

Informal Assessment Questions Used by Secondary School Mathematics Teachers

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Considerable emphasis is placed on teachers' informal assessment of students, yet little has been written about the assessment that teachers actually use in their classrooms and its effect. This paper presents a framework for analysing teachers' use of informal assessment questions. A sample of the teaching of two secondary school teachers is presented to demonstrate the usefulness of this framework. The analysis of these two lessons demonstrates the possibility of discriminating lessons based on teaching procedures from those emphasising more complex conceptual relationships. The potential value of the framework is discussed.

Background

There would be few dissenters to the view that assessment is a vital component of teaching and learning mathematics. However, defining what is implied by assessment would produce a wide range of ideas and opinions. One of the most common understandings of assessment is that suggested by Niss (1993, p. 3):

...assessment in mathematics education is taken to concern the judging of the mathematical capability, performance, and achievement – all three notions to be taken in their broadest sense – of students whether as individuals or in groups, with the notion of “student” ranging from Kindergarten pupils to Ph.D. students.

In spite of this wide ranging definition, some teachers still promote only summative assessment, often meaning paper-and-pencil tests or written examinations, while others also espouse the value of formative, informal or diagnostic assessment. In addition, there is now a growing level of agreement that assessment methods should go beyond formal written forms, whether formative or summative, and enter the informal domain (Schloemer, Cain, & Kenney, 1994). However, while there are a number of frameworks for formal assessment (e.g. *Bloom's Taxonomy* – Bloom, 1956; *SOLO Taxonomy* – Biggs & Collis, 1982 and *Authentic Assessment* – de Lange, 1995) there are few if any for informal classroom assessment. Informal assessment can be defined as all classroom procedures that are not followed by written records, which would include observation of students' work, discussion and question posing, either at the individual or group level.

These informal assessment procedures are common in most mathematics classrooms, with teachers making decisions about their students both during and after lessons. However, without a clear framework it is difficult to ascertain either the foundation or the effectiveness of such informal assessment. Research has shown that the classroom practice of teachers emphasises different assessment procedures (Dennisse, Sharon, & Charlene, 1997). The interactions between teachers and students in the classroom have become a

focus of teachers' professional development, along with teachers' content and pedagogical knowledge. All are increasingly becoming objects of research (e.g. Britt & Irwin, in press; Bromme, 1994; Chinnappan & Thomas, 1999).

The question arises as to what aspects of content and pedagogical knowledge are likely to be primary influences on teachers' informal assessment, and how one can analyse their influence. Skemp's (1985) idea that we construct 'reality' by engaging in mental building and testing of a schema is a useful way of thinking about our conceptual knowledge. Such schemas have been postulated to help us to categorise when problem solving (Sweller, 1992) and to play an important role in teaching, according to the model presented by Chinnappan and Thomas (1999). One of the roles of schemas is described as assisting teachers to discriminate between various mathematical representations of the same concepts, described by Kaput (1987) as involving a correspondence between aspects of two 'mathematical' worlds, the represented and the representing. On the basis of this teachers can build suitable conceptual learning experiences into their lessons rather than taking for granted switches between representations. Following mathematics across these changes of representation forms one of the four basic activities of mathematics described by Kaput (1992) as: syntactically constrained transformations within a notation system, with or without reference to any external meanings; translations between notation systems and co-ordination across these systems; construction and testing of mathematical models, which amounts to translation between aspects of situations and sets of notation; and consolidation or crystallisation of relationships and/or processes into conceptual objects or cognitive entities that can be used in relationships and/or processes at a higher level of organisation.

Teachers who are aware of these activities, and in particular view the keeping intact of concepts across translations between notation systems as a vital part of their classroom activity, should have pedagogical schemas (Shulman, 1986) which encourage informal assessment of student conceptual learning across the representations. Hence we were interested in developing a framework which would enable us to analyse whether teachers' informal assessment procedures (specifically, informal assessment questions) were based on pedagogical schemas. This paper presents this framework and analyses its application in the classrooms of two teachers.

Guiding Principles

In order to develop a framework for informal assessment, some of the issues of taxonomy and authentic assessment given above were seen as essential. Particular emphasis was given to the importance of ideas stemming from Skemp and Kaput on conceptual development through linking different representations, and to Bloom's first category of *Knowledge*. The principles below were seen as essential in this development.

Principle 1: Informal assessment rests on the importance of the interaction between teacher and students in the teaching and learning process.

Principle 2: Informal assessment seeks to inform teachers of the quality and organisation of students' procedural and conceptual knowledge. This then informs their teaching

Principle 3: Informal assessment should cover the main types of mathematical activity in schools (see Kaput, 1992, as noted above).

