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SUCCESSFUL PERCENT PROBLEM SOLVING FOR YEAR 8 STUDENTS USING THE PROPORTIONAL NUMBER LINE METHOD

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A review of the literature to guide the development of a coherent teaching program on percent for Year 8 students revealed varied alternatives and little consensus. Working from the premise that fundamentally percent is a proportion, the proportional number line method was created to assist students experience success in percent problem solving, to also promote conceptual understanding of percent as a proportion, and to embody the multiplicative structure of percent situations. Classroom research indicated that students readily adopted the method.

BACKGROUND

The percent symbol % is arguably one of the most frequently sighted mathematical symbols in Australian society. It can be found in shops, on billboards, in newspapers, on television, and is used for a variety of purposes. Commonly in our society, the term *percent* and the percent symbol are used, for example, to convey messages about discounts and savings in commercial transactions, profits and losses in the business world, interest rates in the banking sector, statistical information in media research reports, and so on. The percent notion is firmly embedded in our society. As a mathematical topic, percent translates directly to the real world. However, percent is often misused or misunderstood when applied to the real world (Watson, 1994). Despite its pervasiveness, the percent notion may possibly be poorly understood.

As a topic within the mathematics curriculum, percent is difficult both to teach and to learn (Cole & Weisenfluh, 1974; Parker & Leinhardt, 1995; Smart, 1980). Consultation of the literature to assist in the development of a comprehensive and coherent teaching program reveals a particularly confusing and entangled picture. Percent literature relevant to instruction can be classified as either that which relates to percent concept development, or that which relates to percent calculations and percent problem solving. Suggested teaching approaches for developing the concept of percent include the following: linking percent to common and decimal fractions (e.g., Hauck, 1954), using 10x10 grids to promote mental images of percents (e.g., Bennett & Nelson, 1994; Reys, Suydam & Lindquist, 1992), investigating the special language of percent used in society (e.g., the use of such terms as 100% attendance, 200% attendance) and building estimation skills through exploration of patterns of simple percent calculations (e.g., Cooper & Irons, 1987; Glatzer, 1984), linking percent to ratio understanding (e.g., Brown & Kinney, 1973; Cooper & Irons, 1987), and studying percent expressions as statements of proportion (e.g., Schmaltz, 1977). The above approaches appear to advocate the building of percent conceptual knowledge upon other prior mathematical knowledge, namely, decimal-fractions, ratio, proportion and therefore appear to suggest that percent can be regarded as any or all of these. It could be argued, therefore, that successful development of percent knowledge from any of these suggested approaches would be dependent upon student familiarity with one or more of these prior topics. If percent can be a decimal-fraction, a ratio and a proportion, developing the concept of percent solely from one of these perspectives would appear to be a narrow approach to percent instruction.

As instructional approaches for developing the concept of percent are varied, so too are suggested methods for performing percent calculations and solving percent application problems. As stated by Ashlock, Johnson, Wilson and Jones (1983), percent application problems are of three types each which contain three elements, as follows: (i) finding a part or percent of a number (e.g., 25% of 20 is x); (ii) finding a part or percent one number is of another (e.g., x% of

15 is 5); or (iii) finding a number when a certain part or percent of that number is known (e.g., 20% of x is 6). In general, percent application problems give two elements in the percent statement and require students to find the third. (Throughout this report, the three types of percent application problems will be referred to as Type I, Type II and Type III problems respectively). Models, strategies and procedures for solving percent application problems, as presented in the literature, include the use of concrete materials and diagrams, such as fraction/percent overlays and elastic strips (e.g., Weibe, 1986), 10x10 grids (e.g., Bennett & Nelson, 1994; Cooper & Irons, 1987), comparison scales (e.g., Dewar, 1984; Haubner, 1992), as well as strategies for the identification of key words and mnemonic devices (e.g., Boling, 1985; McGivney & Nitschke, 1988). The solution of percent application problems using the above methods requires the use of a variety of arithmetic procedures, including common fraction, decimal-fraction, and percent conversions; whole number multiplication and division; and decimal-fraction multiplication and division. Typically, the models and strategies aim to assist students visualise the process of the calculations employed, but also appear to present a particular perspective of percent as a decimal-fraction, a ratio, or a proportion.

