Dynamic scaffolding and reflective discourse: successful teaching styles observed within a project to teach mathematical thinking skills

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The mathematical thinking skills project (Tanner & Jones, 1995) reported that classes which followed a course emphasising metacognitive skills were not only more successful than controls in assessments of those skills, but also in assessments of mathematical development. Ethnographic data revealed significant variations in the teaching style from teacher to teacher and was used to classify the teachers into four groups. This paper discusses the teaching styles of the two most successful groups: the ìdynamic scaffoldersî and the ìreflective scaffoldersî.

Thinking mathematically

Thinking as sense making is deeply embedded in the constructivist viewpoint in which the learner is considered as an active purveyor of meaning (McGuinness, 1993) and within this tradition, a clear distinction may be drawn between mathematical thinking and the knowledge base, strategies and techniques described as mathematics. Learning to think mathematically is more than just learning to use mathematical techniques, although developing a facility with the tools of the trade is clearly an element. Mathematical thinkers have a way of seeing, representing and analysing their world, and a tendency to engage in the practices of mathematical communities.

> Learning to think mathematically means (a) developing a mathematical point of view - valuing the processes of mathematisation and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade and using those tools in the service of the goal of understanding structure - mathematical sense making
> (Schoenfeld, 1994, p60).

Wheeler (1982) claims that it is "more useful to know how to mathematize than to know a lot of mathematics" and suggests that pupils should be taught to function as mathematicians. Whilst mathematical sense making may have its roots in constructivism, developing a mathematical point of view may be more akin to enculturation into a community. Two clusters of positions are identifiable which claim to explain learning in social contexts. According to the constructivist viewpoint ìan individual makes sense of her surroundings, and tests hypotheses and sense making, by means of her actions, and through responses from others (and the environment)î, whereas according to the sociocultural viewpoint ìpupils learn to operate mathematically in contextsî and ìa person's identity in a new context is constructed within that contextî (Dawson, 1994, p24-25).

This can be summarized as being the difference between the individual constructing his/her world by making sense of it, and being constructed by his/her world by participating in it. However the two need not be mutually exclusive and a middle position between individualistic and collectivist perspectives is taken here, in which the teacher and pupils are considered to interactively constitute the culture of the classroom through negotiation and communication (Bauersfeld, 1994). But within the constraints of that negotiated culture, individuals construct. From such a view, pupils should learn by participating in a ìculture of mathematisingî which is characterised by subjective reconstruction of knowledge through negotiation of meaning in social interaction (Bauersfeld, 1988).

Cobb et al (1997, p269) claim that a ìmathematical dispositionî may be developed in an indirect manner through participation in ìreflective classroom discourseî. In reflective discourse, teachers should manage the interplay of social norms and patterns of interaction to create opportunities for pupils to reason for themselves and ìengage in reflective thinking or reflective abstractionî (Wood, 1996, p102-103).

One of the issues which arises is the extent to which the teacher acts as a genuinely, neutral moderator of discussions amongst co-participants or as a director and guide of pupils' learning. There is an obvious power imbalance between teachers and pupils in classrooms and teachers' comments carry great weight. What is significant is the manner in which the power is expressed in action (Cobb et al, 1992, p486).

It is possible to distinguish between two very different forms of interaction which might be described as scaffolding to support pupils' learning: *funnelling* and *focusing* (Bauersfeld, 1988; Wood, 1994). In *funnelling* it is the teacher who is involved in using thinking strategies and carrying out the demanding tasks to lead the discourse to a predetermined solution. The social processes of the classroom hide the mathematical structure, which the pupil may only construct by choosing to reflect on regularities in the actions performed. ìContext- and problem-specific routines and skillsî are likely to result (Bauersfeld, 1988, p37). Mathematical logic and meaning are replaced by the social logic and meaning of the interaction. In *focusing* the teacher's questions draw attention to critical features of the problem which might not yet be understood. The pupil is then left to resolve perturbations which have thus been created (Wood, 1994, p160).

The discourse mode of teaching is claimed to lead to higher levels of understanding and thoughtfulness in mathematics (Prawat, 1991). It is in ìreflective discourseî, that teachers are able to ìproactively support students' mathematical developmentî by guiding and if necessary initiating shifts in the discourse so that ìwhat was previously done in action can become an explicit topic of conversationî and thus ìparticipation in this type of discourse constitutes conditions for the possibility of mathematical learningî (Cobb et al, 1997, p264 - 269). The social character of the discourse may be arranged to lend social status to ìthe disposition to meaning construction activitiesî which is a ìhabit of thoughtî that can be learned (Resnick, 1988, p40).