Principle 4: Informal assessment should take in to account the nature of the subject matter presented in school mathematics and the usefulness of integrating real world contexts as discussed in de Lange (1995).

The Framework for Informal Assessment Questions

The framework for informal assessment questions (FIAQ) presented here was developed using the principles above, in conjunction with the views of experienced teachers. It should be noted that we have taken a broad interpretation of the word 'question' throughout, including in it requests requiring a student response. The FIAQ has identified five levels in the secondary school mathematics teachers' informal *assessment regime*. Assigning levels to informal assessment questions depends on the current situation of the class and the background of the students; what is a higher-level question for one class may be a review question for other students. Sometimes a lower level question may be more difficult than a higher level question, but the degree of difficulty of informal questions has not been a factor determining the hierarchy of levels. Rather, the hierarchy of levels is related to the richness in areas such as making connections, transformations between representation systems, and problem solving in real-world contexts. Such inter-representational connections and transformations are common in mathematics, but links are not always maintained. For example, when moving from a symbolic form of a function to a graphical form it is important to connect sub-concepts such as the dependent and independent variables in each representation, as well as the primary concept of function. Such linking can then assist in the connecting of procedures in each domain, for example in finding the zeros of the function.

Table 1
The Framework for Informal Assessment

Level	Description	Example
1	Questions on subject matter that do not require any mathematical operation or transformation	How do you define a 'set'? State Pythagoras's theorem.
2	Questions of one step or a few steps that do not require any transformations	What is -1 minus -2 ? Solve $2x - 6 = 0$.
3	Questions requiring transformation from one representation system to another without interpretation or extrapolation and/or connecting two or more sub concepts, without reference to a practical situation (3.1) or with reference to a practical situation (3.2).	What is the standard form of 145.28? (3.1) The population of New Zealand is said to be 3.6 million. Write this in standard form. (3.2)
4	Questions requiring transformation from one representation system to another with interpretation or extrapolation and/or connecting two distinct concepts without reference to a practical situation (4.1) or with reference to a practical situation (4.2).	Now you found that, x is less than -2 and x is greater than 3 , for this inequality. How do you represent this answer in a notation system that we used in sets? (4.1) Explain the trend in a (given) time series graph of ice cream sales (4.2).
5	Questions requiring students to make generalisation with or without judgement.	Most proofs in geometry. Given that $n(A)=25$, $n(B)=20$, and $n(A \cap B)=5$, use a Venn diagram to find $n(A \cup B)$ and hence find a relationship among $n(A)$, $n(B)$, $n(A \cap B)$ and $n(A \cup B)$.

The five levels of the Framework for Informal Assessment Questions asked by secondary school mathematics teachers in their regular classroom are given in Table 1. Levels 1 and 2 in the framework examine the use of definitions or steps in procedures, while levels 3 to 5 refer to these connections across representations. Hence, the framework is considered to be capable of differentiating lessons that are primarily procedural, from those that are primarily conceptual.

The Framework for Informal Assessment in Action

To date the FIAQ has been used to analyse the lessons of 10 teachers, from New Zealand and Sri Lanka in classes where the students ranged from years 9 to 12. The class dialogues were transcribed to identify the teacher's informal assessment questions and then these were assigned levels according to the framework. There was a high level of agreement between the two experienced teachers who assigned levels to the teachers' informal assessment. Two of these lessons (teacher A and teacher B, who are both from New Zealand) have been selected to demonstrate the FIAQ. Extracts from teacher A's lesson, which was later identified as procedural, and teacher B's lesson, which was identified as conceptual, are given below. Minor grammatical errors in the dialogue, have been corrected. Most of teacher A's questions fell in categories 1 and 2, while Teacher B used a range of questions, covering all five of the levels of the FIAQ. Figure 1 shows the number of informal assessment questions in each of the five categories, for these two teachers.

Samples of Teacher Dialogue

Teacher A was teaching a Year 9 class in a lesson on rounding. The following portion of dialogue took place close to the beginning of the lesson. It can be seen that the major portion of the teacher's informal assessment fell into levels 1 and 2 described above. These were either questions that did not require any mathematical operations or transformations, or ones that required only one or two operations. There were no requests requiring transformations between representations, or requests for generalisations.

T: Do you know what a decimal number is? L1 [A question on subject matter]

S: Yes.

T: What type of number is called a decimal number? L1 [A question on subject matter]

S: Any number, which has a decimal point.

T: (Writes that statement on the board.)

T: Any example for a decimal number? L1 [A question on subject matter]

S: (Say many decimal numbers.)

T: (Writes on the board.)

