Improving Decimal Understanding: Can Targetted Resources Make a Difference?

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This study investigated whether minimal intervention with new teaching resources could make a difference to children's understanding of decimal numeration. Four teachers were supplied with resources designed specifically to address the common misconceptions children have about decimal numbers, and asked to keep a record of their teaching. Use of the resources was unexpectedly low, but teachers who used them achieved an encouraging improvement in decimal understanding, measured against previous performance of the school over some years, indicating that a small amount of deliberate attention to decimal concepts can make a difference.

The Incidence and Persistence of Difficulties with Decimal Numeration

It is well documented that many students throughout schooling, and indeed many adults, have difficulties with decimals. A central problem (and possible underlying cause of many other difficulties) is that many people lack understanding of decimal numeration. This is a long-standing and international problem. For example, Grossman (1983) reported that, even though over 50% of U.S. students entering tertiary education could add, subtract or multiply decimals, less than 30% could select the smallest decimal from five options.

Recent Australian data from the *Learning Decimals Project* (Steinle & Stacey, 1998) confirm that Australian students have similar difficulties and that misconceptions are widespread and persistent. Using a carefully designed Decimal Comparison Test, they found that the proportion of students able to compare decimals reliably is relatively stable from year 6 (about half) to year 10 (less than two thirds), suggesting that normal teaching is making little difference to the way students think about decimals. The test enables most students' responses to be classified according to the way they are thinking about decimal numeration. Longitudinal data indicates that about half of the students who have a decimal misconception have the same misconception a year later. (Stacey & Steinle, 1999b).

The data above demonstrate that understanding decimal numeration is a complex task. Students have to coordinate their ideas of place value learned in the whole number setting with their understandings of fractions. For example, to compare 0.3 and 0.34, a student who knows that the first is 3 tenths and the second is 34 hundredths has to be able to mentally coordinate the effect of the varying numerators (3 and 34) and the varying denominators (tenths and hundredths). There are other ways of comparing these two numbers, for example by using knowledge of equivalent fractions so that 0.3 can be read as 0.30 and therefore as 30 hundredths, making the comparison easier. Another way is to think of 0.34 as 3 tenths + 4 hundredths. However, all of these methods require coordination of place value knowledge and fraction knowledge.

Some schools and teachers seem to teach decimal numeration well and others not so well. Steinle and Stacey (1998) found that although 52% of grade 6 students were apparent experts, the results by school varied from 0% to 82%. This variability is only partly explained by socioeconomic factors. For this reason, it seems likely that attention to the meaning of decimal numbers of varying length in teaching is a major factor in determining success. It is likely that, given just a small impetus, the understanding of children in this topic could be markedly improved. This study set out to investigate whether this is the case. Could teachers, given minimal support, change their teaching so that most children develop a good idea of decimal numeration over a relatively short time?

Resources for Learning about Decimals

The *Learning Decimals Project* has developed a collection of classroom resources for teaching about decimal numeration, including activities to address common misconceptions. The resources include classroom activities using simple equipment such as cards and the chalkboard, computer games, and a concrete model of decimals which we call Linear Arithmetic Blocks (LAB) (see Figures 1 and 2).

This model was first shown to us by Heather McCarthy, a local teacher, who saw it at an in-service training session given by a person whose name we do not know. LAB is similar to Dienes' Multibase Arithmetic Blocks (MAB) in that it can be used as a hands-on model of decimal numbers. Units, tenths, hundredths and thousandths are represented by hollow tubes of decreasing, and proportionally accurate, length. The unit piece we use is just over one metre long, so that thousandths are just over a millimetre long.

LAB pieces can be arranged in two ways to represent a number, either end-to-end or on what we call an organiser. When placed end-to-end the pieces form a linear representation of decimal numbers, as in Figure 1. This facilitates direct comparison of decimal numbers. The second possibility is to place the pieces on the organiser, as in Figure 2. The organiser consists of upright rods attached to a wooden base, which provide a concrete model for the place value columns. The height of the rods is such that only nine of the relevant pieces can be placed on it, providing an in-built constraint on column overflow. Thus if there are ten or more pieces of any one size, they will not fit on the appropriate rod of the organiser and so ten of them must be exchanged for a single component of the next highest place value.

In this study, we followed the advice of Judah Schwartz who observed that "in the case of education reform, there need to be new curricular artifacts that allow the users to mark the newness of their undertaking" (1994, p. 4). LAB, something that teachers and children had not seen before, served as the marker of this new undertaking.