From a Piagetian viewpoint, adolescence marks the onset of formal thought - the ability to argue from a hypothesis and to view reality as a reflection of theoretical possibilities. Formal thought has been described as a systematic way of thinking; a generalized orientation towards problem-solving with an improvement in the student's ability to organize and structure the elements of a problem (Sutherland 1992). However, these key aspects of problem-solving are metacognitive rather than conceptual in nature. It can be argued, therefore, that formal thought is underpinned by the development of metacognitive skills.

Recent research suggests that cognitive development can be accelerated (eg: Shayer and Adey, 1993; Cardelle-Elawar, 1995). A key feature of these studies has been their deliberate enhancement of metacognitive skills. Indeed, metacognition has been identified by McGuinness (1993) as a primary tool for conceptual development.

Metacognition

Metacognition is a ìfuzzyî and elusive term which is used as an umbrella to cover a range of ill defined interacting categories which share certain family resemblances

(Brown, 1987, p106). It refers loosely to the knowledge and control which individuals have of their own cognitive systems (Flavell, 1976; Brown, 1987). This dual nature includes both (a) the awareness that individuals have of their own knowledge, their strengths and weaknesses, their beliefs about themselves as learners and the nature of mathematics; and (b) their ability to regulate their own actions in the application of that knowledge (Flavell, 1976; Brown, 1987). The former aspect is passive in character and is characterised here as metacognitive knowledge or ìknowing what you knowî whereas the second refers to the ìactive monitoring and consequent regulation and orchestrationî of cognition (Flavell, 1976, p232) and is characterised here as metacognitive skill. The use of the term ìmetacognitive skillsî to distinguish metacognitive activities and processes from metacognitive knowledge is deliberate and consistent with an emphasis on processes that might be improved by training.

The Practical Applications of Mathematics Project

This project (Tanner & Jones, 1994) identified the metacognitive skills of planning, monitoring and evaluating as necessary for successful practical problem solving and suggested classroom practices which would facilitate their development. These included the use of social structures to frame pupils' behaviour and constrain them to act as experts rather than novices, eg by slowing down impulsive behaviour and encouraging the examination of several problem formulations; the development of a discourse in which differences in perspective were welcomed; the use of focusing questions in scientific argument; and the encouragement of reflective discourse through peer and self assessment. One key practice which was developed was referred to as ìStart-stop-goî in which pupils were asked to read the problem and think in silence for a few minutes and then discuss possible plans in small groups before a teacher led brainstorming session which focused attention on key features. At intervals work was stopped for group progress reports to encourage monitoring.

The Mathematical Thinking Skills Project

These classroom practices were taken as the basis of a project to develop and evaluate a mathematical thinking skills course. The thinking skills targeted were metacognitive rather than cognitive. That is the course focused on the processes rather than the content of mathematics in the context of practical problem solving and modelling. It was hypothesised that "near transfer" would be found, meaning that pupils would demonstrate improved performance in modelling situations which were similar in character to those used in the course, but did not repeat the content of the lessons. It was further hypothesised that the development of metacognitive skills would lead to improved learning in mathematics through "far transfer" into the cognitive domain and pupils' performance was assessed in the content areas of mathematics which had not been targeted by the course.

Methodology

An action research network of six secondary schools was established to develop and trial teaching strategies and materials, supported by members of the project team. Two matched pairs of classes were identified in each school to act as control and intervention groups. One pair was in year seven (11-12 years old) and one pair in year eight (12-13 years old). Matched classes were either of mixed ability or parallel sets in every case. Regular participant observation by the university researchers was necessary to record the nature of the interventions made. These observations revealed that the extent to which teachers were able to adopt the suggested approaches was variable.

Three teachers were dropped from the final analysis for failing to follow the course or approaches to any appreciable extent (see Tanner, 1997 for details).

Written test papers were designed to assess pupils' cognitive and metacognitive development based on a neo-Piagetian structure (see Tanner 1997 for details). Items emphasised comprehension rather than recall. The metacognitive skills of question posing, planning, evaluating and reflecting were assessed through a section in the written paper entitled "Planning and doing an experiment". Metacognitive self knowledge was also assessed by asking students to predict the number of questions they would get correct *before* and *after* each section (referred to here as *forecasting* and *postcasting*). The overall test was reliable with Cronbach's alpha of 0.86.

The pilot course and intervention teaching lasted for approximately five months. Regular network meetings were held at which experiences were exchanged, strategies discussed and new activities devised and refined. Post-testing occurred at the end of the course. Delayed testing occurred five months later.

The overall results

Multivariate analysis of variance (MANOVA) was used to analyse the three levels of test (ie: pre, post and delayed) and two types of class (ie: control and intervention). A covariate approach (using pre-test scores as covariates) was used to add power to the analysis by adjusting for the small inequalities which existed between groups at the start of the quasi-experiment. For simplicity, only the multivariate results are given here (Table 1) (but see Tanner & Jones, 1995 or Tanner, 1997 for further details).