S: 9.9999.

T: How do you show that? L1 [A question on convention]

S: 9.9 dot.

T: What does that mean? L1 [A question on convention]

S: 9.9 recurring

All these level 1 informal questions are limited to asking about subject matter of varying kinds, such as definitions and conventions. Another portion from the same lesson:

T: (Writes on board) 35.5283 How many decimal places are there in this number? L2

S: Four. T: Four.

T: How do you round this number to one decimal place? What is the answer? L2

S: 53.5

The two questions above are designated level 2 questions because students have to follow a pre-learned procedure in order to respond to them. There was only one level 3 question in this lesson, as follows.

T: What is the difference between rounding to decimal places and rounding to significant figures? L3.1

This question is assigned to level 3 because it requires the students to make connections between two methods of rounding numbers, namely to decimal places or significant figures. These may be considered as two different representations of the number.

The second example is from Teacher B. This was a lesson on time series, with a Year 12 class and the portion selected came near the end of the class period. The students were given a table showing monthly electricity consumption of an energy conscious family over three years and this provided a context for the mathematics. Analysis of the teacher's questions showed a wider range of levels of questioning than teacher A, including requests for transformations across representations, and one request for a generalisation.

T: So, the question (b) says 'what's the point of doing this? What's the point (of doing moving averages)? L1

S: To smooth the data.

T: (Repeats the student's answer) It probably needs a better explanation than that. To smooth the data and what else does it eliminate? L1

S: Noise

T: Noise. Very good answer there is, 'to eliminate noise and smooth the data. Question c says 'the family has tried to become more energy-conscious over the 3-year period. Have they succeeded? Comment. L4.2

The first two questions above illustrate well the need to take into account the background of the students when assigning the question levels using the FIAQ. In both cases they are only level 1 questions since the purpose of moving averages has been taught in the previous lesson. In contrast, the final question above is at level 4.2. To respond to this students have to link a graphical and a verbal representation, interpreting the data from the graph in the practical context.

In the following discussion the teacher asked a similar question in a different way.

T: If you are more energy conscious, what would be happening to the number of units? L3.2

In this case the students have to transform data from a tabular representation to a verbal representation in a practical context, but without interpreting it. Three other examples of questions assigned to this level are given in the passage below.

S: Using fewer of them.

T: (repeats the answer) From your table it appears that the mean of three is decreasing, but from your graph it's easy to see. The graph of the moving mean is this. It's decreasing. So, it tells

you that ‘yes, they’ve become more energy-conscious. Had you not done this graph, the original graph looks like this (Shows the previous graph of row data). Can you tell from this, if the family has become more energy-conscious? L3.2 [Transformation between representations, but without interpretation]

S: Yes.

T: (Smiles) Yes. It’s a bit dicey, isn’t it? You can see: moving means tell you more. When there is a lot of noise, the moving mean is a really good thing to do. So, I think the family has become more energy-conscious and in comments you can say, because...(pauses) because why? L3.2 [Transformation between representations, but without interpretation]

S: The moving mean is decreasing.

T: Decreasing. Dropping. OK. Now we move on to d. Individual seasonal effects need to be calculated. Because electricity usage is a seasonal thing. We use more electricity in the winter than in the summer. Give me any other example for a seasonal thing. L 3.2

S: Ice cream sales.

This final question requires students to look for another situation where the ‘seasonal effect’ is prominent. Therefore they have to link the concepts (not distinct) ‘seasonal effect’ and ‘variation’ with an appropriate practical situation. Had they done similar examples before, then this question would have been at level 1.

During the lesson, this teacher asked one level 5 question, which required generalisation. When discussing moving averages, she wanted to get the idea of ‘mean of three’ as a student’s response. She asked, “Has anybody come up with an excellent idea to line up moving average against a month?” A student responded, “Find the mean of three x values”. That question was at level 5, since it was about a generalisation, the concept of *mean of three* which is a general idea that can be used to smooth time series graphs.

