TEACHER MEDIATION OF STUDENT CONSTRUCTIONS OF ALGEBRA KNOWLEDGE

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This study suggests that classroom research into students' knowledge construction is incomplete without data about teachers' knowledge constructions and practice. Teachers' knowledge related to algebra and teaching algebra were identified and categorised using three knowledge domains identified by Shulman (1987) as knowledge of epistemology, pedagogy and pedagogical content of a algebra. Diverse and idiosyncratic examples of algebra knowledge were articulated by each of the study teachers in relation to the epistemology and pedagogy of algebra, but examples of pedagogical content knowledge were seldom easily identified.

Previous studies about teachers' knowledge and beliefs have mentioned the relationship between teacher knowledge on student learning (Southwell, 1997; Dossey, 1992), and the need for improved teacher knowledge (Borko and Putnam, 1996; Dossey, 1992; Mitchelmore, 1995; Prawat, 1992; Shulman, 1987). My research looked beyond these observations and suggests that there are important if not critical reasons for not only acknowledging but characterising teachers' knowledge constructions when investigating classroom learning. Shulman states that the teacher

> communicates, whether consciously or not, ideas about the ways in which "truth" is determined in a field and a set of attitudes and values that markedly influence student understanding. (Shulman, 1987, p. 9)

Accepting Shulman's (1987) view, and the views of the researchers mentioned above, suggests that classroom research into students' construction of knowledge cannot be divorced from investigations into teachers' knowledge constructions. Classroom learning is viewed in the present study from the perspective of teacher mediation between a content domain and student knowledge constructions, and emphasises that knowledge construction in classroom settings is the result of a partnership between the teacher and students where knowledge is developed as it is shared. This research built upon the idea of "classroom negotiation of meaning" (Clarke, 1996). Clarke's (1996) study was concerned with two foci. One was the individual students' sense making. The other foci was the "social domain within which teachers' and learners' individual contributions play a key role in social sense-making and in the utilisation of mathematics and science concepts and skills" (Clarke, 1996, p. 4). Clarke's (1996) research emphasises that research into classroom learning involves examination of the "symbiotic relationship of social and individual negotiation of meaning: symbiotic in the sense of being mutually dependent and mutually supportive" (Clarke, 1996, p. 4). This suggests that the teacher is an important element in the highly complex learning environment of the student. Whether researchers adapt constructivist or socio-constructivist views of learning, acknowledgment is made of the significance of individual knowledge constructions (Lerman, 1994). This suggests that if teachers each have their own conceptions of mathematics then the knowledge constructions of teachers must play a crucial role in the knowledge construction of students. Classroom research needs to investigate the nature of teachers' knowledge constructions and particularly teachers' interpretations of specific content domains (such as algebra) and how teachers choose to make these interpretations visible for students via different representational forms.

The research design of this study therefore required the development of instruments and methods for identifying and analysing the forms of teacher knowledge constructions. Because learning cannot be separated from the effects of cultural influences and personal values (Southwell, 1997), beliefs about and reasons for teachers' choices of content, content

representation and style of instruction also formed important aspects of the present research. The diverse nature of the data required for the current study made it obligatory to include in the research methods: Classroom observations, questionnaires and interviews. The data collection instruments created opportunities for teachers to express their knowledge and beliefs about algebra in each of these knowledge domains. This paper provides an explanation of the ways in which this research complements classroom research into students' knowledge construction.

A Methodology for Investigating Teacher Knowledge

A useful understanding of knowledge construction in classrooms must recognise the form of teachers' knowledge constructions and the form in which teachers' express their understandings within a particular content domain. This means that research into students' construction of knowledge:

1. Must acknowledge and incorporate teacher interpretation and representation of prescribed academic content,
2. Must include document the nature of teachers' understandings,
3. Must include research into the form and patterns of teachers' instructional practices and, the identification of the criteria used by teachers to select and construct instructional materials and in making pedagogical decisions that lead to their instructional behaviours.

The methods used in this research were designed to ensure a comprehensive investigation of teacher knowledge with respect to the epistemology of algebra, the pedagogy of algebra, general pedagogy, general learning theory and the learning of algebra. Each of the research components outlined in points 1 to 3 above are described in greater detail below.

