VISUAL REPRESENTATIONS IN FIRST YEAR STATISTICS

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This paper summarises the findings of a study into the extent to which students in introductory statistics courses use diagrams when solving problems. The results of the study showed that many students did not use diagrams but those who used diagrams were more successful. Use of diagrams and university entrance score appeared to be better predictors of success than the level of mathematics studied at secondary school.

INTRODUCTION

At university many students study statistics because of its importance to disciplines such as economics, psychology, medicine, biology, and education. Statistics differs from school mathematics because of its emphasis on interpretation of data, rather than on solution of mathematical expressions. Statistical reasoning involves both visual and algebraic methods; visual or graphical methods are necessary for an initial inspection of the data to determine distribution characteristics, such as normality, dispersion, and outliers, while non-visual methods are predominantly employed in hypothesis testing and calculating confidence intervals. It is important that students can coordinate both visual and non-visual thinking when solving statistical problems.

Students who enrol in statistics with minimal secondary school mathematics may rely on learning procedures to solve statistical problems, without a sound understanding of fundamental concepts. Therefore, they may have difficulty in solving non-routine statistical problems for which they have to decide on an appropriate procedure and interpret its results. By contrast, students with strong mathematical backgrounds may be reluctant to use visual methods, such as graphing or drawing a diagram, because of an emphasis on algebraic methods in mathematics teaching. For example, Vinner (1989) found that tertiary students tended to prefer an algebraic proof to a diagrammatic one, even when the latter, as stated by students was easier to follow. He felt that this preference was affected by the method of teaching where students gained the impression that symbolic solutions were more prestigious than diagrammatic solutions.

Visual representations may be included as part of a problem information or students may produce their own representations as part of the solution process. In research on primary and secondary students' interpretations of graphs that accompanied statistical problems, Curcio (1987, 1996) identified three levels of graph comprehension: literal reading from the graph; comparisons using the graph with an emphasis on reading between the data; and reading beyond the data. If students are at the literal and comparative levels in their knowledge of graphs, they may have difficulty solving statistical problems when they are only given the raw data. However, some research suggests that students may respond at a higher level when the data is presented in raw form rather than in graphical form (Reading and Pegg, 1998). These authors analysed the responses of secondary students to two data reduction questions, one of which presented the raw data and the other a graphical representation. Other authors have questioned whether graphical ability is related to cognitive ability (Roth and McGinn, 1997). These authors suggest that graphical ability is a consequence of practice in using graphical methods in practical and social situations.

Students, who represent statistical data visually, for instance by using a stem and leaf plot or a graph, may be more successful in solving statistical problems than those who rely on algebraic methods. Shaw (1998) investigated whether the use of statistical displays by university students was associated with the use of diagrams in non-mathematical and mathematical problems. The results of her study indicated a relationship between the use of statistical displays and the use of diagrams in mathematical, but not non-mathematical situations. Moreover, students who spontaneously drew diagrams for the statistical problems were more likely to gain higher marks in the final statistical examination than students who did not.

The aim of this paper is to investigate the spontaneous use of diagrams by tertiary students as they solved statistical problems. The problems were chosen so that their solutions would be facilitated by drawing a diagram. The aims of the study were to:

- ascertain if students who drew diagrams to represent data were more successful than those who did not;
- investigate the relationship between ability, level of school mathematics, and the use of visual solution methods on students' success in solving statistical problems.

METHODOLOGY

Students in introductory statistics courses at two Australian universities in different states were given an assessment task comprising three statistical problems and one mathematical problem. At both universities the course was a large (>500) service course for first year students. Both courses covered displaying and summarising data, distributions and sampling distributions, hypothesis testing of means for one and two samples, regression and categorical data. The problems required extrapolation from and interpretation of the data, that is, responses at Curcio's third level. There were four versions of the task, the versions differing only in the order in which the problems were given. Students were asked to provide their University entrance score as an indicator of ability.

The tasks were not given at the same time, or under the same conditions at the two universities. At University 1 the assessment task was given in a lecture midway through the course when they had studied hypothesis testing with one and two samples but had not yet received any instruction on regression. Not all students were assessed; only those attending particular lectures, 186 students altogether. At University 2 the assessment task was set as a take-home assignment and the students were given 5 marks for completing it. Students were asked to spend no more than 30 minutes on the task, which was given at the end of the course, as students were preparing for their final examination. A total of 781 students completed the task.

Each student's response was given a diagram score of 0, 1 or 2 (0 = no diagram, 1 = partial diagram and 2 = correct diagram) and a score for the solution from 0 to 3 where 3 = correct solution. In addition, the students were asked to give the level of mathematics they had studied at secondary school and this was coded from 0 to 3 where 0 = no mathematics at Year 12 and 3 = the highest level of mathematics studied. However, in University 1, students with the highest level of mathematics enrol in a different statistics unit so the populations of students in the two universities were quite different.

