FREE PROBLEM-POSING: YEAR 3/4 STUDENTS CONSTRUCTING PROBLEMS FOR FRIENDS TO SOLVE

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This paper investigates the way in which two children constructed and designed mathematics problems for friends to solve. The two children, of different ages, designed problems for one another and for other friends over a ten week period. The way in which the children engaged in problem solving prior to, and after, formulating or posing a problem was explored. Insights into the children's mathematical abilities were identified through the problem-posing activities.

LITERATURE REVIEW

Problem posing is an important companion to problem solving and lies at the heart of mathematical activity (Kilpatrick, 1987). Stoyanova (1998) defined problem posing as "the process by which, on the basis of their mathematical experience, students construct personal interpretations of specific situations and formulate them as meaningful well-structured mathematical problems" (p. 165). Until recently, few studies have examined the mathematical processes employed by the problem poser when constructing a problem. As Silver (1993),commented:

> despite the interest, however, there is no coherent, comprehensive account of problem posing as part of the mathematics curriculum and instruction, nor has there been systematic research of mathematical problem posing. (p. 66)

More recently, some educators have recognised the importance of promoting problem-posing opportunities in the curriculum (English & Halford, 1995; Silver, 1995). These studies suggested that when proposing a problem to solve, the originator of the problem will usually investigate the type of processes required to solve the problem. Importantly, the solution to the problem is likely to also be considered.

Silver (1995) identified four types of problem-posing experiences that provide opportunities for children to engage in mathematical activity. He argued that problem posing could occur *prior* to problem solving when problems were being generated from a particular situation, *during* problem solving when the individual intentionally changes the problem's goals or conditions, or *after* solving a problem when experiences from the problem-solving context are modified or applied to new situations. The way in which children engage in problem solving prior to, and after, constructing or posing a problem will be investigated in the present study.

Stoyanova (1998) identified a number of categories that could be used by teachers and researchers to identify different problem-posing situations. These categories included: a) free; b) semi-structured; and c) structured problem-posing situations. One of the situations described in the free category included *problems written for a friend.* In such cases, a student creates a problem for a friend to solve. Some researchers (Ellerton, 1986; Mamona-Downs, 1993) have found that, for motivational purposes, it is helpful to have someone in mind when designing problems. In the present study, the friend would be someone in the problem poser's class. Moreover, the friend would attempt to generate a solution to the problem designed for them. Thus, the problem poser would receive feedback on the solution to the problem. The feedback obtained from this stage of the process fosters a reflective component of the problem-solving process.

Ellerton (1986) found that encouraging students to write problems for a friend was a useful way of understanding that person's mathematical ability. In such problem-solving situations the problem poser is forced to consider the individual for whom they are designing the problem. As Stoyanova (1998) commented:

> there is a strong acceptance among researchers and educators of the notion that students' ability in posing quality problems provides a useful indication of potential mathematical talent. (p. 172)

The very fact that a student must consider the mathematical ability of another person when engaged in free problem-solving situations requires reflection and careful planning. In order to complete the task successfully, the problem poser might not only focus on the underlying structures of the problem but also the extent to which the problem solving will be able to interpret the components of the problem. Such metacognitive thinking processes encourage mathematical power (Lowrie, in press).

Problem-posing situation allow children to have some control over the curriculum content and the type of learning activities presented in the classroom. Furthermore, the tasks or the activities children construct may provide insights into their beliefs or attitudes they have toward mathematics. The way in which the problem poser represent problems, for example, may reflect the type of problem-solving experiences they have been accustomed to solving in the classroom. The present study will investigate whether feedback obtained by the problem poser—through interactions with the classroom teacher, the researcher, and other members of the class—influences the type of problems they pose over time.

METHOD

This investigation was conducted over a ten week period (one term of the school year) in a composite Year 3/4 class. Throughout the term the children were provided with opportunities to construct (pose) a range of different problems in free, semi-structured and structured situations. The present study will report on the findings associated with children engaged in free problem-solving situations.

When developing a problem, the students were asked to *write a problem for a friend* or a *particular student in the class*. It was envisaged that these two scenarios would create different types of problem-solving situations for the children. In the first instance, students would choose the person they would write the problem for whereas in the second case the teacher identified the student for whom the problem poser would construct the problem for.

The classroom teacher provided opportunities for the children to engage in problem-posing situations twice a week. Children were encouraged to discuss solutions with each other after they had exchanged and solved problems. Moreover, the children were able to modify problems posed if the task was either too easy or too difficult for the intended problem solver. This reflective stage of the process was seen as an important component of the problem-posing process. The children had not been accustomed to posing problems and as a result cooperative learning experiences and discussion sessions influenced the way in which the children constructed problems as the study progressed. The researcher visited the class on five occasions throughout the term. These visits were at regular intervals throughout the duration of the study and provided opportunities for the researcher to talk to the children and the classroom teacher. Several of the discussions with the two case-study participants were audio taped.

