

# Knowledge and Strategies Students Employ to Solve Open-Ended Problem-Solving Activities

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This paper examines the type of specific understandings and general strategies students of different ages (Grade 3 and Grades 5/6) employ to solve an open-ended mathematics problem. The task required the students to construct a map that would allow a friend to get from a specific destination to their house. Although the Grade 6 students had a more sophisticated understanding of scale and proportion than the Grade 3 students there was not a great deal of difference between their worksamples.

Increasingly, open-ended problem-solving tasks are expected to be part of teaching and assessing in primary schools. Open problems required students to make their own decisions, plan their own routes through tasks, choose methods, and apply mathematical knowledge (Boaler, 1998). These tasks may have many or no specific answers, a variety of solutions or possible representations, and/or more than one interpretation of the problem. Such problems provide opportunities for students to engage in mathematical experiences at a higher level than do typical routine tasks (Lowrie, 1998a). Researchers are using a variety of terms to indicate these tasks including: novel problem solving (English & Warren, 1994); extended mathematical tasks (Leder & Forgasz, 1997); non-routine problem solving (Lowrie, 1996, 1998a; Taplin, 1994); and goal-free problems (Sweller, 1992). Open-ended tasks allow students of different ability levels to work on the same problem and achieve different outcomes. Very good open-ended tasks can generate a variety of mathematically valid responses that differ in the quality of understanding displayed (Clarke, Clarke & Lovitt, 1990). In the present study student's responses are assessed in relation to the mathematical knowledge and general knowledge of the processes (in this case maps) that are displayed in their solution.

## Knowledge and Understandings Used in Open Tasks

Open-ended tasks allow the problem solver to have some control over the curriculum content and the type of learning activities presented in the classroom. Furthermore, the tasks or the activities children construct may provide insights into the beliefs or attitudes they have toward mathematics and the way in which mathematical knowledge is developed. The very nature of these tasks ensures that the problem solver will use different types of knowledge and a variety of strategies to complete a given task. Although numerous studies have found that children are encouraged to use a diverse range of approaches when solving open-ended tasks (eg., Lowrie, 1998b; Sullivan, Bourke, & Scott, 1995) few studies have attempted to monitor the extent to which mathematical understandings and knowledge are developed and enhanced when students solve such tasks. Mayer (1992), for example, argued that whilst open-ended problems have been studied widely over the last thirty years the emphasis has been on the diversity of responses and not on the kinds of mathematical knowledge which are evoked through such experiences.

The ability to access relevant information is central to success in problem solving (Chi, Glaser & Rees, 1982). In most open-ended situations the problem solver needs to access a combination of specific knowledge relevant to the problem and general knowledge that is more

easily transferred from other problem-solving situations (Glaser, 1991). The opportunities to find alternate solutions, justify solution strategies, and reflect on one's actions from immediate feedback are also critical aspects of the problem-solving process (English & Halford, 1995). In this study we assess the extent to which students use specific mathematics knowledge of proportion and scale and general understandings of map representation to complete an open-ended investigation.

## The Purpose

The central concern of the study was to investigate the influence students' general understanding of maps and specific knowledge of scale had on their ability to complete an open-ended task. The following research questions were posed.

- What impact did a student's knowledge of scale and proportion have on the quality and accuracy of the final product?
- Did the students' age (Year 3 to Year 6) influence the way in which they interpreted and represented the problem?
- Did the students' locality (city or rural) influence the way in which they interpreted and represented the problem?

## Method

### *The students*

The investigation took place in two primary schools in New South Wales. The first site involved 21 Year 3 students from an independent girl's school in Sydney. At the second site students from a Year 3 class ( $n = 26$ ) and a selective Year 5/6 Opportunity Class (OC) class ( $n = 29$ ) participated in the study. This school was located in a large rural city. It could be argued that the composition of the sample was diverse in nature, with the children coming from distinct cultural and socio-economic backgrounds. It was anticipated that the students would bring quite different experiences to the problem-solving situations when considering their age and place of residence<sup>1</sup>.

### *The Activity*

The activity was conducted over a week (of four or five mathematics lessons) and consisted of three interrelated sections. These three components included a) a questionnaire; b) an open-ended problem-solving task; and c) a reflective response. The questionnaire was designed as an assessment tool to determine the type of knowledge and understandings the students had acquired on concepts associated with maps. The students were encouraged to describe instances when they had used maps, consider people who relied on using maps and identify important characteristic of maps. The questions were carefully scaffolded in order to encourage the children to remember as much as they could about maps including concepts of scale and proportion.

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<sup>1</sup> The problem-solving activity required the students to design a map of their local area. The way in which the map needed to be represented would be different for children living in the city or rural areas. Moreover, the children's understanding of what constituted a map would have been influenced by their experiences in using particular types of maps.

The open-ended task was presented in the following scenario.