An emphasis in this research was teachers' conscious and unconscious efforts to mediate learning. A major premise was that teachers' unique constructions of their knowledge and beliefs form a significant factor influencing the creation of possibilities for instruction. The thesis presented here views the function of teacher mediation in the learning process as involving first the teachers "interpretation" of mathematics content and the "representing" of this content for students in accordance with the teachers' personal conceptions. Each of these terms "interpreting', "mediating" and "representing" are used for a specific purpose in this study to make clear different aspects of the teaching-learning process. Each of these terms are described below in relation to algebra.

Interpreting

Teachers draw on their experiences with algebra in different contexts over time to form their "interpretations" of "algebra". Part of what teachers regard to be algebra is situated as personal knowledge, knowledge they see no need to communicate, part is public knowledge, knowledge they formalise for the purpose of communicating to others, and part of their algebra knowledge they identify as suitable content for students. For the purpose of teaching, interpretation is predicated on an understanding of the structure of the subject matter and the principles of conceptual organisation of the content domain (Shulman, 1987).

Mediating

Teachers are in the position of having to "mediate" between recognised algebra content and their students' conceptions of algebra. Mediation begins with teachers' understanding of algebra, algebra curriculum, students' stages of conceptual development, how students conceive algebra knowledge and student' understanding of algebra. The mediation process involves teachers translating aspects of their personal epistemologies of algebra into representational forms they believe will be both useful and comprehensible to students. Mediation is conceived, in this

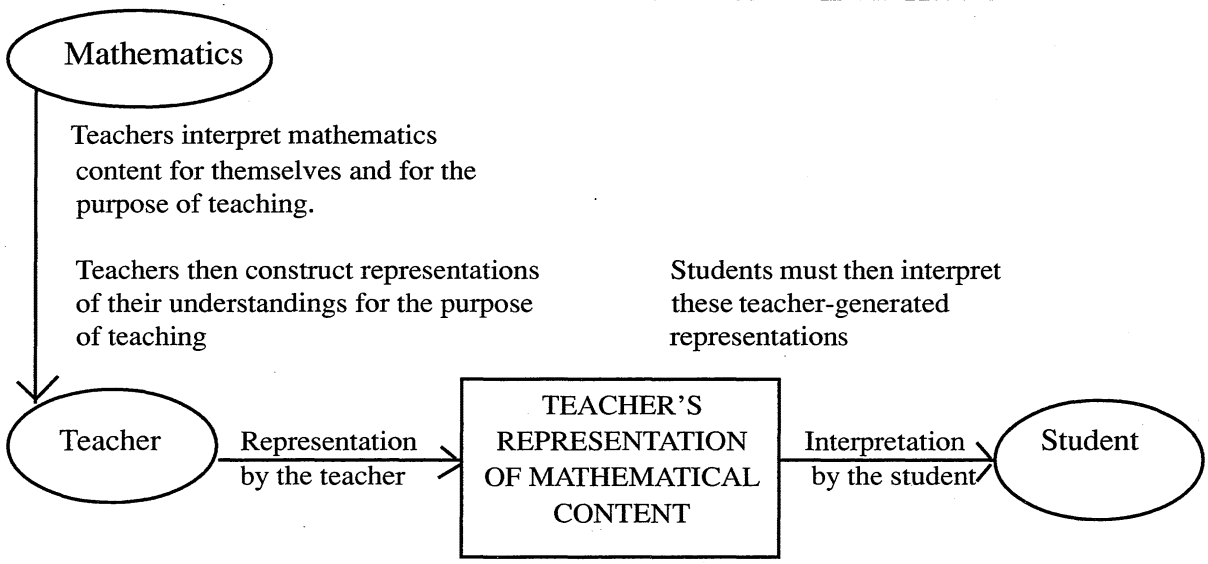
study, as a dynamic process whereby teachers are continually responding to the evolution of students' knowledge constructions.

Representing

The form of teachers' representations of algebra included the algebra content they choose to include, the time allocated to different aspects of algebra, instructional methods (for example, expository styles or activity based), the representational sign systems used by the teacher (for example, words, pictures, patterns, metaphors, and graphs) and also the context in which teachers use different representations (for example, real world problems, and the extension of ideas of arithmetic).

