DO GAMES HELP THE LEARNING OF PROBABILITY?

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This paper reports on a research project which investigated the value of using games to assist Year 7 and 8 students' learning of probability concepts. Games, although generally useful in mathematics for helping children learn, may not automatically be as useful in helping students develop normative probability concepts. The study found that the type of game impacted on the value of using games for learning probability, and that there are implications for the teacher's role when using games for learning.

INTRODUCTION

In the world as it currently is, and will be in the future, concepts involving chance are pervasive. Thus an understanding of chance and randomness is desirable so that we can make the best of the probabilistic decisions that face us in everyday situations.

Although probability concepts are an integral part of life, they have not always been included in the school curriculum at the primary school level. New Zealand's national curriculum statement *Mathematics in the New Zealand Curriculum* has raised the profile of probability in the school curriculum, compared with previous curriculum statements. Because probability learning is introduced at earlier levels than has happened in the past, primary teachers need ways to help ensure that effective learning of probability occurs.

Learning Probability Compared with 'Other' Mathematics

It is readily acknowledged that many of the ideas behind probability are difficult to learn and therefore hard to teach (Ahlgren & Garfield, 1991). Freudenthal (1973), when discussing the types of problems that arise in probability, indicates that pure mathematics does not know of similar analogies; it appears that probability learning may be different from learning in other areas of mathematics. Borovcnik and Bentz (1991) suggest that our desire for deterministic explanations works against the development of an adequate understanding of randomness, which is inextricably linked to an understanding of probability.

In most areas of primary school mathematics, the use of equipment, teaching experiments, and practical activities can be used to demonstrate various concepts. From the specific examples and results obtained, students are encouraged to look for patterns and generalize, thereby developing their mathematical understanding. However, this process of learning through induction may not be as relevant and useful in probability, because of the nature of randomness and random events - the specific examples and results from chance events may not be the expected ones. These 'unusual' results conflict with what is expected, and students are therefore faced with experimental evidence that does not clearly illustrate the concept, and furthermore, it is unlikely that they would realize this. If the student ascribes the result to an overall pattern and views the outcome deterministically, then this unusual result may interfere with the understanding of probability that the teacher is hoping to encourage.

Additionally, the intuitive knowledge that students bring to the classroom can advantage the learning process, but unfortunately, it can also impede the learning (Borovcnik & Bentz, 1991). This appears to be especially pronounced in probability learning, because of the often strong probability intuitions that students develop prior to any formal learning in the classroom (Fischbein, 1987). These intuitive ideas of probability are particularly resistant to change (Konold, 1995). This is problematic for learning because normative probability concepts are often counter-intuitive.

Konold (1991) advocates a multifaceted approach to instruction designed to confront the students' misconceptions. In this approach, students are encouraged to evaluate their current beliefs and how well they fit against: 1) the beliefs of others; 2) their other, related beliefs; and 3) their own observations. The first aspect involves discussion and Konold describes the ways in which teachers can facilitate this by helping to keep the conversation going, and focusing the discussion. However, according to Konold, the most important role of the teacher is to create a classroom atmosphere in which the students can discuss, argue, and convince one another of their views, while at the same time exploring their own views and understandings. The teacher, through the use of probing questions and prompts, encourages the students to analyze the consistency and completeness of their own beliefs – both related to the misconception being considered as well as other related beliefs.

Games for Learning

Games are commonly-used mathematics activities in the primary school. Numerous resources for teachers give details of games which can be used in the classroom for a variety of purposes. Many games are for practising and consolidating skills, but of interest in this study were those which help students learn new concepts. Since games involving chance are a common part of many cultures and therefore part of the real-life experiences of many children, it would seem sensible to use games as a context for learning mathematics, and in particular probability, in the classroom. Also because games involving chance historically brought about the development of probability theory, it has been suggested that games of chance could profitably be used in the classroom to help children learn probability concepts (Biehler, 1991). However, there is a limited amount of research available on the use of games as they relate to the learning of probability concepts for younger students.

Bright (1980) expressed caution about the use of games for probability learning because of the possibility that, as the students become more familiar with a game, they may become more skilful in the strategy required in the game without necessarily increasing their understanding of probabilities related to the dice outcomes. He indicated the importance of carefully examining the players' moves during the game in order to determine how much is attributable to practice at playing the game compared with real understanding of probability. Falk, Falk and Levin (1980) also referred to the possibly passive role of children during the playing of games involving only chance. They suggested that to overcome this, games involving some decision-making from the players should be used to ensure that probability concepts are more likely to be explicitly thought about during the game. Also, the game should be played a number of times. Because of the nature of random events, less likely outcomes could dominate small trials and would not necessarily help develop an appropriate understanding of probability.

