What is Taught versus What is Learnt: The Case of Linear Measurement

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This paper presents the findings of a study that examined the results of 120 students selected according to their performance levels in mathematics in Year 1 to Year 5. The results show that while most high-ability students had a conceptual understanding of length, the majority of the lower-ability students did not acquire important concepts relating to the linear nature of units and took longer to acquire basic measuring skills. This knowledge is essential for its application in perimeter, area and volume, and in topics that rely on the understanding of scales, such as directed number gauges, and graphs.

Introduction

While performance on mathematics tests have shown that some cultural groups in the United States have made significant gains in mathematics, Lindquist (1989, p. 4) commented that it is imperative to "...take a look at our low–ability students." A survey of the research literature reveals that scant attention has been paid to students who perform poorly in mathematics. For example, such students may take longer to complete tasks, may not understand mathematical concepts and may make errors in basic calculations. Lindquist suggests these students need a curriculum designed so that they can make sense of mathematics, instead of an emphasis on learning procedures that they may not understand or see as relevant.

Low-achieving students may face years of frustration studying material that they may see as irrelevant (Tayler, 1995). These students will require specific assistance from teachers, yet there seems to be little information to identify their particular difficulties. Additional research is required to pinpoint the specific problems of low-achieving students, particularly in measurement, because little appears to be known about the development of measurement concepts by these students.

Students, especially low-ability students, may be taught techniques for measuring lengths, rather than the principles that govern the measurement process. Such techniques may not provide the basis for developing ideas of units and scales or a framework for later measurement concepts. If students do not have an understanding of linear units they are unlikely to succeed in more complex areas of mathematics; for example, area and volume in which length units are extended to two and three dimensions respectively. Crucial to an understanding of measurement is the nature of the parts (or units) that are counted (Hiebert, 1984). The cognitive significance of the distinction between counting discrete objects and the 'counting' of a continuous property defined as length was noted by Piaget, Inhelder and Szeminska (1960) and, more recently, in a pedagogical sense by Wilson and Rowland (1992).

Researchers have suggested that measurement instruction should begin with informal units because the use of informal units promotes understanding of measurement processes by helping students to see the relationship between the 'continuous' nature of length and the 'parts' that may be counted (Wilson & Osborne, 1988). However, unless the process of measuring with informal units is linked to the construction of a scale, students may not see the relationship between this process and its formalisation as an abstract representation of standard unit, the centimetre, in the form of a ruler. Although rulers are "...an indirect method of laying down units of length end-to-end" and the numerals on the ruler indicate the number of unit lengths used (Thompson & Van de Walle, 1985, p. 8), students, especially low-achieving students, may not perceive this structure. Thus, even such a common tool as a ruler, may be understood only in a limited way.

This distinction between counting discrete objects and subdividing a continuous property into units is crucial to understanding the importance of 'zero' in its role as an indicator of no length. The importance of 'zero' in the construction of measurement instruments, such as rulers, tape measures, and student-constructed tools should be made clear to students (Thompson & Van de Walle, 1985). Ward's work (1979) with ten-year-old students on the construction of graphs has indicated the importance of the fundamental properties of scales. He also emphasised that students need to understand the point of origin and the iteration of measurement units. Familiarity with the process of 'renaming zero' is an important skill in flexible use of scales, as solutions can then be obtained by counting units or by subtraction. Such skills are important when a length to be measured cannot be aligned with zero, or when changes in measurements must be considered.

The aim of this paper is to investigate the performance of students in Years 1 to 5 on a variety of length measurement tasks using a common measuring tool, a ruler. The students were chosen by teachers to represent the range of mathematical ability in the different classes. The results of a subset of ruler and scale questions are presented; the items are part of a larger study of the development of children's understanding of linear measurement.

Methodology

The design of the study was cross-sectional; 120 students from Years 1-5 (aged 6 to 10 years) were selected from four state primary schools in a medium to low socio-economic area of Sydney. Each class teacher selected six students: one girl and one boy considered 'above average', 'average', and 'below average' in terms of mathematical achievement.