With such controversy in the percent literature on teaching approaches, the question arises as to which method is best, or more fundamentally, should percent be promoted as a decimal fraction, a ratio or a proportion? A comprehensive review of percent literature was undertaken by Parker and Leinhardt (1995) and they provide compelling arguments in relation to these two questions. In terms of planning instruction, they state that to date, there is no single best method for teaching percent offered in the literature. As to the meaning of percent, they state that percent is an elusive, concise concept with multiple meanings and therefore should not be regarded from any single perspective, but that "percent is fundamentally a language of privileged proportion which simplifies and condenses descriptions of multiplicative comparisons" (p. 472). In their discussion of issues pertaining to percent teaching and learning, Parker and Lienhardt appear to suggest that there are three basic components necessary for successful instruction in percent. They state that instruction must serve to assist students to read, interpret and define relationships between percent problem components; that a solid representation of percent is required; and that instruction must aim to build students' understanding of percent as a proportion.

If percent is defined fundamentally as a proportion, it follows that instruction should be based on drawing on students' prior proportional knowledge to build percent knowledge. However, the perceived difficulty of implementing a program of percent instruction from a proportional perspective is that students typically do not have a well-developed concept of proportion at the time they meet instruction in percent (Lo & Watanabe, 1997). The development of the proportion concept and proportional reasoning skills takes a long time and is dependent upon the consolidation of many other prior mathematics topics (Post, Behr & Lesh, 1988). The wisdom of a teaching program which aims to build students' percent knowledge through proportional understanding and reasoning is questionable. If a proportional approach to percent instruction is undertaken, it follows that the material presented must be accessible to students regardless of the level of their proportional knowledge, it must provide the foundation for continual and continued knowledge growth of percent and percent as a proportion, and fundamentally, it must provide students with the means to analyse, interpret and visualise percent situations.

From the literature on percent, selected literature on proportion, and other literature pertaining to teaching and learning mathematics, a new approach to percent problem solving is proposed. It is based on representing percent as a proportion, and designed to enable students to explore all types of percent situations meaningfully and to experience successful percent problem solving.

In this approach, entitled the proportional number line method, the first step is to identify the elements in a given percent problem. As previously stated, percent situations contain three elements, and in this method, they are labelled as the part, the whole, or the percent. Using this method, identification of the part, the whole and the percent as either given or needed in the problem is the initial point of entry into percent problem solving. Identification of elements in this way is similar to using a part-whole schema (e.g., Mahlios, 1988; Wolters, 1983) for identifying addition and subtraction word problems. In this case it can be seen to be a part-whole-percent schema. The purpose of the part-whole-percent schema is to provide assistance with the first stage of percent problem solving which, according to Parker and Leinhardt (1995), lies in reading, interpreting and defining the relationships within percent problems. For representing percent problems, a single vertical number line is utilised in this approach which follows a similar format to the comparison scales suggested by Dewar (1984) and Haubner (1992), where an amount is compared to the percent base of 100 simultaneously in a linear fashion on a dual-scale number line. The dual-scale number line provides a clear image of the proportional relationship of percent situations (Dewar, 1984; Haubner, 1992), and can be used to model all three types of percent situations, including increase and decrease. Once the elements of the percent situation are represented on the dual-scale number line, they can be directly translated into a proportion equation and solved. For this approach, solving the proportion equation occurs using the cross-multiply technique (Rule of Three) via a hand-held calculator. The cross-multiply calculation is utilised simply to enable students to attain solutions to given percent problems. Although it is acknowledged that the cross-multiply technique may be regarded by students as a meaningless arithmetic procedure (Cramer, Post & Currier, 1992), justification for inclusion in this approach is that the proportion equation $^a/_b = {^c/_d}$ embodies the proportionality of percent (Post, Behr & Lesh, 1988) and with appropriate instruction, the cross-multiply technique can be taught meaningfully (Robinson, 1981). It was also seen as a means to alleviate cognitive load necessary to free up mental space (Sweller, 1988), and avoid the possible creation of messy errors which can interfere with problem solving (Noddings, 1990).