The Students

The two students investigated in the study were from a composite Year 3/4 class from a small school in a large rural city. John was an eight year old boy who was considered by his classroom teacher to be a capable student who was particularly good at problem solving. Bettina was a nine year old girl who was considered to be an outstanding student. The two children appeared to be "at ease" when being interviewed by the researcher and were able to express and discuss the way in which they posed the problems in great detail.

When asked to *write a problem for a friend* John enjoyed designing problems for his best friend Tom. It appeared that Tom's mathematical ability was not quite as advanced as his own. Nevertheless, they enjoyed writing problems for each other and worked well in cooperative problem-solving situations. Bettina did not choose a close friend to construct problems for. She enjoyed writing problems for Joe because "he really liked solving hard problems because it was challenging." Like Bettina, he was a very capable student. They also worked well cooperatively and seemed to thrive on each other's desire to learn.

When required to design problems for one another the two children had a reasonably good understanding of the other's mathematical ability. John commented that Bettina was "the best at maths in the whole class" and that "she could get all the hard problems right." Bettina realised that John was "good at mathematics but was only in Year 3 so he couldn't do what some of the Year 4's could do." Throughout the study the two children developed a strong rapport with one another.

The Procedure

The investigation traced the way two of the children in the class engaged in problem-posing activities throughout the term. These two children were selected for investigation for a number of reasons.

- The children were of different ages allowing comparisons across grades and different levels of mathematical ability.
- Both children designed problems for friends who were at a stage of mathematical development similar to their own.
- The children were asked to solve problems for one another.
- A detailed case study analysis (Yin, 1994) of the two children provided the researcher with an opportunity to trace the way in which the children constructed and developed problem-posing tasks over an extended period of time.

The case study format of the study ensured that a rich, detailed, analysis of the students' worksamples and reflections could be gathered over an extended period of time (a ten week period). Interviews with the children were audio taped with information collected about the way in which the students posed problems for their friends and for each other. When asked to *pose a problem for a friend* the two children designed problems for children in their grade (Year 3 or 4). On other occasions they were required to pose problems for each other (thus, one student was required to solve problems for a person younger than themself, the other problems for someone older than themself). Direct questioning (see Lowrie, 1998a) was used to stimulate the children to think metacognitively. Lowrie (1998b) found that when young children were able to interact in discussion sessions that stimulated them to consider "what-if" situations they were able to engage in increasingly sophisticated problem-solving activities. These questioning techniques were used to encourage the children to discuss their thinking processes prior, and after, constructing problems (see Silver, 1995).

Another focus of the present study was to assess whether the types of problems posed by the children changed over the ten week period. It was anticipated that interactions with classroom teacher, the researcher, and other students would influence the way they, and other members of the class, constructed and designed problems over the term.

THE RESEARCH QUESTIONS

Three research questions were formulated for the study.

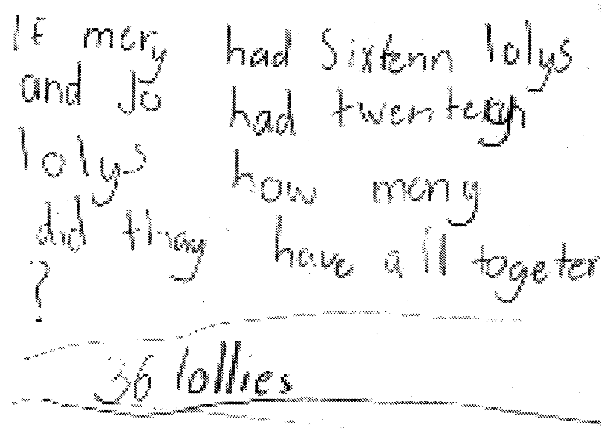
1. Were the children able to consider the needs and interests of a peer when posing a mathematical problem for them to solve?
2. Did a student's mathematical understandings impact on their ability to construct "well designed" problems?
3. To what extent did an individual's personal beliefs about mathematics influence the types of problems they formulated and did the type of problems they pose change over time?