Table 1
The set of Tasks Involving Ruler Use

Task	Description
1	Draw a line 7 cm long above a ruler printed on the page (0 is aligned with the end of the ruler).
2	Draw a 9 cm line using a wooden ruler (0 is not aligned with the end of the ruler).
3	Measure a 6 cm line. (0 is not aligned with the end of the clear plastic ruler).
4	Measure a line that is longer than the ruler—Years 1 and 2 measured a 16cm line, Years 3, 4 and 5 measured a 26cm line—using the same ruler as for Task 3.
	The ruler used in Tasks 5 to 7 was a clear plastic strip on which a centimetre scale had been marked. Different modifications were made to this "ruler" in each task.
5	Measure a 9cm line with a "ruler" that had been cut at the 3.5cm mark. The scale was marked from 4 to 20cm.
6	Measure an 11cm line using a "ruler" showing centimetre marks without a corresponding numeric scale.
7	Draw a 6cm line above a printed "ruler". The marked scale, beginning with 6 and ending with 13 only comprises part of the line segment. A 3.5cm unmarked line extends to the left of the 6 and a similar line extends to the right of the 13.
8	Count five sea horses shown on a card and state what the '5' represents. Then explain what '5' on the ruler represents and identify a single unit.
9	Use 2 paper clips to measure a 28cm line; note there is a fraction of a unit remaining.
10	Make a ruler using paper clips as the unit of measure (given a long rectangular strip of cardboard and a supply of paper clips).

The subset of questions (see Table 1) was designed to investigate knowledge of rulers. The students were given four common classroom tasks (Tasks 1-4) involving length measurement (the technique of using a ruler), as well as four tasks (Tasks 5-8) that required the students to apply their knowledge of rulers in unfamiliar contexts (the understanding of using a ruler). Two additional tasks (Tasks 9-10) were given; these assessed students' use of informal units—in this case, paper clips—as a measuring tool.

The first researcher interviewed individual students towards the end of the school year (September-November) when they had completed a large part of the measurement program for the year. The interview tasks were designed to elicit information about the students' understanding of length measurement. Practical tasks using wooden or plastic rulers were used because the effectiveness of paper-and-pencil items for the assessment of mathematics knowledge has been questioned (Clements & Ellerton, 1995).

For all tasks except Task 2, clear plastic rulers marked in centimetres were used; these were made from overhead transparency film. For Task 2 a wooden ruler was used; on this ruler the zero mark was not aligned with the end of the ruler. The tasks have been grouped to emphasise their conceptual similarities. They were presented in the same order to all students but not in order given in Table 1 (the paper clip items were presented earlier) and the ten tasks were separated by other tasks not listed.

Results

Figure 1 shows the results for the first four tasks, those involving the technique of using a ruler. In general, all groups were successful on Tasks 1-3 by Year 5. The high-and middle-ability groups were successful from Year 3 on but few low-ability students were successful before Year 5. Students in the low-ability group made ruler-placement errors more often than the other groups, and these errors persisted into Years 4 and 5. Older low-ability students