The Assessment task

The four questions used in the assessment task are given below. The fourth question was a modification of a question used by Campbell et al (1995).

Problem 1 **Area under the normal curve**: The age of academic staff at Newport University are normally distributed with a mean of 38 years and a standard deviation of 5 years. What proportion of staff would be expected to be aged between 45 and 50 years?

Problem 2 **Linear relationship between two variables**: It has been claimed that as academic staff get older their tolerance of students decreases. A test of staff tolerance of students has been developed. Ages of a random sample of six staff at Newport University and their tolerance scores are listed below. Are the data likely to support this claim?

Name	Grey beard	Long legs	Mac	Boffin	Blondie	Shortie
Age	52	39	33	25	22	45
Tolerance	28	35	35	50	39	23

Problem 3 **Distribution of a single variable**: The employment history of a random sample of 30 academic staff at Newport University was obtained. Listed below are the number of years that they have worked at Newport University. As a person with statistical knowledge you have been asked to comment on this data.

1	7	5	2	5	6	4	5	5	6	2	2	2	6	6
3	5	6	9	5	5	1	3	4	7	5	2	2	7	6

Problem 4 **Linear algebra**: The blood alcohol readings of two lecturers from Newport University were recorded the morning after an accident. The readings were:

Alison	6 hours after accident: 5 units 8 hours after accident: 2 units	Brett	5 hours after accident: 7.5 units 9 hours after accident: 5.5 units

Assuming a linear relationship, when were their readings the same?

RESULTS

Relationship between Drawing a Diagram and Solving the Assessment Task

It is clear from Table 1 that as would be expected, the results are very different for the two universities.

Table 1.
Percentage of Responses from each University, Categorised by Diagram Usage and Solution Category for each Problem

	University 1		University 2	
	No solution, or inadequate solution	Reasonable or correct solution	No solution, or inadequate solution	Reasonable or correct solution
Problem 1	*Areas under the normal curve*			
No diagram*	48	13	16	31
Correct diagram	15	24	5	45
Problem 2	*Linear relationship between two variables*			
No diagram*	61	23	26	17
Correct diagram	3	13	8	50
Problem 3	*Distribution of a single variable*			
No diagram*	57	7	26	5
Correct diagram	17	19	24	45
Problem 4	*Linear algebra*			
No diagram*	60	19	14	42
Correct diagram	4	17	1	43

* A partial diagram was included in the "No diagram" category as such diagrams were usually inadequate as an aid to obtaining a solution.
No solution, or inadequate solution ¾ score of 0 or 1; Reasonable or correct solution ¾ score of 2 or 3.

The results for University 1 show that in general these students did not draw diagrams and the majority did not successfully solve the problems. For Problem 1, the most familiar to the students, more students drew diagrams (39%), than for Problems 2, 3, and 4 for which the percentages were 16%, 34% and 21% respectively. For this sample of students Problem 1 would have been the most familiar and in lectures and tutorials diagrams would have been emphasised for problems of this type. Problem 2 was given to students from this university before they had been given any instruction in regression and their responses reflect ideas developed at secondary school. The majority of the students (84%) did not draw a diagram and less than a quarter of these students obtained the correct answer whereas 81% of the students who drew a diagram successfully solved the problem.

The results for University 2 clearly show that this sample of students was much more successful in solving statistical problems and many more of these students drew diagrams. Nevertheless, a large proportion of them (approximately 30 to 55%) did not draw diagrams and these students were less successful in solving the problems than those who drew, particularly for Problems 2 and 3. These two problems were both open-ended and less familiar in form than Problems 1 and 4.

Table 2 shows the relationship between diagram usage and correctness; only students who obtained a maximum score of 3 for each problem are included. The results presented in Table 2 show that for all four problems those students who drew a diagram were far more likely to obtain a successful solution than those who did not, particularly for Problems 2 and 3. For Problem 2 many students who stated that there was no relationship between the two variables were influenced by their interpretations of one or two points, rather than looking at the global trend. In Problem 3 most students simply calculated measures of centre and spread without investigating the form of the distribution. Such a response was coded as 1, an inadequate solution. In fact, the data was bimodal with peaks at 2 and between 5 and 6.

Table 2
The Percentage of Students who Correctly Solved (score=3) each Problem, Categorised by Diagram Usage and University

Problem	1	2	3	4
University 1				
Correct solution, no diagram	11	14	1	21
Correct solution, diagram	46	57	18	58
University 2				
Correct solution, no diagram	62	22	0	70
Correct solution, diagram	84	87	15	95

However, it is evident from Table 2 that for University 2 many students who did not draw a diagram were successful in solving Problems 1 and 4. Problem 1 was a standard statistical problem, finding an area under the normal curve, and Problem 4 was a mathematical question, rather than a statistical one. The latter could be answered graphically, algebraically or using ratios. Students who had a higher level of mathematics at secondary school might be expected to be more successful if they chose to solve these problems algebraically. There is some evidence that this is so; 71% of the highest mathematics group were successful for Problem 1 without drawing a diagram, compared with 90% who drew a diagram. For Problem 4 the corresponding percentages are 84% and 98% respectively.