Pretend a friend from out of town is coming to play with you on the weekend. Your friend knows how to get to your school but has never been to your house. Unfortunately you are unable to meet them at school so you decide to post your friend a map that gives directions on how to get from school to your house. The map should allow your friend to find your house easily.

The students were encouraged to consider the mode of transport most commonly used to get home from school as a starting point for designing an appropriate route. Many of the city students lived a considerable distance from school with some children's journey involving several modes of transport.

The third component of the activity was completed after the students had completed their map. This reflective response encouraged the children to describe parts of the problem they found easy or difficult, what they had learnt about the process, and whether or not they would do things differently if presented with a similar problem in the future. Many of the questions were designed in order for the students to think at a metacognitive level.

### *Data Coding*

Three sets of data—including the questionnaire, the problem-solving task and the reflective response—were used to monitor outcomes for the activity. The criteria identified in Table 1 are categorised under the headings of a) general knowledge of maps; and b) specific mathematics understandings.

Table 1

#### *Criteria Used to Assess Students' Responses*

General knowledge of maps	Specific mathematics understandings
Landmarks	Understanding of scale
Key	Directions- N, S, E, & W
Symbolic representations	Coordinates
"Birds eye" [plan] view	2-D representation

## Results

### *Questionnaires*

When asked to describe what they knew about maps a wide range of responses were obtained. Students were aware of a diverse array of maps in the form of street directories, world and country, treasure, weather, railway, mind / concept, planet, orienteering and wildlife maps. Most were able to explain that maps were useful for providing directions to specific locations and to help find travel routes.

The greatest difficulties which the students expected to encounter while trying to draw their maps were trying to fit everything in on the page and trying to remember street names. The Grade 5/6 teacher commented that with her students, there appeared to be very little carry over of what they said they knew about scale to actually applying it to their maps (only one Grade 5/6 student included a scale on their map). In describing these difficulties the older children openly discussed scale and proportion. Some of the younger children demonstrated informal notions of proportion when commenting that it would be difficult to fit everything on the page. Interestingly, the younger children were not as worried about precision in their

drawings as much as the older students. The Grade 5/6 teacher noted that the class was full of “perfectionists” with many of the children enjoying a “healthy rivalry” with peers. This would be expected in a class of very talented students.

### *The Problem-solving Task*

All of the maps produced by the students were analysed and specific features identified and collated in Table 2. All maps drawn were from a “birds eye” or plan perspective view.

Table 2  
*Incidence of Significant Map Features*

Features	Year 3 City School n = 21	Year 3 Rural School n = 26	Year 5/6 Rural School n = 29
Use of grids	24%	0%	0%
Use of streets	90%	96%	100%
Naming of streets	72%	54%	97%
Use of key	0%	12%	93%
Use of scale	0%	0%	4%
Use of proportion	12%	15%	48%
Use of direction	33%	42%	86%
Compass directions	0%	4%	6%
Landmarks			
– Houses	52%	92%	83%
– Railways	24%	0%	21%
– Other	81%	100%	34%
More than 1 page	43%	4%	7%

From the above data, it can be seen that five students from Grade 3 at the city school used grids (see Figure 1) on their maps and nine made use of more than one page. A possible explanation could be that these students had prior exposure to street directories. This is in contrast to the children from the rural school whose parents no doubt did not make frequent use of a street directory. This is a good example of how children bring their prior knowledge/experience to school tasks. The use of a key (see Figure 2) by three of the rural Grade 3 students was interesting since it had not been taught specifically at school. Once again an example of possible prior knowledge/experience was being used in a school context.

A large number of students from the Grade 5/6 class included street names on their maps (see Figure 3) perhaps suggesting a developing awareness of a common feature found on most maps. Another interesting observation was that the city students were less likely to include houses in their maps than the other students were. This may be due to the fact that the majority of these students lived a long distance from their school and concentrated their attention on bus/railway routes and other significant landmarks to find their way in suburbia. Only one student made a rough draft before drawing the map. He explained that “I needed to

tell myself where to start and finish. I put myself in the car and drove there.” The three teachers commented that most students started from one point (eg., the school) and simply progressed to the destination.

