Orchestrating Different Voices in Student Talk about Infinity: Theoretical and Empirical Analyses.

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In this paper we review major theoretical perspectives on differences between students in their thinking and classroom talk, and conclude that Bakhtin's concept of *voice* provides a powerful way of theorising difference that draws attention to the multivocality (inherent diversity) of individual ideas and utterances. It is the orchestration of the diversity of voices that enables new insights to emerge both collectively and individually. We apply analytical tools (derived from Bakhtin) to an episode of collective argumentation focussed on the concept of infinity, and we identify key elements of everyday classroom practice that enable challenging and productive talk to occur.

There is considerable research evidence for the view that specific types of student talk - talk that involves explaining, justifying and representing ideas for others - enables the appropriation of mathematical ways of thinking. Such talk requires certain social conditions to be established within classrooms - social conditions that support students to engage in joint activities, that support students to propose and consider different viewpoints, that support students to explain and justify their ideas to each other, and that support students to evaluate the relative worth of competing proposals. Recently we reviewed evidence arising from our own research programme which indicated that such classroom conditions can be facilitated by following the ground rules and social values inherent in collective argumentation (see Brown & Renshaw,1995;1996; Renshaw & Brown, 1997). Our purpose here is to begin to examine in more detail, using Bakhtin's notion of voice, the dialogues that occur between students during episodes of collective argumentation. In particular, we focus on differences that arise between students as they try to communicate and represent (in words and diagrams) their ideas to each other. We present a range of evidence for the claim, that it is the orchestration of differences in students' talk about mathematics tasks that is generative of new insights and deeper understanding. Orchestration is a key term in our paper because it conveys our view that understanding is a process that involves learning to speak in different voices and being able to move flexibly and smoothly between different voices in order to communicate ideas and convince others of their relevance. Orchestration also conveys the notion that understanding is not achieved by creating insight in isolation, but is achieved by composing ideas with others, learning to speak with others, and learning to incorporate aspects of their performance in ones own utterances.

Theoretical Framing of Differences

Social conflict and cognitive disequilibrium. The view that differences between students are crucial in creating the conditions for change is supported by Piagetian and neo-Piagetian literature which has emphasised the role of social conflict in creating cognitive disequilibrium and conceptual change. Relevant research reviewed by Chapman and McBride (1992) has suggested, however, that conflict per se does not relate to conceptual change. Rather, participation in argumentation - discourse that has the form of a dialogue in which claims are made and challenged by appeal to evidence and reasoning - appears to be the social process that generates cognitive change. This conclusion is supported by research from the cooperative learning literature. Johnson and Johnson (1987) investigated two contrasting formats for group interaction -*concurrence* and *controversy*. The process of concurrence involves maintaining social cohesion within the group by quickly resolving differences and disagreements, whereas the process of controversy involves the exploration and analysis of differences prior to moving towards a joint solution. Research has shown that the process of controversy is productive of deeper understanding than the process of concurrence.

The work of Marton, Asplund-Carlsson and Halasz (1992) is relevant here - though not framed by a cognitive-developmental theory. They focus on intra-individual processes that appear to enable individuals to make progress in problem-solving. The first process - reflective variation - involves considering and alternating between different interpretations of a task. The second process - elaborative variation - describes how particular interpretations are followed and implications are explicated and compared. Their work is consistent with the viewpoint being articulated in this paper, namely, that it is the orchestration of differences in the social domain that enables progress in thinking to occur. Later, we refer to research by Wistedt and Martinsson (1996) who have employed Marton et al's distinctions in considering the different *voices* that can be heard during student discussions.

A sociocultural view - exploratory talk. Argumentation, involving attempts to publicly justify opinions and ideas, needs to be distinguished from quarrelling on the one hand, where the participants simply disagree, and from conflict resolution on the other, where the participants' overriding goal is to reach consensus (concurrence) rather than to search for the best solution to the problem (Chapman & McBride, 1992). Similar distinctions have been proposed also in the neo-Vygotskian literature by Mercer (1995) who defined three social modes of thinking. The first social mode of thinking, *disputational talk*, is characterised by disagreement, assertion and counter-assertion by students as they attempt to work together on a task. Differences in viewpoint, however, do not lead to a pooling of resources or constructive criticism and analysis, but rather to the entrenchment of students within their individual perspectives. At the other pole in Mercer's typology is *cumulative talk,* which is characterised by uncritical compiling and accumulation of ideas. Such talk is equivalent to concurrence-seeking as defined by Johnson and Johnson, where students confirm and repeat each others' contributions as a means of maintaining harmony. So while conflicts are avoided or quickly resolved, there is no attempt to analyse or evaluate the worth of the contributions.