The active metacognitive skills of planning, monitoring and evaluating:

Both the control and intervention classes improved over the period of the quasi-experiment although the intervention classes improved more than the control classes in the post-test and this improvement was sustained in the delayed-test. The effect size was small (0.19), but significant at the 0.1% level for the written tests (Table 1).

As the active metacognitive skills had been taught in practical mathematical modelling contexts, such ìnear transferîmight be considered unsurprising. Its achievement was non-trivial, however, as the pupils were required by the assessments to form their own problems within open situations, plan, identify and control variables, choose simple strategies, monitor their work, collect and organise their data, find relationships, evaluate and reflect on their results. These are the process skills of mathematics and are identified in the National Curriculum for England and Wales as worthy of learning in their own right.

Passive metacognitive knowledge or ìknowing what you knowî

The results here were not clear cut. Although the intervention classes improved in their forecasting more than the control classes in the post-tests and this improvement was sustained in delayed-testing, the effect of type of class was not significant at the 5% level. The postcasting of the intervention classes improved more than the control classes in the post-tests and this improvement was sustained in delayed-testing. This was significant beyond the 5% level but was limited to an extremely small effect size of 0.02 (Table 1).

Mathematical cognitive development

This showed a similar pattern to the active metacognitive skills but a smaller effect. The intervention classes improved more than the control classes in the post-test and the advantage was largely sustained at the delayed-test but the effect size (0.02) was extremely small (Table 1).

The content of the cognitive section of the written paper was not taught directly by the course, and intervention class teachers were careful to avoid practising questions like those on the test in advance. Given that the intervention classes had less teaching in the normal mathematics curriculum over the period of the quasi-experiment, it might have been expected that the control classes would generally have outperformed the intervention classes. This small overall effect is claimed, therefore, to be an example of mathematical thinking skills paying for themselves through far transfer.

Table 1: Multivariate tests of significance for the effect of type of class

Variable	Hotellings	F value	Hypoth DF	Error DF	Sig of F	Effect size
Metacog skill	.235	43.67	2	371	.000	.191
Forecast	.013	2.30	2	363	.101	.013
Postcast	.022	3.78	2	341	.024	.022
Cognitive dev	.021	3.89	2	369	.021	.021

The four teaching styles

Analysis of the qualitative data collected through participant observation led to the classification of the teachers into four characteristic groups according to the teaching styles employed. These were *taskers, rigid scaffolders, dynamic scaffolders* and *reflective scaffolders* (see Tanner, 1997 for detailed descriptions).

The *taskers* focused on the demands of the task rather than the targeted metacognitive skills. The *rigid scaffolders* focused on planning, but rather than helping pupils to develop their own plans, aimed to share their own plans with the class. The scaffolding support provided by their questioning constrained pupilsí thinking, leading them down a pre-determined path. These two groups of teachers were the least successful. The *taskersí* classes showed no advantage over their controls in any test. The *rigid scaffolders* showed an advantage in only the metacognitive delayed test with a very small effect size (0.09) significant at the 5% level. This paper focuses on the other two groups of teachers.

The dynamic scaffolders

The *dynamic scaffolders* made full use of the social structure of ìStart-stop-goî to frame their pupilsí behaviour and constrain them to act as experts rather than novices. This included the granting of significant autonomy to pupils, particularly in the early stages of planning. Their scaffolding was dynamic in character and was based on participation in a discourse in which differences in perspective were welcomed and encouraged. The most significant participant in the discourse was the teacher, who validated conjectures and used focusing questions to control its general direction ensuring that an acceptable whole class plan was generated. The participation framework was equivalent to ìlegitimate peripheral participationî within an apprenticeship model of learning (Lave &Wenger, 1991), the autonomy and responsibility of the pupils being limited by the teacher's desire to negotiate a plan to a pre-determined template. The discourse focused on both procedural knowledge and conceptual knowledge and during the planning and monitoring sessions, articulation and objectification of explanation was encouraged, making the explanation itself the object of the discourse. This was the only form of evaluation or reflection used by the *dynamic scaffolders*, however, and is characterised as ìreflection in actionî as opposed to ìreflection on actionî (Sch^n, 1990).

The *dynamic scaffolders* were very successful in accelerating the development of the active metacognitive skills of planning, monitoring and evaluating in the context of mathematical modelling, that is in near transfer, with a small to medium effect size (0.36) which was significant beyond the 0.1% level (Table 2). However they failed to achieve a significant advantage for their classes in either passive metacognitive self knowledge or ìfar transferî into the content areas of mathematics. It is conjectured that, although active metacognitive skills may be necessary in the learning of new knowledge, they are not sufficient. Metacognitive self knowledge may also be necessary for far transfer.

