Year 3 students' place-value misconceptions: Another look at MAB

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Sixteen Year 3 students were interviewed individually about two- and three-digit numeration concepts, before and after ten groupwork sessions. One group of questions asked students about values represented by each digit in a two-digit number. Responses revealed that some students had not developed the idea that the "tens" digit represents a collection of ones, but believed that it represented only its face value. Comparison of results before and after the groupwork sessions showed that a number of participants had improved understandings of values represented by individual digits at the conclusion of the study. These results have implications for how place-value concepts are developed with place-value blocks.

Introduction

The author has previously reported on a study in progress, in which Year 3 students' understanding of place-value concepts is being evaluated (Price, 1997). The central objective of the study is to investigate differences in learning that result from use of either conventional place-value blocks or computer software. Participants were placed in groups of four students, and each group was involved in ten daily groupwork sessions, in which they used place-value material (blocks or a computer) to model numbers to answer questions about two-digit and three-digit numbers. This paper focuses on results from two interviews given individually to each participant, before and after groupwork sessions.

The Base-ten Numeration System

The Hindu-Arabic numeration system is a place-value system with a base of 10; each individual digit in a multidigit number represents a value equal to the product of its face value and the appropriate power of ten. It enables numbers of virtually any size to be represented unambiguously and succinctly, and is an essential component of many fields, including finance, industry and science. Numbers recorded in the base-ten numeration system are also used heavily in other school curriculum areas, including science, economics and geography. Thus learning place-value concepts is an important foundation for students' future needs, both within and beyond their schooling.

It is appropriate to consider the concepts that underlie the base-ten numeration system that are regarded as essential for students to understand by the end of primary school, and how they can be taught. These concepts can usefully be grouped into three types: basic number concepts, concepts of abstraction and system concepts.

Basic number concepts relate to single digit numbers, and are prerequisites to developing an understanding of the base-ten numeration system. These include knowledge of the "face values" of the number digits 0 to 9, the use of symbols to represent quantities, and basic counting concepts and skills. Each of these understandings is necessary before a child can begin to learn about multidigit numbers.

Concepts of abstraction relate to basic number concepts, and are also prerequisites for grasping multidigit number concepts. At the heart of the use of numerical symbols (whether written, pictorial or physical) is a central abstraction—that numbers can be

represented or symbolised, but can never be perceived directly with the physical senses. Abstract concepts include the concept of number and the twin concepts that symbols represent numbers in written communication and that material objects, such as counters and place-value blocks, may represent quantities by illustrating their numerosity.

Third, *system concepts* relate to the numeration system itself and the rules that govern how it is used to represent quantities. System concepts include the ideas that unit values represented in each successive place map onto successive powers of ten, that the value of a place is determined by reference to the right-most whole-number place and that the value of each digit equals the product of the digit's face value and the appropriate power of ten.

Primary mathematics curricula typically include the development of understanding of these many concepts as a major objective. Benchmarks for inclusion of concepts at various year levels provide for development of the concepts described above over the entire span of primary year levels. Basic number concepts are developed in the first year of schooling, as young students are introduced to the principle of cardination, to the skills of counting a finite number of objects fewer than 10 and to the symbols that represent the numbers 0 to 9. In the succeeding years, students are introduced to the numeration system, starting with two-digit numbers and building up to very large numbers. For example, in Queensland two-digit numbers are introduced in Year 2, three-digit numbers in Year 3, four-digit numbers in Year 4 and five-digit numbers and beyond in Year 5. Teaching about numeration of decimal fractions similarly progresses year by year.

Use of schema in understanding numeration concepts

Cognitive science has provided researchers and educators with a useful notion, the *schema*, which can help to understand what is necessary for a student to develop understanding of a complex concept such as numeration. Schemas have been defined variously by different authors; Derry (1996) noted that "*schema* is a general term connoting virtually any memory structure" (p. 167). Because numeration concepts described earlier are integrated into a complex whole, a schema is needed to relate the various components of the system to each other. Thus for a student to gain a deep understanding of the base-ten numeration system, he or she needs to develop a schema to organise the thinking involved in using the system to solve mathematical problems. Such a schema should enable the student to see connections between aspects of numbers and their uses in novel situations.