A number of arguments can be put forward in favour of LAB over MAB as a tool for supporting the learning of decimal concepts.

1. The values of MAB pieces used for decimals are often confused with whole number values that they previously represented. Since LAB is probably new to teachers and children, this confusion is unlikely.
2. LAB is a simpler model than MAB. The underlying representation of the size of number by MAB is by volume (or mass if the density of material used is constant). One number is larger than another if the volume of the pieces assembled to represent it is larger. LAB, however, represents numbers by the length of the pieces assembled, which in view of children's difficulties with volume concepts (Battista & Clements, 1996) may be more developmentally appropriate. MAB material is also complicated by what users may perceive as a switch from the 3 dimensional block, to the 2 dimensional flat to the 1 dimensional long to the 0 dimensional unit.
3. Third, as noted above, no more than 9 pieces of any one denomination fit on a rod of the organiser, forcing trading ten of one unit for one of the next.
4. Fourth, because LAB has structural similarity to the number line, it has important advantages in being able to demonstrate the density property of decimals (the property that between any two decimal numbers, there is another). It also provides an excellent concrete basis for rounding decimals.

5. Two small, unpublished student studies (Archer, 1999; Condon, 1999) reported several advantages of LAB over MAB in practice. Analysis of lesson transcripts revealed significantly more discussion, conjecture and explanation by groups of children using LAB. Some children reported being confused by MAB but there were no such reports for LAB. Children were also more likely to visualise using LAB than MAB when a model was not available. These encouraging results suggest that LAB warrant further investigation.

Figure 1 (above). LAB pieces laid linearly to illustrate and compare the numbers 0.2, 0.27 and 0.3

Figure 2 (left). Placing pieces on the organiser to illustrate place value and the base 10 structure of decimals

Method

Six primary schools had participated in the longitudinal study of the *Learning Decimals Project* (Stacey and Steinle, 1999a, 1999b) which had three years of data showing the numbers of students who were apparently expert at understanding decimals and the numbers who held misconceptions of various types about decimal numeration. Two of the primary schools had high percentages of apparent experts and so were not suitable for targetted instruction. After eliminating a distant school, we approached the three remaining schools and invited them to participate. One school, through 'M' their Grade 6 coordinator, willingly agreed to be involved.

Two members of the research team visited the school and spent about one hour with the four grade six teachers (M, S, X and Z) who all agreed to participate. M, S and X taught grade six, and Z taught a combined grade five and six class. Each teacher received two booklets and a set of LAB that was demonstrated. One booklet contained lesson plans for using LAB

(Archer & Condon, 1999) and the other contained general lesson ideas for teaching about decimals (Condon & Archer 1999). (Some of the contents are described below.) Teachers were asked to use as many activities as they wished, but record their experiences of five activities from each booklet on simple feedback sheets supplied by the project team. They were asked to report on how they used LAB, and given space to record their observations of children, as well as general comments. They were also asked to provide a written account of their thoughts on teaching decimals during the time of the intervention. Teachers were asked to send back their responses at the end of the following school term (about three months later).

The teachers offered to test their students prior to the intervention, to obtain more accurate data on their students' misconceptions to supplement the longitudinal data already in the *Learning Decimals* database. Ninety-eight children completed the Decimal Comparison Test, a one-page 5-minute test which presents 30 pairs of decimals and asks students to nominate which of each pair is the larger. (Refer to Steinle & Stacey, 1998 for details). The test results were returned to the teachers, with a description of what the test indicated about each child's thinking. Early in term 4, the school was contacted again and a member of the research team visited the school to finalise the post-testing and meet with teachers again to discuss their participation in the project. Ninety-six children were tested on the second occasion, making 87 children who were tested twice.

Results and Discussion

Although all four teachers had agreed to participate, only M (the Grade 6 coordinator who had volunteered the school for the study) trialled the activities and provided a record of what she did. She commented that other curriculum priorities prevented her and the other teachers from completing the task. Thus her reports were limited to a record of three activities using the LAB model, and four activities from the Lesson Ideas Booklet.

At the meeting, teacher S explained that LAB had been used at least once. Class Y had not used them at all. She was on leave at the time of the first meeting and there was no record of her temporary replacement X, who had attended the training session, having used the materials. Despite repeated requests for information after the end of the study, no record ever became available of use of the activities by Z and it seems reasonable to assume there was no use in this class either. Thus we loosely defined three categories of use: Most use (Class M), Some use (Class S) and No use (Class X/Y and Class Z).