Table 2: Multivariate tests of significance for the effect of type of class for *dynamic scaffolders*

Variable	Hotellings	F value	Hypoth. DF	Error DF	Sig of F	Effect size
Metacog skill	0.550	21.27	2	77	.000	0.36
Forecast	0.040	1.52	2	76	.226	0.04
Postcast	0.015	0.53	2	70	.590	0.02
Cognitive dev	0.020	0.80	2	78	.453	0.02

The reflective scaffolders

The reflective scaffolders also used the social structure of ìStart-stop-goî to constrain their pupils to act as experts rather than novices, and dynamic scaffolding in a conjecturing atmosphere to lead the discourse in their classrooms. They granted their pupils more autonomy, however, encouraging several approaches to the problems rather than constraining the discourse to produce a class plan. Pupils thus had to evaluate their own plans in comparison with the other plans in the posing, planning and monitoring phases of the lessons. The participation framework had fewer of the characteristics of apprenticeship, with pupils drawing on the help of the expert and taking a greater responsibility for an end product of their own design rather than taking limited responsibility for an element in the design of a ìmasterî. The characteristic feature of the *reflective scaffolders,* however, was their focus on evaluation and reflection. During interim and final reporting back sessions, scientific argument was encouraged to make the explanation an object of the discourse. Peer and self assessment was encouraged through group presentations of draft reports before redrafting for assessment. They deliberately generated a reflective discourse (Cobb et al, 1997) after activities to encourage self evaluation and reflection on process. Collective reflection does not equal reflected abstraction, but it is conjectured that during collective reflection, opportunities arise for pupils to reflect on and objectify their previous actions as they engage in reflective discourse (Wheatley, 1991; Cobb et al, 1997).

The *reflective scaffolders* were very successful in accelerating the development of active metacognitive skills, achieving near transfer in practical modelling situations with a medium size of effect (0.4) which was significant beyond the 0.1% level of

significance. They also succeeded in accelerating the development of passive metacognitive self knowledge in forecasting and postcasting. The effect sizes were very small (0.07 and 0.14), but significant beyond the 5% and 1% levels respectively (Table 3). They were the only group of classes to achieve this and it is conjectured that this was due to their emphasis on self evaluation and reflection.
The *reflective scaffolders* also succeeded in accelerating development in the content domains of mathematics measured by the cognitive test, again the only group of classes to achieve this far transfer. The size of effect was small (0.21), but was statistically significant beyond the 0.1% level and approximated to a year's development (Table 3).

Table 3: Multivariate tests for the effect of type of class for *reflective scaffolders*

Variable	Hotellings	F value	Hypoth. DF	Error DF	Sig of F	Effect size
Metacog skill	0.652	43.34	2	133	.000	0.40
Forecast	0.073	4.73	2	130	.010	0.07
Postcast	0.161	9.82	2	122	.000	0.14
Cognitive dev	0.272	18.09	2	133	.000	0.21

Discussion

The ìDynamic scaffoldersî were operating a model of cognitive apprenticeship which included authentic tasks, student autonomy and dynamic scaffolding but although they were very effective in teaching for near transfer, they failed to achieve far transfer. Individual construction was subordinated to the dynamics of the apprenticeship model, whereas the *reflective scaffolders* encouraged both cognitive apprenticeship and individual construction. The dynamics of ìStart-stop-goî were internalised through participation in social processes. Learning this procedural knowledge was achieved through an apprenticeship model, organised and controlled by the teacher. Through participating in a scientific discourse led by an expert using dynamic scaffolding, pupils learned to internalise the processes of scientific argument and ìargue with themselvesî. They also learned that mathematics ìmade senseî and that they could ìmake their own senseî of what occurred by making their own tentative conjectures and constructions and linking them with prior schemata.

Participation in reflective discourse encouraged reflective abstraction and the objectification of explanation. It is conjectured that the processes of mathematisation and problem solving, once objectified through individual construction, were thus knowledge rather than mere information and thus capable of being used elsewhere. Collective reflection provided both a social model and an opportunity for reflected abstraction.

The reflective discourse focused on the processes of mathematisation, abstraction and generalisation in the service of understanding structure. Participation in reflective discourse may have encouraged the development of a mathematical disposition or point of view (Schoenfeld, 1994; Cobb et al, 1997) which might be expected to be transferable.

The active metacognitive skills of planning, monitoring and evaluating are generic rather than specific and these skills can survive near transfer to similar modelling contexts. It is conjectured that participation in reflective discourse can encourage objectification and the development of metacognitive self knowledge thus enhancing the transfer potential of such skills. It is further conjectured that the combination of active metacognitive skills and passive metacognitive knowledge supports both the application of old mathematics to new contexts and the learning of new mathematics.

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