RESULTS

Initial problem-posing situations

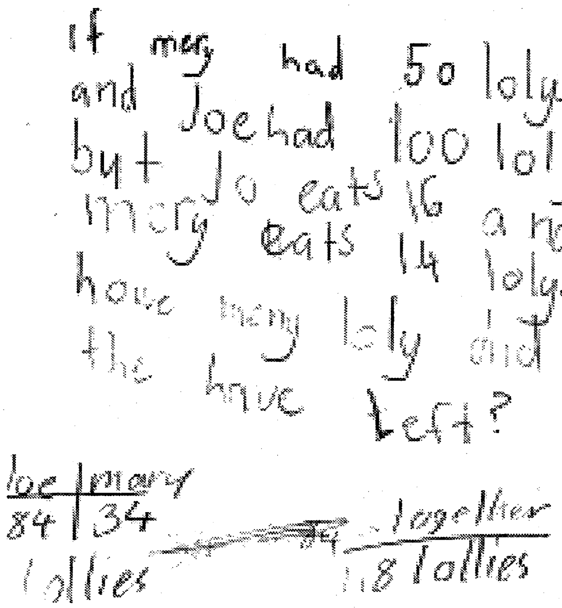
From the outset of the investigation it was evident that John and Bettina were attempting to consider the needs of the problem solver when designing problems for friends in their class to solve. This is in line with Mamona-Downs (1993) suggestion that the type of question asked will be influenced by the person for whom the problem is being designed. When posing a problem for Tom (someone at or near his ability level) to solve John constructed a problem that required the addition of two 2-digit numbers (see Figure 1). In contrast, the problem intended for Bettina was a two-step problem requiring addition and subtraction operations (see Figure 2). The two problems were solved successfully by the respective students.

Figure 1
John's Problem for a Friend


If mery had Sixtenn lolys and Jo had twenteegh lolys how meny did thay have all togeter ?

36 lollies

Figure 2
John's Problem for Bettina



John showed that he had considered the solutions to the particular problems, or at least solution paths, before asking others to solve the tasks. The following section details other problems John had posed for Tom and Bettina respectively.

> Jason saw 17 birds the first day. The next day he saw 14 birds. How many did he see altogether? (problem for Tom).
>
> If Joe had 110 102 lollies and ate 467 and bought 764 144 and ate 14 and then bought 224 213 how many did he have? (problem for Bettina)

The problem designed for Tom was similar to the one proposed at an earlier time (see Figure 1). Similarly, the problem prepared for Bettina required a two-step process but in this instance the numerals were considerably larger than in the first problem (see Figure 2). An audio-taped interview conducted with John on the second visit to the site indicated the extent to which he attempted to adapt the problems to the students needs.

> Tom will have to trade to get the proper answer...Bettina said my last problem was too easy so I gave her big numbers to work out this time. She will have to do lots of trading and think about when to add or take away...I couldn't get this one right but I think she can.

Although he commented that he did not think that he could obtain a correct solution to this problem, John's comments indicate that he had thought about the processes Bettina would need to use in order to obtain a correct answer. Interestingly, he felt that 5-digit numbers substantially increased the difficulty of the problem.

John found the first problem Bettina designed for him to be quite difficult.

> You have twenty dollars ($20). You go to the newsagency and buy 2 pencils (40c), 1 notebook ($1), stationary pack ($10.50), stickers (10c), and a rubber and sharpener ($1). How many dollars did I spend? How much change did I get? (Bettina's first problem for John)

Although he was able to select the appropriate problem-solving strategies to complete the task after she had read the problem to him, he needed some assistance in order to obtain the *how much change did I get?* component of the question. It would be fair to say that the problem was not well worded. With some guidance from Bettina, however, he was able to obtain a "correct" solution.

One of the most pleasing aspects of the problem-posing activities undertaken in the present study was the fact that the children were willing to work with one another to solve the problems. As Bettina commented:

> At first he didn't really understand all of the problem. He was able to add all the dollars together but he found adding the cents tricky. I helped him with that but he really did most of the work himself.

In subsequent problem-posing situations she was conscious of the mathematical language she used in problems posed for him (see Figure 3).

Figure 3
Bettina's Problem for John

Sarah has \$84.00. To go to noahs ark she needs to pay \$10.5 for each friend. How many friends did she invite?

(Clue - she wants no change)

The content and structure of the initial questions were associated with traditional one- and two-step word problems. Each problem had one correct answer. This is not surprising considering the children were not accustomed to developing their own problems. It could be argued that the traditional format of these problems provided the children with a useful proforma.

Insights into Children's Mathematical Ability

As the children became more competent in designing problems, and more comfortable in working with one another, the way they structured and representated problems changed. Bettina, for example, used less language in the problems posed for John. She also provided him with clues such as "she wants no change" when representing the problem (see Figure 3).

The problems Bettina designed were becoming increasingly sophisticated. In the early weeks of the study she did not need to have a solution in order to pose the problem. After three weeks her problems contained information that could only be designed if she had first sought a solution herself. This type of processing is consistent with findings obtained by other researchers (for example, Ellerton, 1986; Silver, 1995). In the problem posed in Figure 3, for example, she needed to select amounts of money that could be evenly shared. Thus, eight friends @ \$10.50 per head would cost \$84.