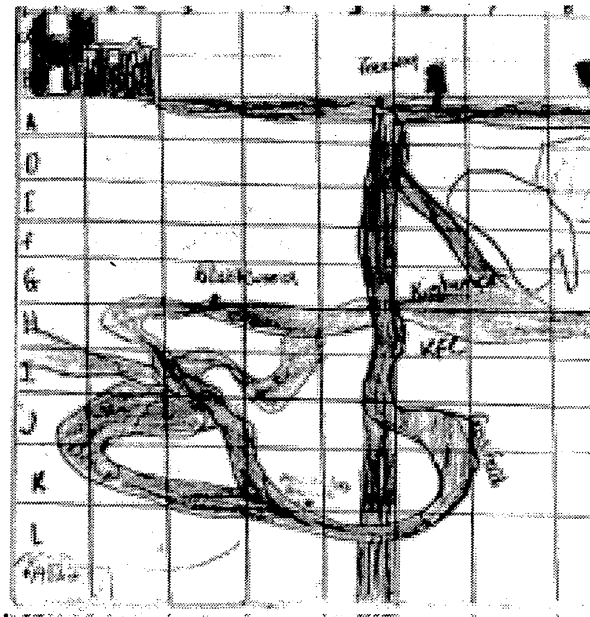


Figure 1. Example of a map drawn by a city Year 3 student

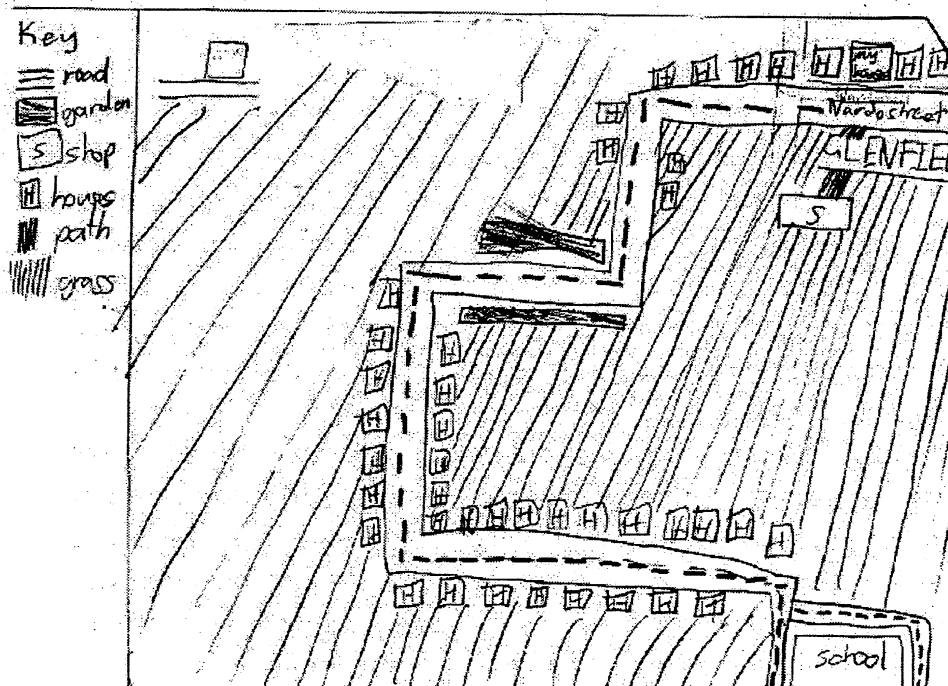


Figure 2. Example of a map drawn by a rural Year 3 student

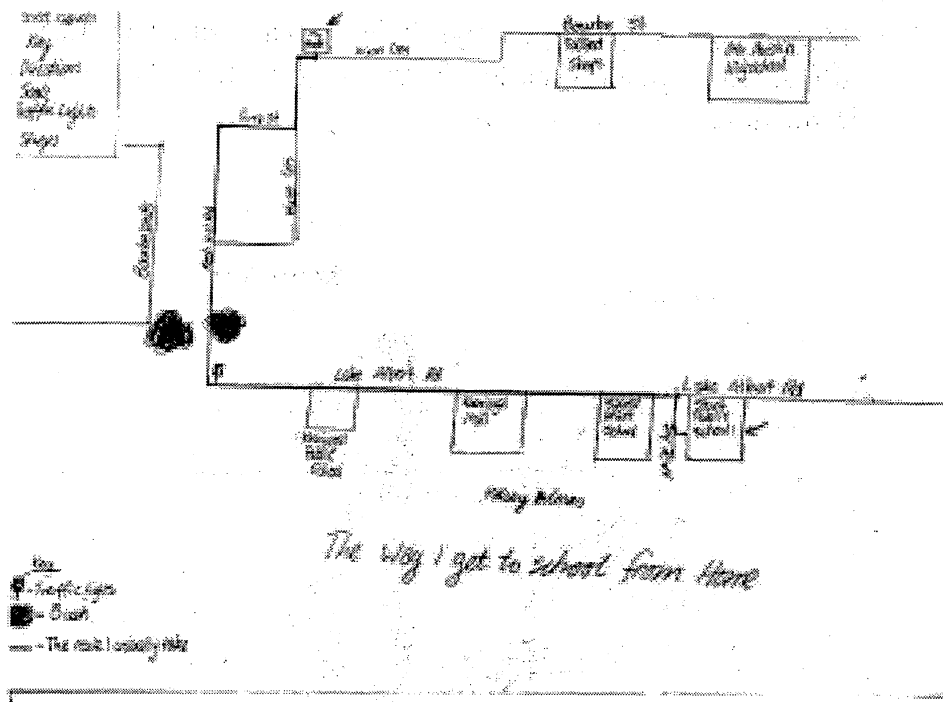


Figure 3. Example of a map drawn by a Year 6 student.