Eade (1988) recommended games that may induce 'cognitive conflict', where the players' intuitions about the game and its outcomes conflict with the empirical results, thereby challenging the students' understanding. Through this, it is hoped that the conflict would be resolved with an improved (normative) understanding.

Burnett (1993) argues that the discussion which arises from the game playing as well as the social context in which it is played facilitates learning. This view is supported by others (eg., Ellingham, Gordon & Fowlie, 1998) who contend that games encourage students to listen to the viewpoints of others and make sense of those interpretations. These group interactions provide the opportunity for the individuals in the group to develop understanding more sophisticated than the original, individual ideas (Wood, Cobb, & Yackel, 1995). Games have the advantage of providing a natural situation for the teacher to be able to question children about their understanding (Ainley, 1990).

Sometimes when students are using games for learning, the players may be able to complete a game without being challenged or required to consider the concepts for which the teacher has chosen the game (Ellingham et al, 1998). They suggest that when the educational aspects and the gaming components are not be fully integrated, students treat it merely as a game rather than a learning experience.

RESEARCH QUESTIONS AND DESIGN

The research project focused on the use of games for helping students learn probability concepts (Burgess, 1999), and this paper reports on part of that research, specifically the following questions:

1. Does the use of games encourage students' thinking in relation to specific probability concepts as well as any misconceptions that the students may have?
2. Is there a difference between the use of a game involving only chance compared with a game involving chance and strategy on students' thinking about probability, and on the interaction between students while they are playing the games?

Eight Year 7 and 8 students from each of two classes were selected by their classroom teachers to be involved in the project, ensuring a range of mathematical abilities within each group. The only information given to the researcher about the students was a grading of mathematical ability, based on a 3-point scale (1 for below average, to 3 for above average).

Since this study was mainly focused on the students' understanding of probability concepts, qualitative methods were seen as the most appropriate for exploring that understanding. A questionnaire was administered, with items based on the objectives from *Mathematics in the NZ Curriculum* – some of which had been used by other researchers in the area of probability misconceptions. All the questions were of multi-choice or true-false type, but most also required written justifications for the answers. Observations and group interviews were used during the subsequent game-playing sessions, which involved four students at a time, playing in pairs. The two game sessions, which each lasted for about 45 minutes, were audio-taped as well as video-taped. The stimulus for the interview questions was both the students' activity during the game, and the discussion which arose from playing the game. The mixture of data collection and analysis methods helped reduce any possible bias, and contributed to the verification and validation of the qualitative analysis.

Two simple games were chosen for the students to play. They needed to be relatively short in duration, so that within the allocated time for the games sessions, a number of repetitions of the games could occur. The repetitions of a short game allowed random outcomes to be observed as well as having the potential for showing the 'longer term' trends of the random outcomes.

The selected games were adapted to enable a type of 'record' to be kept of the outcomes through the use of the counters on the game board, instead of relying on the students' memories of the outcomes to gain some understanding of the (lack of) fairness in the game. Reliance on memory is known to be unsatisfactory in relation to probability-based events; the availability heuristic, as described by Kahnemann and Tversky (1972), leads to 'incorrect' reasoning and decision-making.

The first game involved chance only, where the difference between the numbers on two dice determined which team was able to move their counter. If the difference was 0, 1, or 2, one team moved their counter one space; for a difference of 3, 4, or 5, the other team moved their counter one space. The game board consisted of 12 spaces; the first team to have their counter reach the end was the winner. This game was not 'fair' – the lower differences are more likely to occur than the larger differences. The second game consisted of strategy as well as chance, and the game session was split into two parts: the first part used a game based on the sums of two dice (with the board having the numbers 1 to 12, against which the team could choose where to place their ten counters), while the second involved the differences between the numbers on the two dice (and this board was similar to the first, but had the numbers 0 to 6). The strategy part of the game required both teams (each with two students) to place their counters on the game board, in order to optimize their chances of winning, by being the first team to remove their counters from the board. At each throw of the dice, if a team had a counter beside that corresponding sum (or difference for the second part), they could remove that (one) counter from the board.