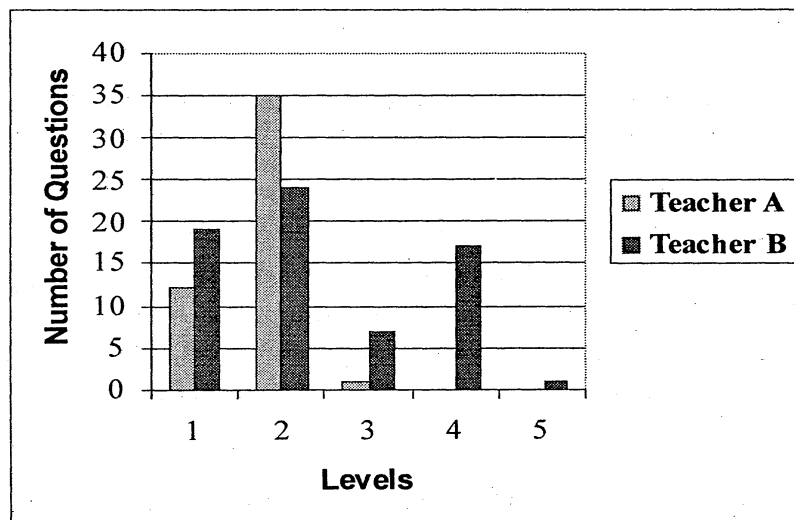


Figure 1. The number of informal assessment questions asked by teachers A and B at each of the levels of the FIAQ

Figure 1 presents a summary of the number of questions in each category asked by each teacher. While Teacher A’s questions were limited almost entirely to Levels 1 and 2 of the FIAQ, Teacher B asked questions across all 5 categories. It should also be noted that Teacher B asked nearly 50% more questions than did teacher A (68 questions versus 48 questions). This quantity is in itself an indication of the interactive nature of the class.

Discussion

These two samples provide examples from one lesson in which teacher's questions were primarily at level 1 and 2 of this framework and another from a lesson that included questions from all levels. They demonstrate the difference between a lesson that appears to be primarily procedural and a lesson in which the relationships between procedures and concepts are being elicited (see Kaput, 1987; Skemp, 1985). Because this framework can demonstrate this distinction, it can be of considerable help to teachers who are trying to evaluate their own practice. It can enable them to see if they are asking questions that provide information on all aspects of students' understanding. This information can then be used together with an analysis of teachers' content and pedagogical knowledge in their own professional development.

There are some things that this framework for analysing informal assessment questions cannot tell. It gives evidence of the informal assessment that can be seen in teachers' questions but it does not cover a variety of other informal assessment procedures other than questions or requests that also add to a formative and diagnostic view of students' understanding. This other informal assessment could include observing students' written work, and noting students' correct or incorrect answers or pauses. It could include non-verbal behaviour of either the teacher or the students that helps provide information on students' understanding.

In addition, on its own it does not tell us about the effectiveness of this informal assessment. To know that would require a fine-grained analysis of both the teachers' and the students' understanding at various times, such as that which might be provided by stimulated recall.

These other aspects may be important, but are less easy to capture than are teachers' questions. An analysis using the FIAQ gives important information about the thrust of a teacher's assessment. From this analysis it is possible to tell if, in a particular lesson, the focus has been on procedures, as defined in this framework, or if the focus has been wider, both exploring students' knowledge of the relationship of concepts and procedures and encouraging them to construct these relationships.

With Kaput, we believe that there are many types of knowledge fostered in mathematics classrooms. Some lessons may need to be primarily procedural. The teacher's question in these lessons might be expected to be categorised as level 1 or 2. Such lessons might cover particular content matter, or be found at certain times in a sequence of lessons when procedures are being taught or reviewed. It might be appropriate in other mathematical topics, or at different points in a sequence of lessons to emphasise the inter-relationships and generalisations that are the focus of Levels 3, 4, and 5 of this classification framework.

We see two major benefits of this framework. Firstly, it can help examine the practices of different teachers who espouse or practice different assessment procedures (Dennisse, Sharon & Charlene, 1997). Most teachers want to improve students' understanding of the relationships between different representations, procedures or concepts as their students develop more inclusive schemas in mathematics. Their professional development is aided

by a close analysis of their own teaching. Using this analysis framework gives them a basis for reflecting on their informal assessment, to see if it meets their own goals as well as it might (see Britt & Irwin, in press).

Secondly, it would be very useful to analyse the teaching of entire units of mathematics, to explore the balance of questioning at the various levels of the Framework for Informal Assessment Questions. This research could inform debate on the order of procedural or conceptual questioning in use. It could be used to explore generalisations about the need to teach procedures in isolation before teaching how they relate to concepts and each other, versus teaching these procedures in the context of higher order concepts as found in solving complex problems.

Both of these uses would enable us to know more about the nature and importance of informal or formative assessment questions. As stated initially, this is seen as a very important aspect of teaching (Schloemer, Cain, & Kenney, 1994) but because it is difficult to capture it has received less attention than has summative assessment. It has been possible to build a framework for analysis of informal assessment, based on the work of several writers (Bloom, 1956; de Lange, 1995, Kaput, 1987, 1992; Skemp, 1985). This framework provides an important step forward in the analysis of informal, formative assessment.

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