Students' Technical Difficulties in Operating a Graphics Calculator[1]

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We report on how students deal with some technical aspects of the operation of a graphics calculator. Clinical interviews were conducted with 25 Year 10-11 students as they used graphics calculators to study graphs of straight lines and parabolas. Three common student difficulties were identified: a tendency to be unduly influenced by the jagged appearance of graphs; a poor understanding of the zoom operation of the graphics calculator; and a limited grasp of the processes used by the calculator to display graphs. Implications for teaching are discussed.

Since graphics calculators (GCs) were first developed in the mid 1980's they have become steadily cheaper, more user-friendly, and more powerful. As a result, they are being increasingly used in mathematics teaching. Although there is considerable research on the use of GCs in the classroom, the majority of these studies simply describe the use of graphics calculators in teaching experiments without differentiating between the role of the GC and the context in which it is used (Dunham & Dick, 1994). In particular, research has yet to identify those aspects of GC use which best facilitate student learning (Penglase & Arnold, 1996). The research reported here is an attempt to rectify these shortcomings.

Background

In precalculus mathematics, a great deal of attention is paid to graphs of linear and quadratic functions. Students learn to recognise the important features of such graphs and link the symbolic form to their shape (line, parabola) and other details (intercepts, gradients of lines, and the direction, axis and vertex of a parabola). They do this by drawing and studying a number of linear and quadratic graphs.

When drawing graphs by hand, examples need to be carefully chosen to minimise complications. Thus, teachers and textbooks usually specify symmetric scales and choose functions whose important features lie near the origin. Learning takes place gradually and higher levels of sophistication are developed slowly over time. In the familiar paper and pencil environment, students are able to exercise direct control over the size and scaling of the graphs which they draw.

A number of papers have been written on how to use GCs in precalculus mathematics (e.g., Asp, Dowsey, & Stacey, 1993). Many writers suggest that a major advantage of the GC is that it allows more freedom to explore (Demana, Schoen, & Waits, 1993). However, such exploration inevitably means that students have to deal with unsymmetrical scales, blank screens or partial views, and non-integer coordinates. Because students are not plotting individual points, and there are no labeled axes or even grid lines, the whole process might appear magical (Dion & Fetta, 1993) and there is clearly a danger that fundamental misconceptions might arise (Mueller & Foster, 1999).

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At the same time, students also have to deal with the non-trivial task of learning how to operate the GC itself. Giamati (1991) studied the use of graphics calculators on students' understanding of transformations of graphs and found that some students were adversely affected by also having to learn how to use the GC. She noted that the students' limited experience in using the GC and their lack of familiarity with its basic operations may have accounted for their lack of progress. Goldenberg (1988) identified several student difficulties related to the GC, including problems with the use of pixels as representations of points and misconceptions associated with scale changes when zooming.

Two further studies are relevant: Williams (1993) remarked on a number of difficulties students have when points of discontinuity are not visible due to the low resolution of the GC screen; while Vonder Embse and Engebretsen (1996) noted the confusion caused by the awkward-looking coordinates which often appear when tracing graphs on a GC. They demonstrated the advantages of being able to create so-called "friendly windows", in which each pixel has an x-coordinate with a small number of decimal places.

The present study was designed to investigate such technical difficulties, specifically those related to (a) pixels, (b) zooming, and (c) the determination of pixel x-coordinates. It is part of a larger study in progress. A report on students' conceptual difficulties is available elsewhere (Cavanagh & Mitchelmore, 2000).

Method

Sample

Clinical interviews were conducted with 25 students, 5 students from each of 5 Sydney metropolitan high schools (15 Year 10 students: 8 girls and 7 boys; and 10 Year 11 students: 5 girls and 5 boys). Students were drawn from higher ability mathematics classes because it was felt that they would be better able to respond to the challenge of the interview tasks and to articulate their ideas clearly. It was assumed that the difficulties which these students demonstrated would also be found in students of lower ability.