The relationship of the acts of "interpreting" and "representing" within the overarching process of "mediation" in classroom settings is illustrated in Figure 1 below.

Figure 1:
A model of Teacher Mediation between Mathematics Content and the Student



The model depicted in Figure 1 postulates a structure for the process whereby the teacher mediates between content and student. What is missing from Figure 1 is demonstration of the responsiveness within the mediation process, whereby the teacher acknowledges student interpretations and consequent mathematical constructions, elicits representations of the students' knowledge, and than adapts or modifies or elaborates the initial representations in ways that promote correct mathematical interpretations and constructions by the students. This "negotiative" process is discussed at greater length in Clarke (1996).

Research Principles

In the course of this research, several principles were developed that guided data collection. These principles appear to have general applicability to research into classroom teaching and learning.

Principle 1. Research into student constructions of knowledge must acknowledge and incorporate teacher interpretation and representation of prescribed academic content.

The mediating role of the teacher can be understood in terms of socio-constructivist theories of learning in which the teacher orchestrates social exchanges that give rise to knowledge

construction both of the teacher and student. The practicalities of this mediating role involve interpretive acts on the part of both teacher and student, and the anticipation on the teacher's part of an audience for their representations. This suggests that teachers and students are involved in what Clarke (1996, p. 2) describes as "negotiation of meaning". It is our contention, based on this research, that comparable attention must be paid to the teachers' learnings as is commonly paid to the learning of the student.

Principle 2. Research into student construction of knowledge must include adequate study of teachers' understandings.

While recognition must be given to the sources of teachers' knowledge construction of academic content (such as might be accessed from textbooks and pre-service programs and inservice training), the resulting teachers' conceptions are not always explicit or visible. Demonstrations of teachers' understandings will include actions such as teaching practices, together with teachers' espoused beliefs and epistemologies. Any study of teacher understandings must distinguish between teacher knowledge and teacher practice. The following list builds upon the work of Shulman (1987) to identify specific indicators of teachers' understandings. These are:

(a) content knowledge;
(b) pedagogical content knowledge;
(c) general pedagogical knowledge;
(d) general instructional practices;
(e) content specific pedagogical practices;
(f) knowledge of learning processes;
(g) curriculum knowledge.

In this study, each of these indicators was studied from multiple perspectives.

Principle 3. Research into students' construction of knowledge must include research into the form and patterns of teachers' instructional practices and the identification of the criteria used by teachers to select and construct instructional materials and in making pedagogical decisions that lead to instructional behaviours.

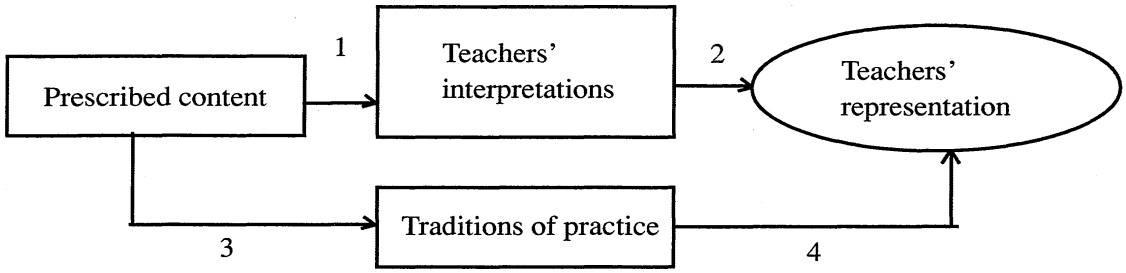
This research needs to address the manner in which teachers reflect upon their knowledge and beliefs about content and practice and the way teachers observe traditions of practice and examples of exemplary practice. Documentation of such reflections and observations should include:

- how teachers construct representations of curriculum content,
- how the representations are delivered to students,
- how teachers assess students' understanding and
- how teachers intervene in students' knowledge constructions.

Teachers may refer to their personal interpretations of prescribed content, or suggested teaching material and activities when they make choices about instructional materials and practices. Teachers' choices will depend, however, upon how they conceive the activity of teaching, learning and what constitutes important content for learning. These choices are illustrated in Figure 2.