In the next section we investigate the effects of school mathematics; overall university entrance score (as an indicator of general ability); and drawing a diagram on students' success in solving these problems

Relationship between Ability, Level of School Mathematics, and use of Visual Solution Methods on Success in Solving Statistical Problems.

A regression analysis was carried out on the results for each university to determine the effect of the three factors on problem solution. Total scores for problem success and diagram usage were obtained for each student by summing the individual scores for the four problems. Additional analyses were undertaken for (a) Problems 1 and 4 and (b) Problems 2 and 3; the same procedure of adding individual scores was used to obtain scores for these problem subsets. The results for the analysis of all four problems is set out in Table 3. There was a high proportion of missing values, particularly for University 1, as not all students included information on their university entrance score when they completed the assessment task.

The results shown in Table 3 indicate that for both universities, university entrance score and drawing a diagram are significant indicators of success on the four tasks. School mathematics, while a significant predictor for University 2, is not significant for University 1. However, these three factors account for about 35% of the total variance in the students' responses. Drawing a diagram was highly correlated with total score on the four problems (0.51 for both universities) but level of school mathematics was not correlated with drawing a diagram (0.02 and 0.03). Correlations for finding a correct solution and school mathematics were 0.11 and 0.19 for University 1 and 2 respectively.

Table 3
Regression Analyses for the Effect of School Mathematics, Ability and Diagram Usage on Success in Solving the Assessment Task

University 1 (n=89)

Predictor	Constant	Ability	Diagram	School maths
Coefficient	-3.32	0.08	0.46	0.32
t-ratio	-2.26	4.56	3.98	0.94
Probability	0.026	0.000	0.000	0.348

R- square = 36.9%, F(3,90)=25.21, P=0.000

University 2 (N=656 subjects)

Predictor	Constant	Ability	Diagram	School maths
Coefficient	4.26	-0.18	0.59	0.65
t-ratio	7.77	- 6.12	16.38	4.45
Probability	0.000	0.000	0.000	0.000

R- square = 34.5%, F(3,652)=114.64, P=0.000

There are several reasons that may explain the discrepant results for school mathematics. First, the students who have the highest school mathematics results enrol in a separate course, so they were not included in the results for University 1. Second, the sample size for University 1 is small relative to University 2 and contains a high proportion of mature age students, some of whom may not have taken mathematics at secondary school and others for whom mathematics is but a distant memory. The age distributions showed that about 58% of University 1 were less than 24 years of age compared with 96% of University 2.

The data were re-analysed to investigate the possibility that students from University 2 with higher levels of mathematics might have been more likely to solve Problem 1 and 4 algebraically than students with lower levels of mathematics. The problems were combined into subsets, Problems 1 and 4, and Problems 2 and 3. Separate regression analyses were undertaken for these subsets for both universities. Table 4 shows the percentages of the variance (R-square) in scores for the problem subsets that is explained by (a) all three factors as predictors, and (b) inclusion of a diagram as the sole predictor.

Table 4
R-square Values for Regression Analyses of the Problem Subsets

	University 1		University 2	
Subset	All factors N=89	Diagram only N=186	All factors N=655	Diagram only N=780
Problems 1 and 4	30.9*	26.0	20.6***	9.8
Problems 2 and 3	56.7+	50.4	42.2++	41.1

* School mathematics and ability significant at $p<0.05$.
*** School mathematics and ability significant at $p<0.001$.
\+ Ability was significant ($p<0.005$).
\++ Neither school mathematics, nor ability significant ($p>0.05$)

The values of R-square shown in Table 4 suggest that knowledge of school mathematics contributes most to the problems that can be solved algebraically, that is, Problems 1 and 4. The values for this subset were quite different for the two universities; for University 2, which had a more mathematically competent enrolment, students were less dependent on diagrammatic methods to solve the two problems. By contrast, for Problems 2 and 3, drawing a diagram was the key predictor of success in both universities.

The means for total and diagram score for each problem subset categorised by ability level (as measured by university entrance score) are shown in Table 5. The ability levels were obtained by categorising the university entrance scores into 4 and 5 categories respectively for Universities 1 and 2, where 1 is the lowest ability category and 4 or 5 is the highest. The scores for the lowest ability categories for both universities on Problems 1 and 4 were slightly anomalous in that they are higher than those of students with a higher level of mathematics. One explanation may be that this category may include a high proportion of mature-age students or those who have not enrolled through the usual admission process. For Problems 2 and 3 this trend can be seen in the means for University 2, but not University 1.