All the students who were interviewed had studied the graphs of straight lines and knew the gradient-intercept equation $y = mx + b$. They were able to sketch the graphs of parabolas given in general form and were familiar with the quadratic formula. They could use this formula to solve quadratic equations and locate the zeros of a parabolic graph.

The students had used a GC, the Casio *fx*-7400G, in mathematics lessons for between 6 and 12 months prior to the first interview. In one school, students owned their own graphic calculators and were able to use them in all lessons and examinations. However, the majority of students had limited access to a class set of GCs owned by the school. Broadly speaking, the students were novice users who had very little experience with GCs and had only used them to display graphs of linear and quadratic functions.

The Interviews

Each student was interviewed individually by the first author for fifty minutes on three separate occasions, each approximately two weeks apart. The students completed a variety of graphing tasks using a Casio *fx*-7400G and were asked to interpret the calculator's output. All of the interviews were videotaped and selected segments were later transcribed so that a more detailed analysis of the students' responses could be made.

The Tasks

The tasks which are relevant to this paper were as follows.

1. Use the graphics calculator to find the intersection of $y = 2x - 1.5$ and $y = 3x + 0.8$.

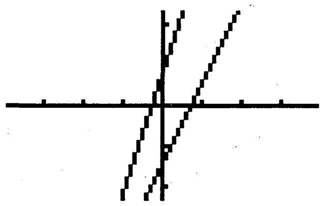


Figure 1. Graphs of $y = 2x - 1.5$ and $y = 3x + 0.8$.

Figure 1 shows the two functions as they are displayed in the initial viewing window. (On the *fx*-7400G, this window is friendly with tick marks at intervals of 1 and pixels at intervals of 0.1.) The two lines have a similar slope and appear to meld together over a number of pixels near their intersection point. Task 1 investigated how students might interpret this apparent contradiction.

2. Display the graph of $y = 0.75x^2 - 1.455x - 1$ on the graphics calculator. Find the intercept with the positive x-axis and the coordinates of the vertex of this parabola.

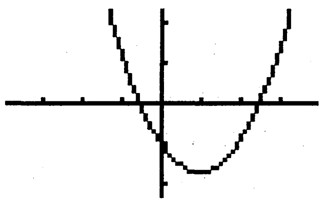


Figure 2. The graph of $y = 0.75x^2 - 1.455x - 1$.

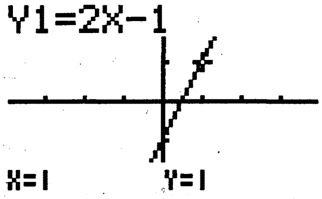
Figure 2 shows the parabola in the initial window. The task was designed to investigate two questions: Firstly, how do students interpret the coordinates displayed on the GC screen when the x-intercept is irrational? Secondly, how do they deal with the situation where the lowest point of a graph is represented by a line of pixels rather than a single one?

3. Display the graph of $y = 2x - 1$ on each graphics calculator. Move the trace cursor to the points (0, -1) and (1, 1). What do you notice? Can you explain what has happened?

For this task, the student was given two calculators—one set to the usual initial window and one set to a window in which both the x- and y-axes were displayed from -10 to 10. (The students were shown these settings before they responded to the task.) On both calculators, it was possible to move the cursor to the point (0, -1). However, on the second calculator, it was not possible to move the cursor until the coordinates (1, 1) appeared on the screen. Because the window was not friendly, almost all coordinates appeared with 4 decimal places. Figure 3 shows the two calculator screens with the cursor on the pixel nearest to (1, 1).

Task 3 was designed to investigate whether the students were aware of the procedure used by the GC to assign x-coordinates to pixels. Knowledge of this procedure is essential if students are to understand how "friendly windows" might be created.

(a)



(b)

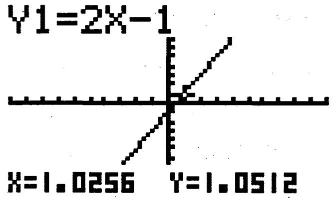


Figure 3. The graph of $y = 2x - 1$ in (a) the initial window, (b) a 10 by 10 window.