Functioning as reflective practitioners, teachers could interpret prescribed content (1) and then develop appropriate representations of that content (2) to sustain student learning. However, it might be that the specification of content in a curriculum document cues the teacher's use of a particular instructional practice (3) (within the traditions of practice for that subject) and that the chosen practice effectively prescribes the representations to be used (4).

Figure 2
Alternate Pathways to Teacher Representations



Conceivably a teacher may devote significant lesson preparation time to lesson designs based on their knowledge of traditions of practice and innovative lessons documented and promoted by "experts". The pathway from prescribed content to teachers' representations of this content along arrows 3 and 4 conceivably bypasses any need for a teacher to first reflect on their personal interpretation of the prescribed content. Teachers' active construction of representations based on their interpretations of prescribed content and their knowledge of their students' states of knowledge is represented by arrows 1 and 2. The importance of teachers utilising both traditions of practice and their personal knowledge is illustrated by the research findings summarised later in this paper.

Principle 4 Pedagogical content knowledge is important

Throughout the research it became clear that it was difficult to identify, using primary data, examples of teachers' knowledge of pedagogical content knowledge. This raised the question: What value to researchers or educators is Shulman's (1987) concept of pedagogical content knowledge? Finding few examples of pedagogical content knowledge in the data set generated by this research suggested that either:

- knowledge cannot usefully be categorised as pedagogical content knowledge,
- the teachers possessed little pedagogical content knowledge, or
- pedagogical content knowledge exists and is important but not in a form that is easily witnessed during classroom observations, interviews or from questionnaires.

The research data suggested that teachers' pedagogical content knowledge would be most evident when the researcher discussed or observed specific incidences of student learning and teacher intervention methods with the teacher. In the case of interviews, the teachers were asked to identify and describe the concepts a particular student needed to have developed to complete a task, this student's current stage of concept development with respect to the task, the concepts the student was failing to understand and how that concept might be expressed to the student. Teachers' responses provided evidence of the teachers' pedagogical content knowledge.

Important Research Findings

Finding 1. The teachers' beliefs about content specific instructional practices were similar despite quite individual interpretations of algebra content.

A finding of this research was that while the teachers' personal epistemologies were quite diverse, their beliefs about content-specific instructional practices were quite similar.

A pattern evident in the research data related to the teachers' views about the importance of "instrumental understanding" (Skemp, 1978) of algebra and the teachers' beliefs about students' construction of knowledge. Each of the teachers studied encouraged rote learning of algebraic procedures and justified their decisions with respect to content and instructional method in terms of providing students with the instrumental understanding the teachers believed students required for their VCE studies. The teachers also explained that "the more able" students would construct their personal understanding of concepts associated with algebra skills and procedures as a consequence of routine practice and would gain no particular benefit from explorations. The teachers' adherence to teacher centred, expository, drill and practice teaching strategies was consistent with the teachers' reported beliefs about how students learn and develop concepts.

The findings above suggest that the form of algebra teaching conceived by the teachers in this study could be carried out by teachers with minimal pedagogical content knowledge and minimal teacher reflection about reasons for students' difficulties, misinterpretations or insights. This is a significant finding with respect to facilitating a better understanding of the practices of the mathematics classroom and of students' knowledge construction in classrooms.

Finding 2. There was often little association between the teachers' reported beliefs about the pedagogy of algebra and the teachers' observed instructional practices.

For the teachers investigated there was often little association between the teachers' knowledge and beliefs about broad algebra concepts they reported to be important aspects of a conception of algebra and the algebra they actually taught. These contradictions included:

1. The teachers' observed focus for instruction tended to be concerned with instrumental rather than relational understanding (Skemp, 1978) even though they reported valuing relational understanding.
2. The teachers expressed a belief that students need a context to make sense of algebra yet the teachers:
    a. usually taught algebra out of context;
    b. often taught applications as a separate topic or activity without relating the application to skills learned in other topics;
    c. made few obvious attempts to link algebra topics to each other or to other student knowledge.

The research data identified the contradictions between the teachers' algebra instruction and comments about how students learn algebra derived from:

1. the teachers' limited knowledge of the epistemology and pedagogy of algebra.
2. the teachers' limited knowledge of the concepts students need in order to develop other algebra concepts; that is, learning hierarchies as these relate to algebra.

3. the teachers' attempts to adopt content and suggested activities set out in curriculum documents without reflecting on their intended function and how the new content or activities might make algebra more accessible to students.