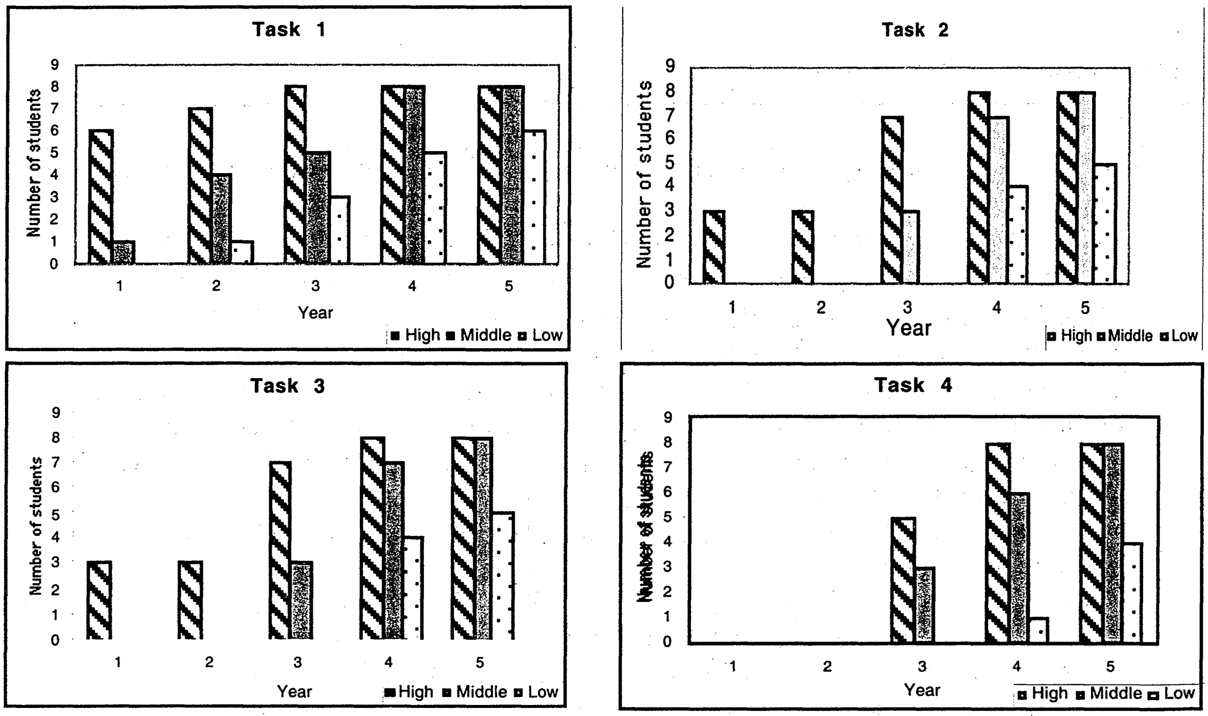

Figure 1. The results for Tasks 1-4—the technique of using a ruler.

were more likely to express concern or confusion over where to start measuring, at zero, at one or at the end of the ruler. Even in Year 5 some of these students could not measure a line that was longer than the ruler. Faced with Task 2, Erin (Year 5), when asked why she started to measure from zero replied, "Oh that's easy. You start at zero because the line hasn't made a centimetre yet, the line hasn't started." Soula (Year 4) demonstrated by drawing a line, marking zero, and moving along the line to where she said 'one' was. High-ability students were able to refer to movement along a line; e.g., "… you go along it to get to 1 …"; or to use a physical movement; for example, finger or pencil. These explanations indicate that these students have acquired an understanding of the underlying concept of the linear nature of length measurement.

The children were asked whether they talked about the length of objects that they measured in their lessons. Peter from the top group in Year 5 replied: " No, we just used to count stuff, you know, …like shoes and our hands, and them paddle pop sticks. I used to get things wrong 'cause I used to start at 1 like we counted. But I worked it out." Peter demonstrated how he had worked it out by using a ruler: "See? It's like you start at one, you get to two, but that's not two 'cause it's just one."

Figure 2 shows the results for Tasks 5-8—the understanding of using a ruler. Although Task 5 (the 'broken ruler') and Task 6 (the 'empty ruler') have appeared in research studies and basic-skills tests, there are very few similar examples in student textbooks. These tasks require students to have an understanding of how rulers are constructed and to apply their knowledge of what constitutes a unit.

All high-ability students in Years 4 and 5 successfully completed Tasks 5 and 6 but only about half the middle-ability group and almost none of the low-ability group were successful. Thus, for the previous set of tasks (1 to 4) the performance of middle-ability students was similar to the top group in Years 4 and 5. However, their performance on these tasks seems to indicate that they have not acquired an understanding of a unit scale. The distribution of results for Task 7 was similar to those for Tasks 5 and 6 but with a slightly higher difficulty level. However, Task 8, "the sea-horses" showed a very different pattern of results. This task indicates that it is not until Year 5 that high- and middle–ability students can explain the measurement unit. Only one low-ability student in Year 5 could point to the centimetre units on the ruler.