Pre- and Post-test Results

A total of 87 children from four different classes were tested twice and the data below is only for these students. All were grade six classes except for Class Z, which was a combined grade 5 and 6 class. Data collected earlier by the *Learning Decimals Project* over a period of three years (1994-1996) indicated that, for this school, 41% of grade 6 students were apparently expert in decimal notation. In the cohort of 1999, 46 children (53%) tested as apparent experts, a slightly higher proportion than previously.

Table 1 shows the results of the Decimal Comparison Test before and after the intervention for each of the four classes. Improvement occurred for the classes that used the resources, particularly the improvement from 33% to 63% expert for Class M. No improvement was evident for the two teachers for whom there was no record of any intervention. This very clean result indicates that deliberate attention to this topic is likely to make a substantial difference to students' understanding at Grade 6, which will in turn put them in a much better position to understand secondary school mathematics.

Table 1
Number and percentage of experts by class on pre- and post-Decimal Comparison Tests

	Class M		Class S		Class X/Y		Class Z		Total	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Number of students	24	24	23	23	20	20	20	20	87	87
Number of experts	8	15	19	23	10	10	9	9	46	57
Percentage of experts	33%	63%	83%	100%	50%	50%	45%	45%	53%	66%
Percentage gain	30%		17%		0%		0%		13%	

In an attempt to identify any trends in how students' knowledge changed, the data for classes M and S were combined. The Decimal Comparison Test classifies students according to the misconceptions in four major categories: the apparent experts; students who exhibit various of the longer-is-larger misconceptions; students who exhibit various of the shorter-is-larger misconceptions; and unclassified students who do not show any known pattern in their responses. Table 2 reports the changes in classification that occurred over the intervention period.

Table 2
Numbers of students by classification from pre- to post-test for classes M and S (N=47)

Pre-test classification	Post-test classification: Apparent experts	Longer-is-larger	Shorter-is-larger	Unclassified
App. experts	27	0	0	0
Longer-is-larger	2	0	0	1
Shorter-is-larger	1	0	0	2
Unclassified	8	3	0	3

All 27 students who were initially classified as apparent experts retested similarly, which suggests that once students reach expertise, they generally retain it. This finding is consistent with other studies using this classification scheme (e.g., Stacey & Steinle, 1999a). Over half of the non-expert students (55%) tested as apparent experts after the intervention period. This is an encouraging result given that Stacey and Steinle (1999a) found that over a period of about six months only about one third of non-expert students in a very large sample moved to expertise. Another encouraging result is that none of the students with misconceptions retained them, either moving to apparent expertise or into the unclassified category. Examination of individual test papers of the students who moved into the unclassified category revealed small improvements in understanding for all three.

Teacher M's use of LAB and the Lesson Plans

As discussed above, Teacher M was the only teacher who provided a written record of what she did with her class and made the most use of the resources. Since she achieved significant gains in performance, we examined more closely what she did.

LAB Activities. The teacher selected the initial three activities. These included familiarisation with the pieces and the organiser; creating decimal numbers with the pieces; placing LAB pieces on the organiser and writing down the number; and comparing decimals of different length. She commented that "the children reacted favourably to the material" and that "most children found this work to be very valuable for their understanding of decimals."

Lesson Ideas. The teacher selected the initial four activities (*Number trails, Decimal Skip Counting, Number between, Stickers game*), details of which can be found in the booklet. *Number Trails* and *Decimal Skip Counting* both involve starting with a particular number and adding or subtracting a constant number, preferably mentally. For example, a beginning class might start at 0.3 and successively add 0.1, producing the sequence 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and then erroneously 0.10. An advanced class might start with 0 and successively add 0.125, obtaining the sequence 0, 0.125, 0.25, 0.375, 0.5 etc. Both of these activities can be used to expose misconceptions, including difficulties with column overflow (eg. the number after 0.9 is 1 and not 0.10). *Number Between* starts with the teacher nominating two endpoints of a segment of the number line and children have to nominate any number between the endpoints. The nominated number and one of the previous endpoints become the endpoints of a new segment. As the activity continues, the number line is divided into smaller and smaller segments. Thus children gain an understanding of the value of numbers from their relative position on the number line and an appreciation of the density of decimal numbers. *Stickers* involves students arranging themselves in order from smallest to largest according to the decimal number they have been given on a sticker. Depending on the numbers chosen, the common misconceptions about the size of decimal numbers can be addressed by these activities, as well as issues such as the function of zeros in different positions in a numeral. The teacher reported that students participated enthusiastically in all activities. She noted the effectiveness of *Number Trails* for addressing column overflow problems, and commented that LAB was useful for helping children to visualise the numbers and understand, for example, why 0.9 plus 0.1 is 1.0 and not 0.10. She also noted that column overflow problems became less evident as children continued to play the game. She reported that children "learnt heaps" from *Number Between,* with children of all abilities participating enthusiastically.