The third social mode of thinking is *exploratory talk,* which is similar to the notion of argumentation and the process of controversy. Talk in this mode is characterised by constructive and critical engagement with the ideas proposed by the partners in the group. Ideas are challenged and defended by proposing justifications, explanations and alternatives. In exploratory talk, claims are publicly accountable and reasoning is visible and therefore open to evaluation and critique. Mercer's neo-Vygotskian theoretical framework emphasises the need for students to gradually appropriate the ground rules of exploratory talk so that their own thinking takes on a dialogical and self-argumentative form. So, engagement in exploratory talk or argumentative discourse, is theorised to have quite general cognitive effects by both neo-Piagetians and neo-Vygotskians, the former describing the effects in terms of cognitive restructuring, and the latter in terms of the internalisation of social forms and processes.

A sociocultural view - collective argumentation. In previous papers (Brown,1994; Brown &Renshaw,1995; 1996; Renshaw & Brown,1997) we have reported research on a process akin to exploratory talk, namely, *collective argumentation.* In one study (Brown, 1994) we assessed the talk of students in collective argumentation using the analytical device of *transacts*, which gives a measure of the extent to which students' talk, and specifically their explanations and justifications, makes reference to previous contributions in the discussion. Using transacts as a indicator of the quality of argumentation within groups, we found that students showing most change in their mathematical thinking were in groups where talk was of a higher quality and more responsive to the partners' ideas.

While our purpose is not to conduct a reductive analysis of the sub-components of collective argumentation, it is clear that the first phase of collective argumentation is important for differences between students' ideas to be revealed and explored in talk. Collective argumentation involves students working in small groups (varying from 2 to 5 members per group) where initially they individually "represent" a problem (using pictures, diagrams, drawings, graphs, algorithms, numbers etc) and then "compare" their representations with those of other group members. This phase of individual representation and comparison provides the potential for differences in understanding to be exposed and examined. Subsequent talk by the students regarding the appraisal of representations is guided by the keywords - "explain", "justify", "agree". Finally, moving from the small group to the classroom group, the thinking within each group is validated as it is presented to the whole class for discussion. Evidence from disparate theoretical traditions indicate, therefore, that differences in thinking between students creates the context for productive talk, but the theoretical terms that should be used to describe such differences, and what types and levels of difference optimise learning remain issues of contention.

Differences That Facilitate Understanding

Cognitive-developmental tradition. Piagetian and neo-Piagetian research has been guided by a stage developmental theory in describing differences between students. With regard to their understanding of a specific concept, like number or weight, students have been classified as non-conservers, transitionals, or conservers. Alternatively, across a range of concepts ("decalage"), students can be developmentally classified as transitional between pre-operational and concrete-operational thinking if they show conservation of only a few concepts, or as concrete-operational thinkers if they conserve a range of concepts across a range of materials (continuous and discontinuous materials, regular and irregular objects). Finally, they might be classified as formal thinkers if they show more generalised, hypothetical and abstract thinking. The units of analysis here are the students (classified according to stage) rather than ideas, the assumption being that students are restricted in their thinking according to hierarchically organised stages. Change for students has been found to be optimal when the partners in the discussion are "one-stage different".

Interweaving concrete and abstract concepts. Our own research on collective argumentation has been framed by sociocultural theory which opens up a number of alternative ways to describe differences between students, and to consider when differences are optimal. Initially, we drew on Vygotsky's work which emphasised the contextual embeddedness of conceptual understanding. In everyday activities at school and home, children use concepts with the assistance of partners, who can enable them to apply concepts in quite abstract and general ways prior to gaining conscious awareness and personal control. Thus, at a particular point in time, children may engage in various forms of conceptual thinking depending on the challenges and support offered by others. Here, difference in conceptual thinking is not determined by internal structures, but is dynamic and created interactively as students work with ideas introduced by the participants in the dialogue.