The idea of assisting students to develop powerful schemas to understand the complexities of the base-ten numeration system is very attractive to mathematics educators. However, anecdotal and research evidence of the understandings of students reveals that for many students such organised mental structures for multidigit numbers have not developed. Rather, many students seem to have little idea of the connections among numbers and number ideas, and instead rely on memorised procedural rules to answer mathematical questions (e.g., Fuson & Briars, 1990). There is therefore a pressing challenge for mathematics teachers to teach in a way that helps students to develop understandings that can be applied to new situations, based on understandings of the structures inherent in the numeration system. The research in progress described in this paper is showing that use of place-value blocks by young students does not necessarily result in the development of powerful models of the base-ten numeration

system, and that teachers need to monitor carefully how such materials are used by themselves and by students.

Modelling numbers with concrete materials

In attempting to help students to develop accurate, flexible understandings of the numeration system, primary teachers use a variety of concrete materials in their teaching of mathematics. The extent of this use is indicated by work conducted by Gilbert and Bush (1988) in the US, and by Howard, Perry and Conroy (1995) in NSW. Both these studies showed frequent use of concrete materials by the surveyed teachers. Gilbert and Bush found that 65% of respondents reported using concrete materials at least once per week; in Howard et al.'s study 62% of respondents reported using concrete materials "often", with only 2 teachers out of 249 reporting that they did not use them at all.

Place-value blocks as analogues of numbers

One material that needs to be singled out in the context of helping students to develop number understandings is place-value blocks (known in Queensland as Multibase Arithmetic Blocks [MAB]). As pointed out by English and Halford (1995), the structure of place-value blocks mirrors the structure of the base-ten numeration system analogically, with the size of each block being proportional to the value it represents, compared to the unit cube.

Place-value blocks have been recommended for teaching numeration concepts since they were developed in the 1960s, and are "probably the most commonly used analogs in the teaching of numeration and computation" (English & Halford, 1995, p. 105). This statement is supported by Howard et al. (1995), who reported that the material most commonly used by their respondents was "base 10 material", used by 84% of respondents. Howard et al. noted that this frequency of use was probably related to two factors. First, 96% of teachers responding to the survey stated that they believed that concrete materials "benefit children's learning". Second, "the syllabus and many commercial mathematics programs encourage the use of such material" (p. 6).

Research results

This confidence in place-value blocks' effectiveness for teaching numeration concepts, expressed by both academics and classroom teachers, has to be tempered by other comments, however. Research into the effects of instructional use of place-value blocks has not produced the positive, unequivocal results that were hoped for. Hunting and Lamon (1995) suggested that many variables may affect results from use of materials, including the type of material, length of time used and teacher training. One fact that needs to be considered by teachers wishing to use concrete materials, pointed out by Hunting and Lamon (1995), is that the mathematical structure of multidigit numbers is not contained in any material; the structure of the numbers themselves has to be constructed in the mind of the individual student. Thus there is no guarantee that use of physical materials will lead to better understanding of numbers. Baroody (1989) added support to this point with his statement that learning can only take place when the "[learning] experience is meaningful to pupils and . . . they are actively engaged in thinking about it" (p. 5).

Thompson (1994) suggested that research studies needed to give attention to the broad picture of the teaching/learning environment to discern reasons for the success or failure of instructional use of concrete materials:

> These contradictions [among findings of different studies] are probably due to aspects of instruction and students' engagement to which studies did not attend. Evidently, just using concrete materials is not enough to guarantee success. We must look at the total instructional environment to understand effective use of concrete materials—especially teachers' images of what they intend to teach and students' images of the activities in which they are asked to engage. (p. 556)

The study reported on here used a descriptive approach to data gathering, relying on videotapes of groupwork sessions to reveal important aspects of the learning environment and its effects on students' learning.

The Research

The research study reported on here investigates Year 3 students' understandings of multidigit numbers. Part of the study involved individual interviews of the participants, in which the student was asked a series of questions probing their understanding of two- and three-digit numbers. The students' responses indicated important mis-conceptions about two-digit numbers held by several of them.

The main part of the study comprised ten consecutive groupwork sessions, in which groups of four students worked on a series of questions on cards, designed to revise two-digit number concepts, and to teach initial three-digit number concepts. Two different "treatments" were used, with half the groups using conventional place-value blocks, and half using computer software, to help them answer the questions.