Solving percent problems using the proportional number-line method occurs in four steps. The first step is the identification of the component parts of the problem situation; the second step is representation of the percent situation on a dual-scale number line; the third step is the translation of the percent situation to a proportion equation; and the final step is calculation of the proportion equation and hence determining the solution to the problem. The four steps in the proportional number line method applied to each of the three types of percent situations are exemplified as follows:

1. Percent situations contain three elements, which can be identified by using the terms *part, whole, percent*. In any percent problem, two elements are given, and solution requires finding the third. The three types of percent problems give the three different combinations of the three elements as follows:

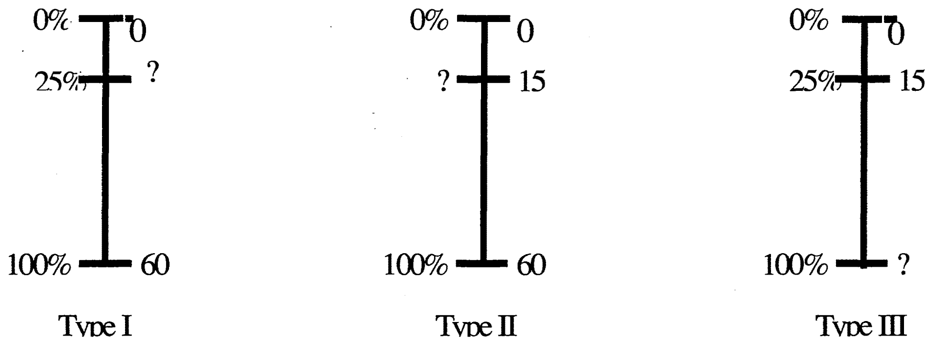
Type I e.g., 25% of 60 = x part = x, whole=60, percent = 25%
Type II e.g., x% of 60 = 15 part = 15, whole=60, percent = x%
Type III e.g., 25% of x = 15 part = 15 , whole=x, percent = 25%

2. The percent situation can be represented on a dual-scale number line, which is a vertical line with the percent scale located on the left and the quantity scale located on the right. Figure 1 shows the positioning of the part, whole and percent elements on the dual-scale number for each of the three types of percent situations.

3. The percent situation can be written as a proportion equation taken directly from the number line. For each of the three percent situations displayed in Figure 1, the proportion equations are as follows: $^{25}/_{100} = {^?/_{60}}$ for Type I; $^?/_{100} = {^{15}/_{60}}$ for Type II; and $^{25}/_{200} = {^{15}/_?}$ for Type III.

4. The Rule of Three procedure enables solution of the proportion equation, and therefore the percent problem. Application of the Rule of Three to the proportion equations generated in Step 3 would proceed as follows: 25 x 60 ÷ 100 = 15 for Type I; 15 x 100 ÷ 60 = 25% for Type II; and 15 x 100 ÷ 25 = 60 for Type III.

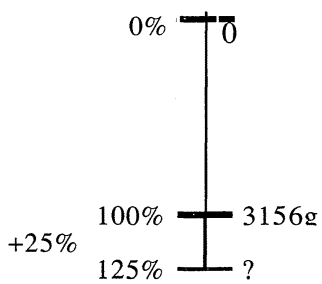
Figure 1
Representation of Type I, Type II and Type III Percent Situations on the Dual-scale Number Line.



In the above description, the three different types of percent problems (Type I, II and III) are given to show the different placement of the elements on the number line according to the problem types. In this approach, students do not need to identify whether the percent problem is a Type I, II, or III problem; they merely need to identify elements of the problem in terms of part, whole or percent. For placement of elements on the number line, students need to recognise that the "whole" corresponds to 100% and thus these two amounts are on the same level either side of the number line. The percent and part are on the same level, with the percent located on the scale between 0% and 100% and the corresponding part at the same level on the other side of the number line.

The dual-scale number line of the proportional number line method can be seen to lend itself to representation of percent increase problems. When used in this way, it clearly exemplifies the additive and multiplicative nature of percent increase situations. Figure 2 shows representation of the following percent increase situation: *A baby's mass increased 25% in 3 months from its birthweight of 3156g.*

Figure 2
The Dual-scale Number Line Representing the Additive and Multiplicative Nature of Percent Increase Situations.