During collective argumentation, we've found that students will represent a problem in quite disparate ways. Some students, in an attempt to represent a problem mathematically, draw a diagram that foregrounds irrelevant and marginal features. A partner might draw a similar concrete representation but s/he appears to have a better grasp of the mathematical significance of the drawing. A few students will work with quite abstract and sophisticated representations that summarise and foreground the central mathematical details of the problem. To coordinate their actions as they work towards a joint representation, students must speak and act in ways that are responsive to the words and actions of the other participants. In such dialogues, individual contributions can be paraphrased, reinterpreted, or particular words replaced by more general terms. In this manner, collaborative interaction can enable students to enter into more abstract and general ways of speaking about, and acting towards objects - that is, it can open up a zone of proximal development. Deeper understanding does not require replacement of the more local and concrete concepts with the more abstract and general concepts, but the weaving together of these two forms in a process of mutual transformation. In summary, the Vygotskian perspective frames difference as a contrast between the local personal experiential and concrete versus the general abstract and formal. However, progress in thinking requires the integration of these two forms of knowing rather than the replacement of one form by the other.

Voice - speaking as a member of diverse language communities. Recently we have begun to move beyond Vygotsky's focus on weaving together everyday and scientific concepts, to explore Bakhtin's theory of *voice.* He formulated a theory of language which emphasised the active, situated, and functional nature of speech as it is employed by various communities within a particular society. In conversation with others, *personal voice* is given to words that are employed by a wider community of users. So, in any individual utterance, a number of *voices* can be heard - the voice of the particular person but also traces of the voices of other community members who had previously employed those words to convey their own meanings. This provides a powerful method of theorising difference. Difference, diversity, or multivocality occurs not only between speech partners, but within individual utterances as speakers use the words and phrases of others to construct and convey an idea. Who is the author of the idea? At one level it is the speaker, but the speaker depends on the appropriation of others' contributions to not only compose the idea itself but to convey it to others. So, insight and understanding arises from the orchestration of difference - the effective composing and recomposing of different voices that produces a performance (explanation, justification, solution to a problem, etc) that is appreciated and accepted as valid by the community.

Privileged voices - everyday, school, and mathematics voices. Schooling is not neutral, however, in providing spaces for all student voices. Schooling privileges particular ways of talking. For example, Wistedt and Martinsson (1996) analysed a session of joint problem solving by groups of 11 year-olds who were working on the problem of dividing 100 into three equal parts and expressing their answer as a decimal. Different voices were identified in the children's discussions - including the discordant everyday voice of practical reasoning - "*But why do we have to make three bookstands. He just needs two, one on each side of the books.*" This student attempts to speak outside the privileged voice of schooling - redefining the problem so that the dilemma is resolved practically rather than mathematically. His utterance might also be read as a critique of the task students are asked to solve in schools. The student's partner reasserts the voice of schooling by saying - "*Yes, but now they've made it in this weird way so we will have to solve it as it is, won't we.*" This student argues that they have to accept the premises of school mathematics tasks, that is, the purpose of the task is not primarily practical (producing actual objects such a bookstands), but educational - to learn strategies and concepts related to division and decimals. Note also that she makes clear her solidarity with the first speaker by using the contrast of "*them*" versus "*us*" ("*they've made it in this weird way so we*) , and distancing herself somewhat from the schooling voice by noting it was "*weird*". The subtle power of this utterance is to both reject the first speaker's practical voice and to invite his participation in her proposed new voice of school mathematics. In this way, the practical voice was marginalised and the students turned their attention back to the task - now interpreted as a school task where the major concern was to produce an acceptable answer. This school voice can be heard in the comment by one student "*I don't think you can solve this task if you're not allowed to write 33 plus 33 plus 34*" Here the student's major concern is to produce an acceptable answer - "*if you're not allowed. to write...*" The term "*allowed* " has connotations of following directions and relying on an authority (the teacher) to determine correct from incorrect ideas - in other words the traditional voice of schooling.

The move from a school mathematics voice per se to a mathematics voice per se is quite subtle, but it occurs as the students move beyond their concern to simply produce an acceptable answer, and begin exploring and speculating about the nature and limitations of the number system. Wistedt and Martinsson provide examples of the students employing the voice of mathematics as they face the puzzling problem of expressing one-third as a decimal - 0.33. They realise that the number system has limitations in expressing certain quantities - "*You can go on forever. It will never come out even. But the more 03's you write, the closer you will get*" . This also provides the space for them to reconstruct the notion of error or remainder - "*33.33 plus 33.33 plus 33.33 and then you add up ... you'll get a nine here and another nine and another... and then plus 0.01 And then it will come out even. There will be a small error that we will have to add*". Note the student's use of the pronoun "*you*" which indicates that she is speaking in the general voice of the mathematician - that is the procedures and outcomes she is reporting are not personal opinions but general claims about the nature of such problems.