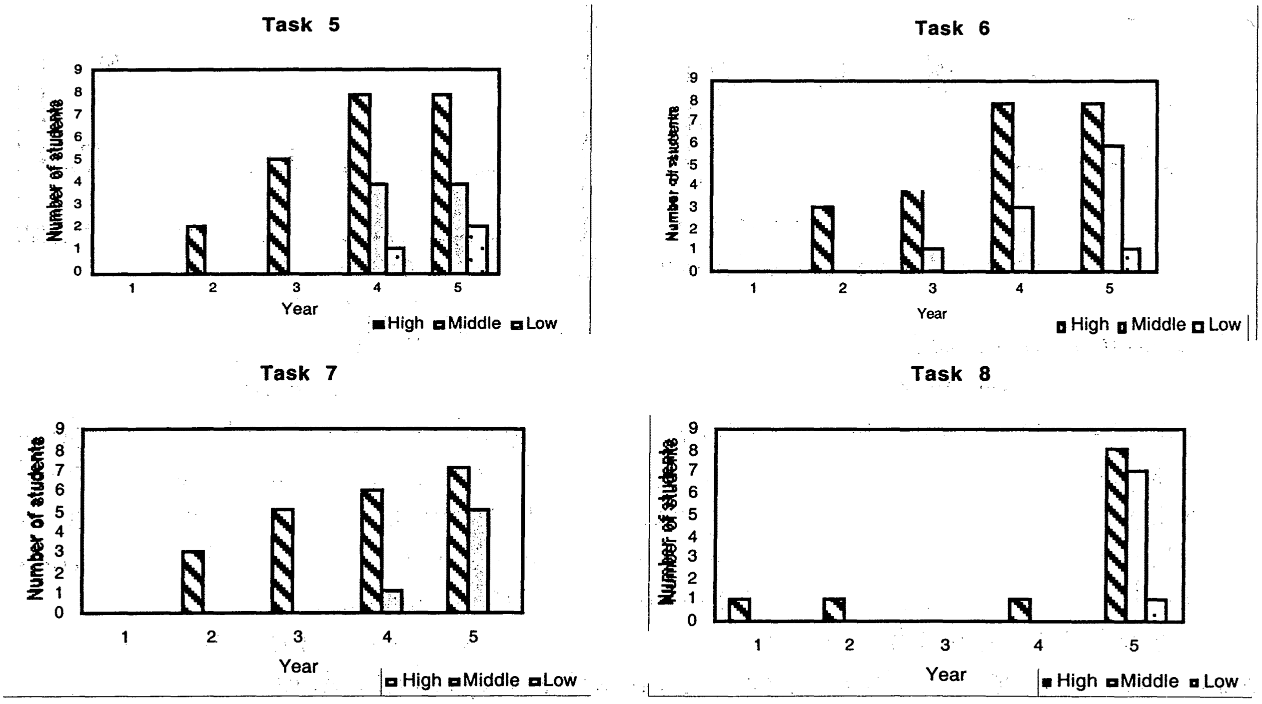

Figure 2. The results for Tasks 5-8—the understanding of using a ruler.

A common feature of students' explanations was a lack of insight into the nature of the units used to measure length. A number of students referred to procedures they had learnt and were confused about where to begin measuring in unfamiliar situations. For example, Jenny (Year 5) said that she started from zero "...because that's how you get the answer." William (Year 4), said, "You start at nought because Mrs. K. said to start there."

Asking children what the numbers refer to on a ruler (Task 8) was quite revealing; many students said that rulers "tell where lines end". Overall, 19 of the 72 students in Years 3 to 5 (26%) indicated that the "5" referred to 5 centimetres and showed a line 1 centimetre long on the ruler. The remaining students said that the "5" referred to the "5" on the ruler, or the unit mark the "5" was next to. Many of the latter students (54%), when prompted to show the "5" centimetres counted the unit markers, some unable to resolve the fact that they had counted six unit markers. Twelve students (17%) counted the spaces, recording what they had counted by colouring the spaces between the unit marks. This evidence seems to have important implications for both the teaching and testing of student understanding.

Measuring length with informal units could be considered a common task for younger students. However, if there are insufficient units, in this case paper clips, to align end-to-end along a line, most students cannot successfully measure the length of the line. The reason appears to be that students do not understand the structure and precision of unit iteration. For Task 9, measuring a line given two paper clips, high-ability students were increasingly successful from Year 2 to Year 5 because they measured accurately. Almost half (14) of the 33 successful students started to measure by using the two clips as the measuring unit. All but two of them switched to an 'alternating' strategy where they would mark and move one paper clip at a time. Comments were "It's easier this way"; "You can count them more easy this way"; and "...you don't get spaces this way." Middle-ability Year 5 students did this task well but only one low-ability student was successful.

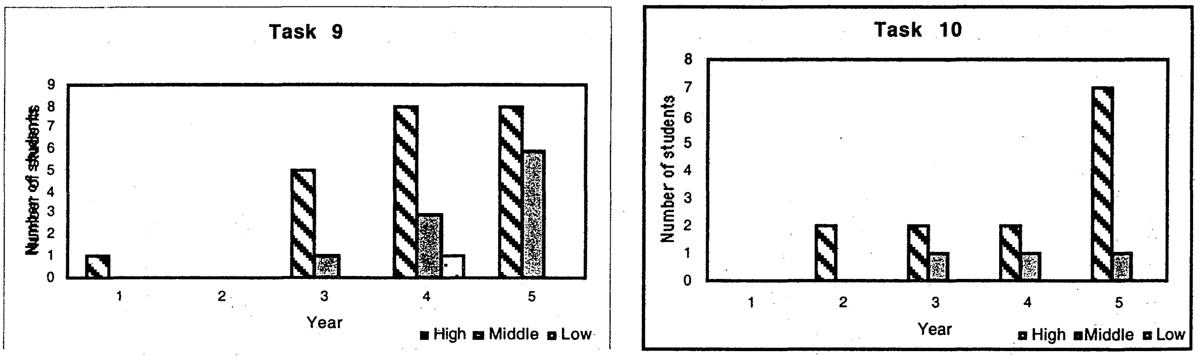

Figure 3. The results for Tasks 9 and 10—the use of informal units.