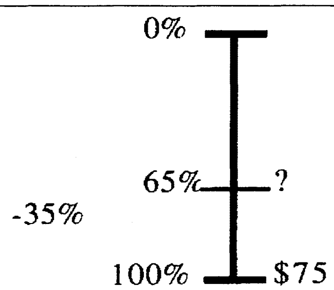


From Figure 2, it can be seen that this percent increase situation can be interpreted two ways. The baby's new mass can be seen as its original mass plus 25% more. It's new mass can also be seen as 125% of its original mass. Therefore, two calculation procedures are possible. One approach is to find 25% of the original mass and add this amount to the original mass as a two-step calculation. Another approach is to calculate 125% of the original mass (using the Rule of Three: $^{100}/_{125} = {}^{3156}/_{?}$) to determine the new mass by using a one-step procedure.

The proportional number line method can also be used for percent decrease situations. In Figure 3, the number line depicts the following problem: *A shirt costing $75 was reduced 35% in a sale. How much would the shirt cost now?* To place 65% on the number line, the problem needs to be clearly interpreted using complementary percent knowledge: a 35% discount is the same as 65% of the original amount. A subtraction takes place, and thus the number line model shows the subtraction. The situation can thus be interpreted in two ways: the shirt is discounted 35%; or, the shirt now costs 65% of the original amount. Similarly, calculation can also proceed in two ways. Either, 35% of the original cost can be found, and subtracted from that cost; or 65% of the original cost can be calculated using the Rule of Three ($^{65}/_{100} = {}^{?}/_{75}$). Using a subtractive application takes percent discount problems from a two-step procedure to a one-step procedure. The number line thus organises elements within percent problems, particularly increase and decrease problems, so that interpretation can occur.

Succinctly, the proportional number line method contains a percent schema for interpreting percent situations in terms of their component parts, a representation which embodies percent as a proportion, and a structure for symbolisation of the percent situation as a proportion. The approach appears to be a simple, yet powerful way to represent the proportional nature of percent situations, and more importantly, for providing the basis for developing students' understanding of the additive (and subtractive) and multiplicative nature of percent increase and decrease.

Figure 3
The Dual-scale Number Line Representing the Subtractive and Multiplicative Nature of Percent Decrease Situations.



To explore the potential of the proportional number-line method, the approach was incorporated into a teaching program on percent and trialed with Year 8 students. As part of a larger study, a series of teaching experiments were conducted in authentic classroom situations with instruction provided during the students' timetabled mathematics classes over the period allocated to percent instruction (2 school weeks -approximately 10 lessons of 40 minutes duration). For the purposes of this report on implementation of the proportional number line method for percent problem solving with Year 8 students, results pertaining specifically to this method with one class will be presented here.

THE STUDY

Students in this study attended an inner city, single-sex secondary school. The class was non-streamed, therefore a range of ability levels were represented in the class. For the study the students were presented with a teaching program consisting of five teaching episodes on the following topics: the concept of percent; the Rule of Three, interpreting and solving percent problems; percent, common and decimal fraction equivalence; the language of percent increase and decrease. For the concept of percent episode, the language of percent used in the real world was explored and classroom activities included searching newspapers, discussing terms such as discount, profit, loss, bank interest, and also looking a common percent phrases such as 110% effort, 200% attendance, and so on. The focus of the second episode was on the Rule of Three and relating it to solving equivalent fractions (proportion equations). Two episodes were devoted to the proportional number line method. In these episodes students were presented with a variety of percent problems with which they could utilise the proportional number line method.

Some examples of the relatively routine problems presented to students were as follows:

> In a school of 875 students the measles epidemic infected 72%. How many students were infected?
>
> The school has 453 girls which is 62% of the total school population. How many students are at the school?
>
> On my Science test I got 58 out of 80. What percent did I get?

Students were also presented with more complex percent problems such as the following:

> A car dealer bought a new car from the manufacturers for $9 500. At what price must the dealer sell the car to ensure he makes a profit of 90%?
>
> Following a minor goldrush and a baby boom, the population of a small town increased from 225 to 910 in two years. What was the percent increase for that town?