Results

Task 1. All 25 students were able to zoom out or scroll the window down until the region near the intersection of the two lines was displayed on the screen. However, only 7 students (28%) recognised that the low resolution of the screen and the fact that the two lines had very similar slopes was the reason why their images melded together over a number of pixels. The remaining 18 students (72%) expressed surprise at the image they saw and explained that they were expecting to see a distinct, easily identifiable pixel at the point of intersection. When this single pixel did not appear even after they had zoomed in, 5 students (20%) explained that the pair of lines briefly ran alongside each other and had more than a single intersection point. Of the others, 6 students (24%) continued zooming and tracing until they obtained coordinates which they felt were "nice" (integer or near-integer values), while 7 (28%) simply chose a point close to the centre of the group of pixels where the lines appeared to merge.

Task 2. All of the students zoomed in to find the intercept and vertex more accurately. They all expressed the belief that the relevant point could be found exactly provided one was prepared to zoom in a sufficient number of times. All of the students also expected that the horizontal line at the base of the parabola would become shorter each time they zoomed in until eventually one pixel would be seen at the vertex. Only 3 students (12%) correctly stated that the scale of the graph they saw displayed on the GC screen had changed after they had zoomed. The other students spoke of zooming as a distinctly different operation from any rescaling the graph.

Task 3. Explaining why it was not possible to locate (1, 1) on the second calculator proved to be an extremely difficult task. In fact, only 2 students (8%) gave a satisfactory explanation which included a reference to the relationship between the number of pixels across the screen and the window settings. Nine students (36%) were unable to answer at all, and the remaining students often gave the impression that they made a particular response because they could think of little else. For instance, 6 students (24%) regarded the unequal scaling of the axes as the source of the problem; 3 students (12%) thought that the problem was due to the linear function itself, and that if another line which contained the point (1,1) was graphed then it would be found; 3 students (12%) said that there was a pixel assigned to (1,1) but the low resolution of the screen meant that it could not be seen until they had zoomed in a number of times; and 2 students (8%) thought that the settings of both coordinate axes caused the problem rather than relating it to the x-axis alone.

Discussion

Students' responses to the tasks suggest three key areas of technical difficulty in using graphics calculators.

The Low Resolution of the Screen

Throughout the interviews, the students regularly commented on the jagged appearance of the graphs they saw on the GC and demonstrated that they were aware of the pixel approximations associated with the low resolution of the screen. For example, most students were able to explain why the calculator displayed the parabola shown in Figure 3 with a horizontal line of pixels near its vertex. However, even though they recognised the inconsistencies which arose from time to time between the position of the cursor and the coordinates, the students were still likely to base their answers exclusively on the visual representations of the highlighted pixels. Furthermore, despite the fact that the students claimed to have more confidence in the coordinates displayed on the screen, they needed constant reminders to consider these values before deciding on an answer.

A related issue concerns the students' ideas about the nature of points. In coordinate geometry, the concept of a point is essentially one of location or position on the number plane. It does not make sense, mathematically speaking, to regard a point as having other properties like size or shape. However, many of the students did attribute such characteristics to points on the GC screen. Goldenberg and Kliman (1988) commented on the tendency of students to speak about the size of points and described this misunderstanding as a "bead necklace metaphor" which "allows ideas of scale to be applied to points ... allows them to be conceptually magnified, lined up in a row, and so on" (p. 5). We noted similar responses from the students whom we interviewed.

One unexpected but positive aspect of the low screen resolution is the fact that students were able to contrast the patterns of pixels in the graphs of linear and quadratic functions to make inferences about their gradients. The majority of students concluded that, whereas the slope of a line remains constant, the slope of a parabola changes as you move along the curve and increases as you get further away from the vertex. The relatively small number of pixels on the GC screen might therefore provide a useful visual support for students in the early development of differential calculus concepts.