RESULTS AND DISCUSSION

The questionnaire revealed that most of the common probability misconceptions, which have been described by various researchers, were held by the 16 students involved in the research, with most of the students demonstrating more than one type of misconception. For 12 of the sixteen students, a dominant type of reasoning emerged for each of those students: four displayed normative (correct) reasoning, while the others mainly used one misconception, such as the outcome approach (Konold, 1995), the representativeness heuristic (Kahnemann & Tversky, 1972), or the equiprobability bias (Amir & Williams, 1994), as their dominant type of reasoning. Overall, the predominant type of reasoning used by the students was the outcome approach – all students used it in some form, even those whose main type of reasoning was normative. Often there were inconsistencies between answers from a student, or inconsistencies between an answer and the justification given for that answer. These results provided a base against which to compare their actions and comments in the subsequent game playing sessions.

Students' thinking about probability was quite clearly stimulated by playing the games. Prior to playing game one the first time, three of the four groups of students indicated that they believed (incorrectly) that the game was fair, and in the other group, there was disagreement as to which team would have the advantage. After playing this game a number of times, there was only one group that still maintained their viewpoint that the game was fair; the other two groups had been convinced by the empirical evidence, that the game was not fair. The discussion within these groups ensured that they moved towards a shared understanding about the lack of fairness in the game. The only group to have predicted that the game was unfair, came to an agreement quite quickly as to which team had the greater chance of winning. The reasoning used within this group was normative, and relied to some extent on the empirical results from the games.

The one group that did not change in their belief about the fairness of the game, were obviously affected by unusual results in two of the games: both games were very close, with the first having a probability of only 0.05 of occurring, and the second game had a probability of 0.03. In between these two games was one where the result was much more conclusive, as was the result from the fourth game. Through all this, the group maintained their belief that the game was fair. The empirical evidence, particularly with two close results out of the four being quite unlikely, was not useful in helping the students develop their understanding of probability. At times like this, the teacher's role is crucial to help students work through the results and clarify their misconceptions, which may otherwise be consolidated.

Comments from students, unprompted by the researcher, during the games also indicated that they were thinking about probability concepts. For example, in relation to the possible differences when throwing two dice, one student suggested after a few throws that it was not fair because her team was losing:

> Because there's more chance of getting a one or a two than having a higher number like a three or a four or a five.

Another student (from a different group), who had demonstrated various misconceptions in the questionnaire, referred to a belief in luck and a causal approach to probability, through the following comment:

> I think yellow is very unlucky for them, because the board is yellow, one dice is yellow, and their counter is yellow. And they're not winning at all.

The negotiation of meaning within the groups was not straightforward however. For instance, the influence of some rather flippant comments by particular individuals could not be underestimated, as these tended to cause some of the less assertive students in the group to contribute less to the discussions. For instance, one student stated (jokingly):

> The dice must be rigged....there are probably magnets inside the dice that make them fall that way.

Also, some comments could be considered influential, in that they altered the 'path' of the discussion, leading the group away from developing the normative understanding that had been occurring. For example, one group had been using a reasonable amount of normative reasoning, but then a suggestion came from a student that the colour of the counters may have something to do with the results - a causal approach to probability. This altered the discussion for some time, with the students 'exploring' other possible influences on the outcomes, before finally reverting to the reasoning which was used previously.

Games are claimed to be useful for encouraging students to ask questions and reflect on responses, hence to make new deductions and inductions. Although the students were encouraged to talk and discuss their ideas throughout the game sessions, the level of involvement of the students varied, as measured by the number of times each spoke. For example, although the mean number of contributions per student over the two game sessions was 27, the minimum was 10, and the maximum was 41. There was no signficant difference between the mean number of contributions for each of the three ability groups.

There was significant variation between groups in the amount of interaction within the group: some groups played the games with minimal verbal interaction, while there was a large amount of discussion in other groups. This was measured by the researcher identifying "spontaneous interactions" – students' comments relevant to the game and unprompted by the researcher. The spontaneous interaction was identified as having ended, when there was a signficant pause, a change in topic of the conversation, or intervention through a question from the researcher. The number of comments for each spontaneous interaction was determined, as well as the number of spontaneous interactions per game session, and this is summarised in Table 1.