### Reflective Responses

Having completed their maps, the students were asked a number of questions to ascertain what they had learnt from doing the exercise. A number commented on the need to learn the names of the streets they use as they travel between their homes and school. This suggested that the students were concerned about the specific content rather than the actual process of designing the map. Some students expressed the need to first try to visualise a rough copy of a map in their head. Others suggested that they needed to be more accurate. Seven (14%) of the Grade 3 students demonstrated an understanding of informal scale; however these younger children were not able to explain why scale could have helped make their map more accurate. In contrast, almost all of the Grade 5/6 students commented that they should have calculated a scale before attempting to draw a map. Over 50% of these students recognised that some of the distances would need to be guessed but accepted that this would still be far more accurate than their original drawings.

The knowledge students had acquired about maps was similar—with the exception of understandings associated with scale—despite the fact that some children were three years older than other students. In most instances, the Grade 5/6 children could not, however, transfer their knowledge of scale into this new problem-solving context. Similarly, the Grade 5/6 children employed the same type of processes to complete the problem even though they appeared to have more sophisticated processes at their disposal.

### Conclusions

As anticipated, the Grade 5/6 students had a more sophisticated understanding of scale and proportion than the younger children did. Interestingly, this effected their ability to complete the task to their satisfaction because they were worried about constructing something that was not accurate. On the other hand the Grade 3 students did not have such a

well grounded understanding of these mathematics concepts and so were much more satisfied with their final product because they thought it served as a good *informal* measurement which looked reasonably accurate.

The Grade 5/6 students were able to articulate what they had learnt from the process and were able to offer insightful comments about ways of making their maps more accurate in the future. Moreover, their maps contained an increased degree of symbolic representation. Overall, however, their maps were not significantly different from the more basic “mud maps” produced by the Grade 3 students. Despite the fact that the older students would have been exposed to the teaching of specific concepts such as space and scale, we felt that fundamentally the ability of all three groups of children to carry out the mapping task was essentially similar. Overall most of the maps produced by the Grade 3 students were quite accurate and could be used to find directions between A and B. No doubt these children were able to link their evolving mapping skills to prior experiences and knowledge gained outside the classroom context. These students were able to draw on their life experience rather than relying on what they learnt from school. When the children were presented with new open-ended problem-solving experiences, such as drawing a map, they reformulated their existing cognitive framework only if the new knowledge was connected to knowledge already in their memory. Children must be encouraged to actively construct knowledge into their existing framework for meaningful learning to occur.

We were able to detect subtle differences between the maps produced by the Grade 3 students from the city and rural school. Some of the maps drawn by the students from the city school resembled a street directory—five contained grid lines and nine were drawn on a number of pages that were attached together. This was in contrast to the rural students' maps that did not have such features. It would be reasonable to suggest that the students who lived in the city were more likely to be exposed to such maps than their rural counterparts. Thus, the lived experiences of the students influenced strongly their representations.

It could be argued that this open-ended task did not provide the older students with the flexibility and scope to demonstrate the specific mathematics knowledge they possessed. However, these students were able to verbalise what they would do differently if given a similar task in the future in a much more sophisticated manner than were the Grade 3 students. They were able to reflect upon the problem-solving strategies they had used and recognised that they should have considered scale and proportion more closely in their representations. Consequently, this investigation shows that the problem-solving process involves much more than an analysis of the completed product (worksample). Although the Grade 5/6 students were not able to represent their knowledge and understandings through the mapping activity, the open-ended investigation enabled them to learn much from the process.

### Implications

Most differences in the worksamples could be attributed to normal developmental processes of abstract thinking, spatial awareness and estimation skills rather than an ability to apply mathematical knowledge to new situations. The manner in which the children represented their work was influenced strongly by experiences gained outside the school context. This study has shown that we need to be very clear about the purpose of open-ended problem solving and its associated worksamples. Consequently, we would argue that:

- A worksample (ie., the product) is not sufficient as evidence of student outcomes in a particular content area. The completed task should be put in the context of what school and life experiences the student brings to the task and the metacognitive reflections the student is able to convey about the physical product; and
- Open-ended tasks may not provide opportunities for students to represent what they know about a particular content area if the novel nature of the task limits their ability to apply understandings to such new contexts.<sup>1</sup>

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<sup>1</sup> *Acknowledgements*. The authors would like to thank the three teachers involved in the study for their valuable contributions—for allowing us into their classrooms to gather data and for their helpful input into discussions.