It was evident from M's feedback that the resources were not always used in the way intended, nor to their full potential. This is not intended as a criticism; teachers need to use resources more than once in order to exploit their full potential. However it does point to the need for development, both of the resources (e.g., making instructions more specific; including more detailed explanations) and for teachers themselves (e.g., to increase their understanding of decimals and how to teach the topic effectively). An action-research model that involves teachers trialling and refining the activities and meeting on a regular basis to discuss their progress may be needed, but it would be much more labour intensive than the low intervention model trialled in this study.

Discussion with Teachers

The follow-up meeting and discussion was conducted at lunchtime (the only available time) and due to heavy commitments, only three teachers could attend (M, S and Y). The general impression was that of a school that expected a lot from the teachers, who had been very busy with extra work, such as orientation programs for secondary school.

The group asked to discuss the sorts of problems children have with decimals and the strategies they found effective in overcoming them. The problem first mentioned was column overflow (that 17 tenths, for example, is not 0.17) and M repeated what she wrote on her feedback sheet about the effectiveness of *Decimal Skip Counting* (see above). S mentioned that she did in fact make use of the LAB in this activity, and that it appeared to trigger conceptual change:

> Actually, it was good to show what happens [when there are more than nine of any denomination] because there's nowhere to go so you've got to take it to the left concretely. And I remember doing that at the time because some of them said, "Oh yeah!"

M's description of using the activity *Number Between* in the interval between 0 and 1 is also suggestive of conceptual change. She remembered it powerfully demonstrated that small decimals (i.e., zero point something) could never be less than zero:

> I just remember putting the number line up on the board and showing them where nought was, and one, and then getting them to halve. Remember one of the activities was getting them to write a fraction in between each time, and they were getting smaller and smaller and smaller and they suddenly realised they could never get smaller than nought.

This discussion brought up a related issue; that pressures of competing tasks and responsibilities interfere with good teaching:

S: You need to talk to them don't you, and say, well, why do you do this?
M: That was coming out with that maths conference we went to. They said that teachers are too busy giving work and looking at results.
M: What we need to do is talk to them
S: Talk to them.
M: —and get them to think about, and even getting them to write why they got a certain answer.

These pressures may partly explain why the teachers who initially agreed to participate failed to do so. As discussed earlier, the only teacher who made a significant effort to use the materials was the initial contact. She clearly felt an obligation to complete the task. The other teachers, faced with competing commitments and responsibilities, opted out. M reported that, having already taught decimals earlier in the year, they found it difficult to justify spending more time on the topic when they were expected to be covering other areas of the curriculum. An intervention in the period when the teacher plans to teach decimals may have increased participation.

Finally, it emerged in the discussion that teachers did not make use of the pre-test results which the project team supplied to learn about children's difficulties or target children with particular misconceptions. This was the case even though an explanatory sheet was sent along with the pre-test results, and the project team also invited them to ask for clarification. This emphasises the need for professional development to assist teachers identify both students' difficulties and conceptual change when it occurs. It is also consistent with a personal communication from Kathleen Hart, the director of the large CSMS project in Britain around 1980. Very large numbers of children were interviewed and good descriptions of children's thinking were carefully prepared for their teachers. Hart believes that there was no evidence that any teacher ever used any of this information.

Conclusions

This study investigated the impact of resources specifically designed to address decimal misconceptions. The intervention was minimal, in an attempt to model as closely as possible what happens in practice when new resources become available. Use of the resources was unexpectedly low, although this genuinely seemed to be due to other commitments rather than disappointment with the materials themselves. The teachers who used the resources with their children achieved an encouraging improvement in decimal understanding: over half of the non-experts tested as apparent experts after the intervention. Thus children can be given a strong foundation in decimal numeration but it requires deliberate attention from their teachers.[1]

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