Zooming

The students did not have a good understanding of the zoom operation of the GC. As Goldenberg and Kliman (1988) also found, there was a marked tendency among the students to disassociate the zoom operation from any change in the scale of the graphs they saw. Instead, the students most often described zooming in terms similar to the use of a magnifying glass on a physical object when, as one continues to zoom in, previously obscured details are gradually revealed.

Students generally failed to recognise scale changes. We found that they had a tendency to regard scale in *absolute* rather than *relative* terms. That is, the students interpreted scale as either the distance between the tick marks on the coordinate axes or their value, but they did not consider scale as the ratio of distance to value (see Cavanagh & Mitchelmore, 2000).

The students' poor understanding of the zoom operation was evidenced in other ways as well. For example, when zooming in to locate the intercept of the parabola in Task 2, many students expressed surprise when they saw the same coordinate values repeated over and over again. This encouraged them to think that there must be a limit to the number of times one could zoom in before the GC screen would eventually "run out of points". The students also showed little appreciation of when it might be appropriate or helpful to use the zoom facility of the GC and when it was not. They invariably attempted to solve any problem they encountered by zooming and persisted with this approach long after it should have been clear to them that it was unproductive.

The Procedures used by the GC to Display Graphs

Students seemed to have little understanding of how a GC produces a graph. The results of Task 3 show that very few students knew that the x-coordinates a GC assigns to the pixels depends on the specified range of x-values to be displayed. Responses to Task 1 and other tasks not reported here indicate that most students did not realise that the GC displays the calculated y-coordinate (i.e., the value calculated from the given function and the x-coordinate of the pixel) and not the y-coordinate of the pixel where the cursor is positioned. The students also often failed to recognise when the y-coordinate was rounded and tended to regard the coordinate values as exact. Other, informal observations we have made suggest that many students also do not know that (in the default setting for Cartesian graphs) the GC calculates and displays one pixel in each column of pixels and then "joins up" these pixels to display the graph.

Students also did not know how to change the spacing of the tick marks displayed on the axes of a graph using the *scl* parameter in the window setting. In tasks not reported here, all the students we interviewed assumed that the *scl* value was connected with the scale of the axes, causing considerable confusion.

Implications and Conclusions

Kissane, Bradley, and Kemp (1994) argue that students should develop the technical skills required to operate a GC effectively. While it may not be necessary for all students to have a highly developed technical understanding, our work suggests that an appreciation of some basic aspects would be helpful. These include an understanding of the zoom operation of the GC and the link between zooming and scale; the methods used by the GC to assign values to the columns of pixels; the calculated nature of the y-coordinates displayed when tracing; and the fact that these y-values are often decimal approximations rather than exact values.

The discussions we had with students during the interviews indicate that they can gain insight into the technical operation of the GC but that this does not come naturally to the majority of students. We agree with Dick (1992) that students would benefit greatly from confronting the limitations of the technology and attempting to explain them, and feel that teachers should not structure their lessons or choose examples to avoid the kinds of technical difficulties we have described. However, we clearly need to take care in how we challenge students' misconceptions.

Students may benefit from understanding how the increment in the adjacent columns of pixels across the screen is calculated. This knowledge might help them recognise why a particular coordinate value is not represented by a pixel on the screen, and assist them in creating their own friendly windows rather than being constrained by the small number of default window settings that are available on the GC (Dowsey & Tynan, 1997).

The links between changing the window settings, rescaling the axes, and zooming needs to be made clearer to students. Our work suggests that students regard these operations as distinctly different and do not understand the effects of zooming on the range and scale of the axes.

Misconceptions may lessen with greater exposure to GCs (Ruthven, 1990). We found that the students who owned their own GCs more frequently exhibited a critical awareness of the calculator's output and were less likely to be confused by its technical limitations.

Finally, one of the things which struck us most during the interviews is that we should never assume too much about what students perceive when they look at the screen. What may seem obvious to the mathematically experienced may not be equally apparent to novice learners. For instance, more than one student struggled to explain how the coordinate axes like those in Figure 3 (b) could both range from -10 to 10 and yet look so different; they had simply failed to notice the rectangular shape of the viewing window.

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