appeared to be performing at a comparatively lower level in 1997 than predicted from the Grade 1 results.

In Table 4 the focus is on the number of children whose results place them on, below or above the 25th percentile of the 1997 results which are categorised in relation to their Grade 1 results.

Table 4: Number of children in lower quartile in 1997 and in 1994/1995 (n=32).

	1997 < 78%	**1997 = 78%**	**1997 > 78%**	**Total**
Grade 1 < 58%	**3**	**1**	**0**	**4** (13%)
Grade 1 = 58%	**1**	**0**	**4**	**5** (15%)
Grade 1 > 58%	**3**	**1**	**19**	**23** (72%)
Total	7(22%)	**2** (6%)	**23** (72%)	**32**(100%)

Although these results must be treated with caution as this is a small sample of 32, the group is representative of the results for the larger group of 278 Grade 1 children interviewed over the last six years. The results in Table 4 indicate there were three students with results in the lower quartile in both Grade 1 and 1997. There was also one student whose 1997 score was in the lower quartile in 1997 but whose score was on the 25th percentile in Grade 1 and one student whose result was on the 25th percentile for 1997 but in the lower quartile for Grade 1. Thus there were five students ranked on the 25th percentile or in the lower quartile for both sets of results. That is, 15% of students tested had not improved their class ranking in mathematics testing over since being tested in Grade 1. There were also students whose results in 1997 appear to be less than their Grade 1 results indicated. There were three students who were above the 25th percentile in Grade 1 but in 1997 were in the lower quartile. There was also one student whose Grade 1 results were higher than the lower quartile but in 1997 was on the 25th percentile. This means that of the nine students on or below the 25th percentile in 1997, four were on or above the 25th percentile in earlier testing.

This highlights a need for some additional assistance for two groups of students tested in Grades 3 and 4 in 1997. That is, those who have not improved their ranking since the Grade 1 testing, and those that appear to have been less successful than their Grade 1 results would indicate. Further analysis will attempt to determine reasons for this lack of success.

Discussion

Of concern to the researcher is the small group of students who were found to be in the lower quartile when tested in 1997. Of most concern is the small group of students who do not appear to have succeeded to the extent that their Grade 1 results indicated. The numbers in the sample are too small to be considered representative but two of the three boys concerned were in Grade 4, the other in Grade 3. This means that the two boys in Grade 4 were performing at a lower level than all but 4 of their peers from both Grade 3 and 4.

This research has highlighted the differences in children's mathematical knowledge and the type of whole number strategies they use when solving tasks set in different contexts. Testing in Grades 3 and 4 revealed that all children had improved considerably in their mathematical knowledge, and were using more sophisticated strategies than they had used in Grade 1. Preliminary results from the Grades 3 and 4 testing have shown that children who are successful with mathematics at this level are flexible in their mathematical thinking and use a variety of strategies, including invented strategies. However, there were Grade 3 and 4 students who were still experiencing difficulties with the verbal counting sequences. There were several students who used the 'count-back' and the 'count-on' strategies while some used 'invented' strategies (see for example Mike's response in Table 3). However, children like Barry (see Table 3), struggling with mathematics, rely on rules and procedures even when these are inefficient and unreliable. Hiebert and Lefevre (1986) have highlighted the difficulties of relying on rules and procedures:

> Procedures that lack connections with conceptual knowledge may deteriorate quickly and are not reconstructable; they may only be partially remembered and combined with other subprocedures in inappropriate ways; they often are bound to the specific context in which they are learned and do not transfer easily to new situations; and they can be applied inappropriately without the benefit of a validating critic to check the reasonableness of the outcome (pp. 21-22).

Conclusion

It would appear from our preliminary study that Grade 3 and 4 students "at risk" need special assistance to succeed at this level. Although all students had showed improvement in their mathematical knowledge and the types of strategies they used since the Grade 1 testing some children had improved to a lesser extent than their peers. There were also students who were not achieving at the level predicted by their Grade 1 results.

For students "at risk" of not succeeding at mathematics, one of the greatest difficulties is that they do not possess either rich mathematical ideas or the powerful strategies that will enable them to use their mathematical knowledge to improve and enhance their mathematical thinking. A *Mathematics Intervention* program at Grade 3 and 4 would not only need to determine the child's counting stage as for the Grade 1 program, but would also need to concentrate on developing more efficient strategies for computational tasks. All students need to develop strategies to enhance their understanding of number concepts rather than being left dependent on rules and procedures which make no sense to them. Students should be encouraged to share their own "invented strategies" with each other and use and discuss alternative strategies for solving mathematical problems in the social context of their classroom.

This research program is continuing. The need for additional assistance and provision of special programs for students "at risk' with the 1998 testing of Grade 3 students, who were tested in Grade 1 in 1996, is being examined. The challenge is now to develop a *Mathematics Intervention* program for Grade 3 and Grade 4 students based on mathematics education research. This program should include the research of Steffe et al. (1993, 1988) into the five counting stages and that of Gray and Tall (1994) into strategy choice. The research of Carpenter et al. (1998), which highlights the need for students to develop flexibility in their mathematical thinking and a range of strategies so they become less reliant on rules and procedures, should also be included.

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