The responses of older low-ability students were more like those of the younger students. They were more likely to ask for more clips, showed less interest in accuracy, and were unaware of or unable to name the fractional part. In spite of the emphasis placed on informal units in the early years of school, a large proportion of students seem not to have learnt the concepts the use of informal units was designed to teach. More students were proficient with a ruler than with informal units, although two factors made the informal task difficult: provision of a limited number of units and inclusion of a fractional unit.

The distribution of results for the final task, Task 10, construction of a paper clip ruler, was quite similar to that for Task 8 (the "sea-horses"). Only about half of the students (49%) in Years 3 to 5 were able to make a ruler where the unit used was the length of a paper clip.

None of the lower ability students could construct such a ruler. The latter seemed unaware that the length of the paper clip was to be used; instead, they created units of arbitrary length using the paper clips as markers. The successful students were all aware of the need to mark off the length of the paper clips and paid attention to the detail of the ruler's scale; for example, "That's because you have to make them as long as the paper clip". Students who created arbitrary units, although aware of the features of a ruler, were unable to maintain the length of the informal unit.

Discussion and Implications

When the three sets of results (Figures 1, 2, and 3) are compared, there appears to be far greater change for all three groups on the ruler-technique and informal-units tasks. For the four tasks involving understanding a ruler there is a substantial change from Year 1 to Year 5 for the high-ability students, a more moderate change for the middle-ability students and almost no change for the lowest group. More improvement would be expected because only about half the Year 5 students gave correct responses on these tasks. The results are consistent with an hypothesis that teachers focus on the techniques for using a ruler rather than on understanding how a ruler is constructed.

The results clearly show that most students were able to measure and rule a line by Year 5, but they were less successful on tasks that required an understanding of linear units, such as Tasks 5-7 where the numeric scale did not start at one or was not given. In these more complex tasks many students reverted to counting from one or counting unit markers. Few students could explain the units on a ruler or construct a measuring tool with paper clips (Tasks 8 and 10). They seemed unaware of how to construct a ruler using the 'length' of the paper clips as the unit, even though many of them were able to find an object's length of by counting how many paper clips fitted along it.

It became clear that the common components of linear measurement—in this case *length* as represented by a line and *linear sub-units* of that line had not been made explicit when informal units had been introduced. The outstanding feature of the responses of the better students was that they seem to have extracted this information for themselves.

Conclusion

These results from this study show that in spite of the ease with which most students measured with rulers and counted informal units, many students did not understand the relationships between linear units and a formal scale. This is particularly evident with the lower students who take longer to acquire basic technical skills and who are unlikely to have developed an understanding of concepts they need for more complex measurement. The main errors made in the tasks involving ruler techniques concerned alignment. Such errors might arise from the emphasis on counting from one or, as found by Gravemeijer, McClain and Stephan (1998), confusion arising from methods of counting informal units such as paces.

Students used two main measurement strategies in this study: (a) counting informal units, unit marks, or unit spaces; or (b) aligning the ruler and reading the scale. Correct use of either strategy did not indicate that students understood linear measurement in the more complex tasks. An explanation for students' reliance on procedures might be the way that measurement is often taught. Worksheets and textbook exercises frequently involve counting informal units or the techniques of ruler use. Such an emphasis on techniques does not develop fundamental measurement concepts of unit size and the structure of the unit iteration. To develop such knowledge would seem to require practical experiences of measuring and marking units; followed by discussion about key aspects of such student-constructed scales. If students do not understand how scales are constructed, they will not have the basic knowledge to relate measurement of length and number lines, nor have the foundation to develop area, volume and other higher order mathematical applications.

While the majority of older students could use informal units to 'measure' a length, the process of counting them appears to have resulted in different interpretations from those expected by teachers. The emphasis on counting may obscure the *linear nature of the unit of measure* if it is not made explicit when informal units are introduced. A possible explanation would be that students attend to the action of placing or counting discrete units rather than their lengths. According to Wilson and Osborne (1988) neither zero nor the iteration of line segments can be made explicit when informal units themselves are counted, thus reducing the possibility that students are able to make the important link with the underlying linear unit concept (Hiebert, 1989).

In 1984 Hiebert pointed out that "...many elementary school students are quite proficient in a variety of standard measuring skills but lack an understanding of some of the basic concepts involved." (p. 19). In 1990 Webb and Briars wrote that teachers should know firstly that students are able to reliably and efficiently apply procedures, but more importantly they should know "what a student knows about the concepts that underlie a procedure" (p. 11). This research suggests that the situation is similar in Australia; few students interviewed could reliably demonstrate a deeper understanding of the concepts that underpin linear measurement. These findings have significant implications for teaching about the relationship between the construction and representation of informal and formal linear units.

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