Analysis of a Fragment of Collective Argumentation

The notion of *multiple voices* seems particularly powerful in capturing the flow of dialogue between students, and providing an insight into the way differences in the dialogue are integrated - orchestrated - to enable new understandings to be co-constructed. To examine these issues further we turn to an analysis of an episode of collective argumentation where the problem of representing infinity is addressed by students in their groups. The students in Wistedt and Martinsson's study had visited the notion of infinity when they discovered the recurring decimal in expressing one-third. In our study, the notion of infinity was introduced explicitly so we could observe the way that children co-constructed arguments to support their particular representation of infinity, and to identify the different voices that could be heard during their discussions. In this paper, we focus particularly on the last phase of collective argumentation where students are recounting and presenting their thinking to the whole class for discussion and validation.

The students were in Year 7 from a Brisbane primary school, and had used collective argumentation as part of their classroom learning processes across all curriculum areas for more than two school terms. In this particular episode they were asked to co-construct a definition of infinity that they could share with the class. The majority of the interaction below occurred after 15 minutes of individual representation and small group discussion, when the teacher invited groups to share their responses with the class.

The Transcript and Analysis

Section 1: Turns 1 - 6

Prior talk involved the teacher introducing the topic, with a number of question-answer sequences between teacher and students. Groups then worked on the problem of representing infinity for about 15 minutes prior to class presentations

Susan, Andrea and Kelly come out to present to the class. They have represented 'infinity' by drawing a line segment on the whiteboard.

1 Susan:	In the dictionary infinity means an infinite number, or time . . . and we thought the soul was infinite, it would go . . . If this line was infinite it would go forever, and God's love for his people is infinite because he will love us forever.
2 Teacher:	What do you think of that class?
3 Anne:	The line is wrong because. . . the line (that the group has drawn) has two ends.
4 Simon:	It's not a true line . . . a line goes on forever.
5 Teacher:	How could we represent that? I understand your point, saying that this (referring to the group's line segment) is not a line, it's a line segment. How could we represent an actual line?
6 Anne:	You could draw a circle.

In this section of transcript (Turns 1-6) the students and teacher introduce a number of privileged voices, that actually open up alternative possibilities for the direction of the dialogue. There is the authoritative and abstract voice of the dictionary, which enters via Sarah's reading of the definition. Talk of the "soul" and "god" makes the voice of religion audible (this arises because "infinity" is studied in this Catholic primary school both in religious lessons and in mathematics), while the drawing of the line segment and the comment "if this line were infinite it would go on forever" brings the voice of mathematics into the conversation. Anne's comment (*"The line is wrong"*) directs the attention of the students primarily to the mathematics of the situation, and away from exploring further the other possibilities. At Turn 6, Anne's suggestion that a circle would be a good representation of infinity reveals why she rejected the line - a line has two ends while a circle has no ends.

Section 2: Turns 7 - 17

Another group, Carey, Melanie and Margaret approached the black-board and drew a circle, and explained that it goes on and on like time. Ten turns at talk involving the teacher and other children clarified the idea that a circle or any closed figure could represent infinity because the line never ends and can be overlapped an infinite number of times. Then the teacher summarised their talk and recalled the line segment introduced by Susan

7 Teacher: Going back to Susan's idea. How could we take that line segment and represent it as an infinite line? With no beginning and no end. One way is with a circle, but as Simon said, there seems to be something missing. Susan, do you want to take back control of the conversation.
8 Susan: Bonnie.
9 Bonnie: Well I did a spiral and it keeps on going around and around and when it finishes, it doesn't actually finish. It keeps on going around when it finishes, it just goes around again.
10 Susan: Margaret.
11 Margaret: You could put two arrows on it (the line segment) to show that it goes on forever.
12 Teacher: Would you like to do that? (To Susan)
Susan and Kelly put an arrow on each end of the line segment.
13 Teacher: What's that represent Margaret?
14 Margaret: It just represents that a line can keep on going that way (points) and that way (points), so that it doesn't end.
15 Teacher: It doesn't have a . . .?
16 Margaret: It doesn't have a beginning and it doesn't have an end.
17 Teacher: So, what are some of the important qualities of the idea of infinity? (To class)

In this section of the transcript (Turns 7 to 17) the students are guided to consider the various representations of the concept of infinity proposed so far - circles, closed figures of any shape, spirals, and lines. In clarifying how a line segment might be drawn to better convey the idea of infinity, Margaret suggests adding two arrows at each end to indicate that the line goes on forever. This enables Margaret, with some support from the teacher, to highlight that infinity has neither a beginning nor an end. Discussion and appraisal of the different representations seems to sharpen the students' awareness of the details of the concept - not only doesn't it end, but it also doesn't have a beginning.