Participants

Participants in the study reported here were selected from the population of Year 3 students (aged 7-8 years) at a primary school in a small Queensland rural town. Participants were selected at random from two groups: students of either high or low mathematical achievement, based on the previous year's *Year Two Diagnostic Net*, a state-wide test used in Queensland to identify students at risk in the areas of literacy and numeracy. Students were assigned to groups that were matched for ability, on the assumptions that maximum learning would be possible if children in each group were of similar mathematical ability. Each group comprised two girls and two boys. Four groups were used, with one high- and one low-achievement level group using each of the blocks or the computer.

Interview questions

The particular interview questions reported on here were used to ask each student about the values represented by the digits in a two-digit number. Similar "digit correspondence" questions have previously been used by Ross (1989) and Miura, Okamoto, Kim, Steere and Fayol (1993). The student is asked to count a number of objects between 10 and 100, and asked about the value represented by each digit.

The two interviews in the present study included parallel questions, using different numbers. The digit correspondence questions from the first interview are shown in Figure 1. Students were asked to count 24 pop sticks, write the symbol for 24 and

explain the values represented by each digit in the written symbol. In the second interview the same questions were asked, based on a group of 37 pop sticks.

- Show the student 24 pop sticks.
- Ask the student to count them (correct if necessary), and to write the symbol for the number.
- Circle the "4", and ask "Does this part of your 24 have anything to do with how many sticks you have? Please show me."
- Circle the "2", and ask "Does this part of your 24 have anything to do with how many sticks you have? Please show me."

Figure 1 Digit correspondence questions from first interview

Results

Results from the first digit correspondence question above are summarised in Table 1. Of the 16 participants, 14 knew that the "4" in "24" stood for 4 of the sticks, but only 7 could correctly state that the "2" in "24" stood for two collections of ten sticks.

Table 1 *Summary of results of digit correspondence questions (first interview)*

Question: *Does the "4" in your "24" have anything to do with the number of sticks?*	
Response	**Number of Responses**
4 sticks	14
No	2
Does the "2" in your "24" have anything to do with the number of sticks?	
Response	**Number of Responses**
20 sticks	7
2 sticks	8
No	1

All participants but two knew that the "4" in "24" represented 4 sticks. The two students who differed both said that the symbol 4 did not have a referent or referents among the sticks. One student, Kelly, said that "you don't have 2 sticks there and you don't have 4 sticks - that's what I know". She agreed that the symbol "24" represented the group of 24 sticks, but believed that the individual digits did not refer to any particular number of sticks. The other student, Simone, who claimed that the "4" had no referent appeared to have a confusion that was related to place-value blocks. Simone said that there were tens among the sticks, but no ones. She indicated 2 sticks as standing for the symbol "2", but said that a "one" would have to be smaller than a single stick. It appears that she was confusing the shape of a stick with a "ten block" in place-value blocks. Evidently she had learned to associate the long shape of the ten block, rather than its size relative to a one block, with ten. A similar confusion was observed in one of the boys, who asked if he should count the sticks as "1, 2, 3, ...", or as "10, 20, 30, ...".

The most obvious confusion held by these children, however, was that the "2" in the symbol "24" represented only its "face value". This conception of values represented by individual digits in a multidigit number has been reported widely in the

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literature (e.g., Bednarz & Janvier, 1982; Miura et al., 1993). Ross (1989) described it in this way:

> Students interpret each digit as representing the number indicated by its face value . . . . but these objects do not truly represent groups of ten units to students in [this stage]; students do not recognize that the number represented by the tens digit is a multiple of ten. (p. 49)

The participants in this study who revealed this misconception were asked by the interviewer to confirm that the symbol "24" represented all 24 sticks. He then asked about the remaining 18 sticks. Answers given by these children included that the other sticks were "left-overs", were "not included", were "just extras" or were "just left out". These children were asked to explain this, but none were able to do so. A few children did not attempt to explain the 18 extra sticks, but maintained two mathematically-contradictory beliefs about the symbols. They explained that the symbol "24" represented all the sticks, and also said that the "2" and "4" represented only 2 and 4 sticks respectively, but could not integrate these two conceptions into a unified idea.