Figure 4
Bettina's Problem for a Friend

You have 72 lollies. You give some to Mr. Kennedy, Tanita and keep some for yourself.
Tanita has double what Mr. Kennedy has. You have 3x more than Mr. Kennedy. How many lollies does Mr. Kennedy have? How many does Tanita have? How much do you have?

In the problem represented in Figure 4, which was much more difficult than the type of problems she was designing for John, also required forward planning in its construction. This is a quite difficult ratio problem for children in Year 4 and was again designed for Joe. By indicating that there was a total of 72 lollies to be shared, and that three people were to receive different proportions of the total Bettina must have calculated that the amounts would be 36, 24 and 12 respectively. As Bettina commented "I had to know what the answer would be otherwise the problem wouldn't work." It could be argued that the way in which the problem was structured provided useful insights into Bettina's mathematical ability.

CONCLUSION AND IMPLICATIONS

This study investigated the way in which primary-aged children engaged in problem-posing situations that required them to pose problems for friends. Although the students had not been exposed to such problem-solving activities in the past the problem originator was able to consider the needs and interests of a peer when posing mathematical problems. Not surprisingly, a student's mathematical knowledge had an impact on their ability to generate a variety of problems. It was more difficult for John to construct challenging problems for Bettina to solve because he did not posses the necessary knowledge or mathematical processes to extend her in a problem-solving environment. His only option was to use larger numbers in his problems. It was much easier for him to design problems for someone at or near his level of development. Although this finding may have implications for classroom practice, it needs to be recognised that John did gain a great deal from designing problems for older children. One positive consequence from these problem-posing activities was the fact that he was able to work cooperatively with a peer who challenged him to engage in mathematics at a higher level than he was accustomed to.

These problem-posing experiences also helped the children think about the problem-solving process in more sophisticated ways. In designing problems for friends to solve, the children were required to consider the type of understandings and processes needed to complete a task and also decide whether these understandings and processes were appropriate for the problem-solver to complete. These experiences also provided opportunities for the children to talk to one another about their solution methods in cooperative situations. When designing new problems for their friends to solve, the two children considered one another's comments and insights on a regular basis. From a classroom perspective, problem-posing activities help children to more fully understand the problem-solving process and help the teacher to gain insights into the children's mathematical ability.

REFERENCES

English, L. D., & Halford, G. S. (1995). *Mathematics education: Models and processes.* Hilsdale, NJ: Lawrence Erlbaum.

Ellerton, N. F. (1986). Children's made-up mathematical problems: A new perspective on talented mathematicians. *Educational Studies in Mathematics, 17,* 261-271.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123-147). Hilsdale, NJ: Lawrence Erlbaum.

Lowrie, T. (1998a). Lowrie, T. (1998). Using technology to enhance children's spatial sense. In C. Kanes, M. Goss., & E. Warren (Eds.), *Teaching Mathematics in New Times* (pp. 319-328). Mathematics Education Research Group of Australasia Incorporated. Griffith Uni Print: Brisbane, Australia.

Lowrie, T. (1998b). Developing metacognitive thinking in young children: A case study. *Gifted Education International 13,* (1).

Lowrie, T. (in press). Developing mathematical power. *Australian Primary Mathematics Classroom.*

Mamona-Downs, J. (1993). On analysing problem posing. In I. Hirabayashi, N. Nohada, K. Shigematsu, & F. L. Lin (Eds.), *Proceedings of the 17th International Conference for the Psychology of Mathematics Education, Vol. 111* (pp. 41-47). Tsukuba, Japan: International Group for the Psychology of Mathematics Education.

Silver, E. A. (1993). On mathematical problem posing. In I. Hirabayashi, N. Nohada, K. Shigematsu, & F. L. Lin (Eds.), *Proceedings of the 17th International Conference for the Psychology of Mathematics Education, Vol. 111* (pp. 66-85). Tsukuba, Japan: International Group for the Psychology of Mathematics Education.

Silver, E. A. (1995). The nature and use of open problems in mathematics education: Mathematical and pedagogical perspectives. *International Reviews on Mathematical Education, 27*(2), 67-72.

Stoyanova, E. (1998). Problem posing in mathematics classrooms. In A. McIntosh, & N. Ellerton (Eds.), *Research in mathematics education: A contemporary perspective* (pp. 164-185). Edith Cowan University, WA: MASTEC.

Yin, R. K. (1994). *Case study research: Design and method (2nd ed.).* Newbury Park, CA: Sage.