In the episode on equivalence the focus was to promote students' understanding of percent as a number written in a variety of equivalent forms (common fraction, decimal fraction, percent).

A variety of data collecting instruments were used in the study including pre-, post- and delayed posttests, teacher generated field notes and reflections upon program implementation, student worksamples, ad hoc interviews with students, interviews with observer, student diaries. Pre- and posttests were administered prior to and immediately after the instructional period. The delayed posttest was administered approximately eight weeks later. The pretest contained items which required students to perform percent calculations and solve percent application. The posttest followed a similar format to the pretest. The delayed posttest was a condensed version of the pre- and posttest with fewer items but of similar type to those on the pre- and posttest. The teaching program was analysed in terms of (i) its *effectiveness* in promoting students' growth of percent knowledge and permanence of that knowledge, and (ii) its efficiency of implementation in actual classrooms with respect to such factors as time, cost, resources, and preparation requirements.

RESULTS AND DISCUSSION

The class mean scores on the pre-, post- and delayed posttests were 12%, 77% and 76% respectively. Pretest results indicate that prior to instruction, students' percent problem solving and percent calculation skills on all three types of percent problems were poor. Immediately after instruction, students' performance increased considerably, and from delayed posttest scores, successful performance was maintained. Analysis of test papers revealed that the majority of students continued to draw the dual-scale number line to assist with organisation of the elements within percent problem situations. Other students dispensed with the number line and wrote proportion equations directly from the problem posed.

As part of the teaching experiment, students were required to write entries in a mathematics journal at the conclusion of lessons. Comments from students' journals relating directly to the proportion number line method included the following:

> One thing I learnt in maths today was how to work out percent problems. I found out that part means percent, well they mean the same thing.
>
> The new way we learnt to do percent problems is a lot easier than the way we learnt last year, so now I can do percent problems a lot easier and quicker.
>
> I use that way (number line method) instead of my own way because it is so easy.
> Using the new way is easier.
>
> I think I am getting better at percentage now because I find it easier to work them out with the number line.

From the above comments it appears that students valued the proportional number line method, particularly for its simplicity and ease of use, and that it enabled successful calculation of percent problems and equations. Many students appeared to regard the method as superior to their previous methods used for percent calculations as they readily abandoned their own methods and adopted the proportional number line method.

At one point during the teaching experiment, students were working on related percent activities, and one student's worksheet showed spontaneous transfer of the key processes inherent in the proportional number line method to another related activity. For this activity, the student was working on the conversion of percent amounts in order to construct a pie chart. One of the equations written on the student's worksheet was the following: $^{65}/_{100} = ^{x}/_{360}$. Instruction in the proportional number line method appeared to enable this student to make connections between percent and proportion and to link this knowledge to other proportional situations.

CONCLUSION

Trialing the proportional number line method for percent problem solving with students in a real classroom has assisted in determining the contribution of this method for the development of Year 8 students' percent knowledge and problem solving skills. The study has shown that the proportional number line method assists students to perform percent calculations and solve percent problems, including percent situations relating to percent increase and decrease. The students in this study readily adopted the method, and regarded it as a better method than their own previous methods. Delayed posttest results indicated that students retained this method and could readily use it for solving percent problems after a break in percent instruction. Such results indicate the degree to which this method was internalised by the students.

The creation of the proportional number line method occurred after consulting a variety of literature on teaching and learning percent. Its purpose was to enable students to engage in successful percent problem solving to fully explore the domain of percent and also to lay the foundation for linking percent knowledge to proportional knowledge. In this study, overt knowledge of the link between percent and proportion was apparent in one student. Further research will determine the extent to which the proportional number line method promotes understanding of the fundamental proportionality of percent.

As a result of this study, it appears that the proportional number line method is a useful method for assisting Year 8 students to operate in the domain of percent. Incorporated within a teaching program on percent, the method appears to be effective in promoting the development of students' knowledge of percent and efficient in that it can be readily implemented in the classroom. It also provides the means to base the teaching of percent upon a proportional perspective and thus assist teachers to lay the foundation for broadening students' conceptual understanding of percent as a multifaceted topic.

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