Table 1
Number of Comments per Spontaneous Interaction during Games

	Game 1				Game 2			
	Group 1	Group 2	Group 3	Group 4	Group 1	Group 2	Group 3	Group 4 *
Average number of comments per spontaneous interaction	2.4	2.0	1.8	2.2	2.5	1.7	1.7	2.1
Number of 'spontaneous interactions' during game session	32	24	25	24	21	21	13	29

* *Note: Group 4 consisted of only three students for game session 2, which may have affected these results.*

Game one involving chance generated 105 interactions overall, whereas game two, involving strategy and chance, generated only 84 such interactions. However, it can also be determined that the average number of comments per interaction is almost identical for the two types of games - 2.1 compared with 2.0. This indicates that the greater number of interactions during the first game must have been shorter in duration than those in the second game. Three groups reduced the number of interactions from game one to game two. For one group (number three) this can probably be attributed to one student who was concerned that he was going to miss some of his class's physical education lesson, and consequently he contributed almost nothing to the second session. For the other two groups, there is no obvious reason for the reduction.

It was necessary, when comparing the games, to consider the quality of the interactions, and this was done by examining the transcripts for evidence of changes in the students' understanding as a result of playing the games. One group particularly showed an 'improvement' in their understanding from game one, where they did not move at all towards a normative understanding, compared with game two, where some normative reasoning was used. As a group, they accepted that some numbers occurred more often than others, and they identified some of the numbers that did occur more often. But against this, one student, with a strong tendency towards the equiprobability bias, argued using the outcome approach. She also indicated that some of it was due to luck. Neither the rest of the group nor the empirical evidence from the games was able to 'shift' this student's understanding. Some students' comments indicated that they were more involved in the second, strategy-based game:

> In these games you are thinking of what move you are going to make, or what numbers you need.
>
> In the other game [game one], we didn't get to choose the numbers so it didn't make you think too much about it.
>
> It has made us think more than in a game like Monopoly, where you just think about how much money you have to hand out or something.

However, an opposing view was put forward by one student:

> I think it made me think harder because it was more of a competition, sort of. And you wanted to win, and you realized it was an unfair game.

Games of strategy and chance, which have been suggested as possibly being more useful to the learning of probability than games of chance only, showed some interesting aspects in regard to the strategies used by the students. The students tended to improve their chances of winning from one trial to the next through the choices they made, but rarely did a team take notice of what the opposition had chosen, in order to further optimize their chances. They tended to focus only their own choices. The nature of random events meant that a well-chosen strategy did not necessarily guarantee success. The random outcomes did not always favour the team with the better choice of numbers. This contrasts with the claim that games give feedback on the "consequence of actions" (Inbar & Stoll, 1970). There were games played in which a 'poorer' choice of numbers resulted in a win for a team over the opposition who had a 'better' choice, in relation to the relative probabilities.

CONCLUSIONS

There was evidence of students holding contradictory beliefs of which they were not aware. This may occur because the beliefs were expressed in different contexts, in which case the students may consider the problems to be different. For effective learning to take place, the students should be made explicitly aware by the teacher of these contradictions, to have cognitive conflict induced. This would enable the students to consider their conflicting ideas and have an opportunity to resolve them.

Students do not necessarily transfer their learning and understanding from one context to another. Most groups concluded (correctly), as a result of playing game one, that the dice differences were not equiprobable. But they did not transfer this knowledge into the second game that used dice differences, as their choices for placement of the counters in the second game did not reflect the understanding that they had developed previously. A teacher would need to make the students explicitly aware of these, so that the students can address and, hopefully, resolve them.

It is important that students are not left on their own to play such probability games, as it may happen that without the influence and input of the teacher, a group does not become as actively involved as is desirable. Also, the importance of the teacher listening to the students to gain an understanding of their concepts is obvious when comparing the results of the questionnaire with the views expressed by the students during the discussions. The questionnaire results were limited by not being able to follow up on what the student meant with some of the responses, but through being involved in the games' discussions, the teacher could easily seek clarification if necessary.

The design of the games was intended to provide a type of record of the results, as these are known to be important so that students can see long-term trends, rather than relying on memory (which is when students are prone to use the availability heuristic). The design of the games did not meet this requirement sufficiently, as there were instances of inadequate recall of the results by the students, indicating use of the availability heuristic. The design of each game needs further refinement to overcome this problem, or some other strategy involving recording needs to be incorporated into or alongside the game.

Active involvement of the students along with the intention of learning is required to ensure maximum benefit. So, although games are strongly advocated because of the way that they assist with the mathematical development of children, they may be inadequate on their own: it is the teacher's planning of the educational process which is essential.

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