Second interview results

In the second interview, after the participants had spent ten sessions working with materials modelling two- and three-digit numbers, participants' responses to digit correspondence questions differed. The students were asked to count 37 pop sticks, and to explain the "3" and the "7". Results are summarised in Table 2. There was no significant difference between students who had used blocks and those who had used the computer, so these are not reported here.

Table 2 *Summary of results of digit correspondence questions (second interview)*

Question: *Does the "7" in your "37" have anything to do with the number of sticks?*	
Response	**Number of Responses**
7 sticks	14
No	2
Does the "3" in your "37" have anything to do with the number of sticks?	
Response (initial)	**Number of Responses**
30 sticks	6
3 sticks	7
No	3
Response (final)	**Number of Responses**
30 sticks	11
3 sticks	4
No	1

The main difference between responses on the first and second interviews was that in the second interviews a number of participants changed their responses when the interviewer asked further questions to probe or challenge their initial response. Initially the number of students who showed the correct number of sticks for the symbol "3" was still low—in fact it was one less than the number who answered the equivalent question

correctly in the first interview. However, as the interview progressed, an additional five students agreed that the "3" represented 30 sticks. In some cases this was due to the student changing their opinion without direct prompting from the interviewer. In other cases, students altered their answer after the interviewer asked further questions. Similar questions had been asked in the first interview, but at that time students did not alter their answers significantly. It appears, however, that in the second interview, although initially more than half of the participants were confused about the values represented, with further questioning they were able to give a correct response. This is illustrated by the following transcript excerpt:

I: *Does this part [7] of your "37" have anything to do with how many sticks you have?*

Maxine: Yes.

I: *Can you show me?*

Maxine: [puts out 7 sticks]

I: *Does this part [3] of your "37" have anything to do with how many sticks you have?*

Maxine: Yes.

I: *Can you show me?*

Maxine: [picks out 3 sticks]

I: *Yes, alright. I'll just put those there. So this 7 is for 7 like that and 3 is for 3 like that. What about those [remaining 27 sticks]?*

Maxine: [pauses, then counts out remaining sticks then puts them with the 3]

I: *[laughs] Now you are saying that that 3 is for these?*

Maxine: [nods]

I: *How many are there?*

Maxine: 30.

Maxine apparently realised that the symbol "3" stood for the remaining sticks, though initially she only used 3 sticks. At least two other students realised that there was a contradiction between the symbol "37" representing all 37 sticks, and their belief that the single digits represented only 3 and 7 sticks respectively. They resolved this difficulty by saying that the single digits did not represent any sticks individually, but only when combined into the two-digit number, as shown in the following transcript:

I: *Does this part [7] of your "37" have anything to do with how many sticks you have?*

Hayden: No.

I: *I doesn't? OK, can you tell me what that 7 means?*

Hayden: It's a part of 30 ... it's a part of like in 30 it's a part like ... you count to 30 and then you count 7 more and it ends up 37.

I: *And what does this 3 here mean?*

Hayden: It's up ... it's up to 30 ... like if you count up to 30.

I: *Uh-huh. Can I show you something? If we have 7 sticks like that [puts out 7 sticks], could we say that 7 is for 7 like that?*

Hayden: No, because that ... that's not like [picks up 3 sticks] ... that's only 11 [puts them down next to the 7 sticks].

I: *Why have you got those 3? That's for the 3 is it?*

Hayden: No, those aren't for the 3.

I: *It's not for 3? So the 7 is not for 7 either?*

Hayden: No, because it doesn't make um 37. It only makes 11.

I: *But the whole number written down like that is for all of them?*

Hayden: Yep.

I: *But if you take just the 7 it's not ... you can't take part of them and say that part is for that?*

Hayden: No.

Hayden clearly knew that the two-digit symbol represented all the sticks, but could not find an explanation that allowed for each digit to represent some of the sticks, because together they would not represent the entire group.

Discussion

The results of the second interview seem to indicate that several of the students were in a "period of transition" regarding their understanding of two-digit number symbols. Clearly their understanding was still immature with regards to digits representing collections of ten ones, but they were able to express more accurate explanations of the value of tens digits with some assistance.

The results of this study are showing that (a) the use of place-value blocks needs careful planning, and (b) students' on-going constructions of meaning for multidigit numbers need to be closely monitored. A teacher of young children cannot assume that students are making sense of number representations in the same way that the teacher is.

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