Section 3 Turns 18 - 22

At the end of this discussion and summary Simon and Angela came out to present their ideas.

18 Angela: We drew a clock and we had, um, about, an infinite number of handles, because time goes on for an infinity. That's how we represented that, because time goes on.
19 Teacher: I didn't understand that phrase, could you say it again please.

20 Angela: Well, we drew a clock and we had an infinite amount of handles, the little things that go around, because time never stops. It just keeps going around.

Teacher clarified with Simon and Angela the term for the hands of the clock, and then Angela continued

21 Angela: Time has no beginning and no end like numbers. And we had the dictionary meaning which says this - infinity has the state of being infinite, infinity of the universe, infinity of space, time, quantity - so infinite space, so, it's so that you can't describe it. Um, (infinite) mass is the concept of increasing (mass) without volume, so we thought that we would make a meaning of our own. So we thought that infinity means everlasting number, object and the universe. So infinity is an everlasting thing.

Teacher recalled the key ideas from Angela's presentation.

22 Angela: Infinity can(not) be determined or explained over a vast amount or period of time, because it is an everlasting idea. And I made this up. I think the word infinity is similar to life. No one can fully explain it and just like infinity it has many definitions. We can't really explain life and we can't really explain the word infinity

The contribution of Angela and Simon (Turns 18 to 22) to the discussion of infinity reveals the capacity of students in this class to work with multiple representations and to move between them flexibly - privileging one voice or another depending on their purpose. Angela and Simon reported four representations including a drawing of a clock with a large number of hands, a dictionary definition, a personal paraphrase of the dictionary, and in Turn 22, Angela adds an analogy she has constructed - infinity is like life - because neither concept can be adequately explained. In this episode, we can hear in the student voices that of the epistemologist . Angela not only could entertain multiple representations of infinity, but had begun to consider the notion that any representation of an abstract idea such as infinity is inadequate. This voice is heard first in her talk about the dictionary definition "*...so, it's so that you can't describe it*" This indicates that the dictionary definition was not much help in building an understanding. She reports that they then translated the dictionary terms into their own words, "*...so we thought that we would make a meaning of our own*" They substituted the term 'everlasting' for 'infinity' which is a small change but one that in Jay Lemke's terms is crucial in "making texts talk" and linking abstract and minimalist definitions to personal understanding. However, for Angela the term '*everlasting*' retains similar connotations of mystery and inexplicability as the term '*infinity*' - to paraphrase Angela in Turn 22 - "infinity like life is an everlasting idea that can't be explained".

The voice of the epistemologist is heard in Angela's recount of her group's work as she reports the multiple representations that she and Simon had composed and compared in terms of adequacy and accessibility. In addition to this overarching voice, Angela articulates a series of voices moving from the transituational voice of the dictionary, to the social voice of her group - "*so we thought that we would make a meaning of our own*" - then to the personal voice - "*..And I made this up. I think the word infinity is similar to life.*" However, she doesn't stay within the personal voice. After claiming her authorship of the idea, she moves into the generalised voice of the expert (scientist, philosopher, mathematician) - "*No one can fully explain it and just like infinity it has many definitions. We can't really explain life and we can't really explain the word infinity*". "No one" and "We" convey Angela's intention to speak authoritatively not on behalf of her small group, or personally, but generally on behalf of humankind.

Conclusion

The effort to translate ideas into ones own words, to make personal sense of the concept of infinity, did not happen by chance. The collective ways of thinking in the class have been scaffolded by the teacher over a period of months as he introduced and supported the students in using collective argumentation. There is specific evidence of this scaffolding process in Angela's talk where she uses the word "so" which appears to be a ventriloquation of the teacher. The teacher often prefaced his transformation of student contributions (summarising or paraphrasing) with the word "so". For example, he says " *So infinity means to us an everlasting number*" There were five such occurrences in this episode of collective argumentation (not all are included in the fragment of transcript). Angela also uses "so" as a marker of changing voice. While apparently insignificant in itself, the use of "so" by both the teacher and Angela to mark a change in voice, suggests that the students in this class have begun to move beyond reproductive learning where they rely on an expert to validate their answers, to a co-constructivist view of learning in which a diverse voices can be heard as students try collectively to compose and orchestrate their local understanding of mathematical ideas

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