The consistencies and inconsistencies evident in the research data suggested an additional seven findings worthy of discussion. Each of these additional findings arose from analyses of the data from the perspective of the teacher as mediator during the student-content interactions that happen in mathematics classrooms. This perspective for the analyses made possible some extremely useful reflection on classroom learning and suggested implications for the classroom teaching and learning, and for further research. It is not possible to address all these additional findings within the scale of this paper, however, it was felt useful to list them below:

1. The teachers' curriculum choices were seldom influenced by the teachers' knowledge and beliefs of the epistemology and pedagogy of algebra but were often influenced significantly by the teachers' beliefs about "good teaching",
2. Teachers need to assist students to make the transition from procedural to structural conceptions of algebra, and recognise the importance of students' understanding of arithmetic for developing an understanding of the structural aspects of generalised arithmetic and algebra,
3. Teachers' descriptions of school algebra emphasised the manipulative aspect of algebra but they did not attend to related structural aspects,
4. The teachers' diagnosis of student difficulty with algebra did not relate to concepts identified in the literature as important for learning algebra,
5. There was a need for teachers to make connections within algebra during their teaching,
6. Teachers need to understand how students construct knowledge and
7. There was a need for the teachers to make connections between algebra content and content beyond algebra.

One general conclusion could be drawn from the combination of findings arising from this study: Teacher reflection is important.

An important finding of this research was that the teachers' often lacked the detailed knowledge needed to both identify specific student difficulties and construct situations that might assist students to overcome their difficulties. In the absence of such pedagogical content knowledge, the teachers in this study had developed beliefs that effective algebra teaching involved taking students through prescribed sequences of instruction along with the incorporation of suggested activities described in the *Curriculum and Standards Framework* (Victorian Board of Studies, 1997) documents and other teacher valued resources.

Generally the teachers believed that task selection did not require their detailed knowledge as this was the responsibility of course advisers and textbook writers. Their teaching focus was curriculum implementation as advocated by curriculum authorities.

Figure 2 above can be used to explain why these dedicated teachers were disappointed by the learning outcomes of their students. Each teacher explained that many students in their classes had conceptual difficulties with algebra but the teachers were seldom able to identify specifically the concepts the student had difficulty with. The teachers usually explained student difficulties in terms of readiness to learn abstract concepts, lack of attention or lack of practice. The more experienced teachers sought out other resources that might provide more effective teaching material. It became clear throughout the research that the

teachers were attending to the process of teaching rather than learning. Paying attention to traditions of practice and new and innovative instructional materials and activities meant that the teachers were distracted from their own knowledge constructions and those of their students and hence saw no need to draw upon their personal knowledge to comprehend their students' learning. Remediation of student misconceptions or lack of understanding requires that teachers develop reflective behaviours with respect to their knowledge of content and of the learning process and their students' knowledge construction.

This suggested that the development of pedagogical content knowledge needs to occur in the context of student learning and has no meaning without it. Pedagogical content knowledge is developed as a consequence of teaching and engaging in the teaching-learning process. This interpretation of the data has important implications for teacher education programs and for research into the meaning constructions that occur in classrooms.

CONCLUSION

This research suggested reasons why it is important to recognise and incorporate teacher interpretations and representation of prescribed academic content when researching students' construction of knowledge. It was suggested that data about teachers' knowledge constructions will both add context and detail to classroom research into students' construction of knowledge. Without the inclusion of teachers' knowledge construction the interpretation of students' knowledge constructions may be flawed or incomplete. Acknowledgment has not been widely given to possibilities of students' derivations of meaning from their teachers' personal representations of their (the teachers') idiosyncratic interpretations of particular algebra content. A finding in the current study was that the teachers studied did have individual conceptions of algebra content. The fact that this small sample of teachers differed in their epistemologies of algebra suggest that classroom implications of this diversity are much wider than research currently suggests. This raises an interesting question for future research: To what extent do we want teachers to express their individual conceptions of content in their teaching, and to what extent do we want teachers to deliver "the same content"? We would argue that if teachers' have a detailed knowledge of the structure of algebra, differences in their conceptions are likely to be ones of emphasis and depth of understanding rather than of the basic nature or spirit of the concepts and skills.

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