Table 5
Mean Total and Diagram Score for Each Level of Ability. Standard Deviations are Shown in Brackets

Ability level	N*	Problems 1&4 Solutions	Problems 1&4 Diagram score	Problems 2&3 Solutions	Problems 2&3 Diagram score
			University 1		
1	7	2.6 (1.7)	2.3 (1.0)	1.3 (1.3)	1.1 (1.1)
2	18	1.8 (1.8)	1.4 (1.5)	1.7 (1.6)	1.0 (1.4)
3	26	2.6 (2.4)	1.9 (1.4)	2.4 (1.5)	1.3 (1.4)
4	37	3.6 (2.1)	2.4 (1.4)	2.4 (1.6)	1.4 (1.5)
			University 2		
1	34	4.3 (1.6)	2.1 (1.1)	3.4 (1.6)	2.3 (1.2)
2	103	4.2 (2.1)	2.2 (1.4)	3.3 (1.6)	2.5 (1.4)
3	126	4.7 (1.7)	2.3 (1.4)	3.4 (1.5)	2.6 (1.4)
4	185	5.1 (1.5)	2.3 (1.4)	3.6 (1.5)	2.7 (1.3)
5	221	5.5 (1.1)	2.1 (1.4)	4.0 (1.5)	2.9 (1.3)

* Number of students included in each ability level.

CONCLUSIONS

The results for these two universities clearly show that students who draw a diagram as part of the solution process were far more successful than students who do not. The majority of students in University 1 did not draw diagrams, despite being encouraged to do so throughout their course. A far higher proportion of students from University 2 drew diagrams and were overall far more successful in solving the four problems. However, as the administration of the problems were so different for the two samples, it is impossible to make comparisons. Perhaps, the students who completed the problems as a take-home task may have drawn diagrams on scrap-paper (despite having been asked to include all working). Moreover, this sample included a far higher proportion of younger students with higher levels of school mathematics as well as higher university entrance scores. These students may have been confident to change their solution strategy. Thus, they drew diagrams when they realised it was necessary, otherwise they solved the problems algebraically.

Why then do students not see that diagrams are a useful problem solving strategy? One reason that came out of interviews with a small sample of students (Shaw and Outhred, 1999) was that students were not sure that a diagram was worth the effort it took to draw it, rather than not knowing what to draw. Students seemed to want to calculate statistics, such as means and standard deviations without first obtaining a feel for the data. None of the students who were interviewed suggested a diagram might be an easier method of solving some types of problems.

We thought that students who had studied more mathematics at secondary school would be more likely to draw a diagram and also be more successful in solving the problems. However, the analysis of the data did not seem to support this hypothesis. Tertiary entrance score was found to be a better predictor of success than level of secondary school mathematics although this relationship did appear to be affected by the type of problem. Problems that were more readily solved by algebraic methods seemed to be related with level of school mathematics, whereas the problems that involved interpretation of data sets did not seem to be related. The relatively poor performance on the problems that required interpretation of data would suggest a need for the inclusion of open-ended tasks based on real or simulated data sets so that students discuss different perceptions of the same information.

Since students who draw diagrams are more successful than those who do not, greater emphasis should to be given to integrating the use of diagrams into the teaching of introductory statistics. In addition, the construction and use of diagrams may need to be incorporated into course materials and assessment. An investigation of the teaching approaches of the two universities would seem to be warrented to determine if this might contribute to the differences in the proportions of students who use diagrams.

REFERENCES

Campbell, K. J., Collis, K. F. and Watson, J. M. (1995). Visual processing during mathematical problem solving. *Educational Studies in Mathematics 28,* 177-194.

Curcio, F.R. (1987). Comprehension of mathematical relationships expressed in graphs. *Journal for Research in Mathematics Education,* 18 (5), 382-393.

Reading, C. & Pegg, J. (1998). Exploring understanding of data reduction. *Proceedings of* the *Twenty-second International Conference on the Psychology of Mathematics Education, 3* (pp187-194). Stellenbosch: Program Committee.

Roth, W-M. & McGinn, M.K. (1997). Graphing: Cognitive ability or Practice? *Science Education,* 81(1), 91-106.

Shaw, P.F. (1998). Why don't students display data? *Paper presented at ICOTS5*, Singapore.

Shaw, P. F. and Outhred, L. (1999). Students' use of diagrams in statistics. Paper submitted to *the Twenty Third International Conference on the Psychology of Mathematics Conference,* Haifa, Israel.

Vinner, S. (1989). The avoidance of visual considerations in calculus students. *Focus on learning problems in mathematics, 11* (2), 149-156.