Graphical Representations of Statistical Associations by Upper Primary Students

Jonathan Moritz
University of Tasmania
<Jonathan.Moritz@utas.edu.au>

Students' graphical representations of statistical associations are explored through analyses of responses from 97 students in grades 4, 5, and 6. The task involved representing (a) the association "People grow taller as they get older," (b) this association with an age modifier, and (c) this association with sex differences. Three levels of representing bivariate associations and of representing multivariate associations are described: unsuccessful, partial, and complete. Issues for developing student representations are discussed.

Graphs are commonly used to convey at a glance large amounts of statistical data, particularly data involving bivariate and multivariate associations, which are otherwise difficult to convey without reporting formal statistics such as t-tests or regression lines. Graphs produced by students may convey not only statistical data, but also how students understand statistical data, and how they understand bivariate- and multivariate-associations. Although many studies have explored students' interpretations of graphs (for a review, see Leinhardt, Zaslavsky, & Stein, 1990), it is only recently that students' representations have begun to receive attention by mathematics education researchers (e.g., Mevarech & Kramarsky, 1997; Moritz, 1999; Moritz & Watson, 2000; Watson, 2000).

Mevarech and Kramarsky (1997) asked 92 grade 8 students to represent the following four claims of association between time studying and achievement grades in school:

1. [increasing function] the more she studies, the better her grades;
2. [constant function] no matter how long she studies, she always gets the same grade;
3. [curvilinear function] up to three hours, the longer she studies the better her grades, but beyond three hours, she becomes tired and her grades become lower; and
4. [decreasing function] when she studies more, her grades decrease.

Approximately 55% of students appropriately represented claims 1, 2, and 4 using a labelled two-axis graph, whereas the curvilinear function of claim 3 was correctly represented by only 38% of students. About 10% of students represented generalised ideas of graphs or represented relations between the variables in an idiosyncratic way. Three common alternative conceptions were evident in responses that represented (a) only a single point in a graph, (b) only one factor in each of a series of graphs, and (c) an increasing function irrespective of task requirements. These conceptions were evident in approximately 20%, 20%, and 10% of students' responses respectively (students could exhibit more than one conception). The first two conceptions involve eliminating variable data to reduce complexity of the representation. The last conception indicates that constant, curvilinear, and decreasing functions may be more difficult for students, for example involving appreciating that as one measure changes, the other measure may remain constant, and thus representing a repeated value on that measure.

A number of studies (Moritz, 1999; Moritz & Watson, 2000; Watson, 2000) have reported on responses from students in grades 6 to 11 to represent in a graph "an almost perfect relationship between the increase in heart deaths and the increase in the use of motor vehicles", as claimed in a newspaper extract and presented in a written survey task. Moritz and Watson (2000) described a framework of four levels according to degree of success in representing the association. At each level, different categories of response were evident. At the level of *No Association*, some responses were pictures and others were basic graphs that displayed no trend and no labelling of variables. At the level of *Unsuccessful Association*, some responses were labelled graphs that did not show an association. Also at this level, many responses were single comparison graphs, often bar graphs comparing a single value of heart deaths with a single value of car use; the reduction of each variable to a single value appeared to be a means of reducing the complexity of the graph. At the level of *Partial Association*, some responses displayed a trend but did not specify the appropriate variables, and others represented a double comparison involving two values for each of heart deaths and car use. At the level of *Complete Association*, some responses showed bivariate association of heart deaths versus car use, and others were a series comparison of heart deaths and of car use displaying similar time-series trends.

The current study aimed (a) to extend to upper primary school students the findings of Mevarech and Kramarsky (1997) concerning students' representations of increasing and curvilinear functions, and (b) to extend to different tasks the findings of Moritz and Watson (2000) concerning levels of conceptual development leading to the ability to use a series comparison representation to compare two bivariate trends.

Method

A survey item, shown in Figure 1, included two parts (B1 and B2, together forming part B) involving bivariate association based on items by Mevarech and Kramarsky (1997) and a third part (M) involving multivariate association to extend the work of Moritz and Watson (2000). The item was administered during November 1998 to whole classes as a written survey along with three other items. Each of the four items was presented on a single A4 sheet of paper. One item asked students to tell a story about an unlabelled, unscaled line graph involving a non-monotonic negative trend, and two other items were on unrelated topics. The two graphing items were always the second and fourth items in students' surveys, with approximately half in each order.

> One day at school, the class began to talk about how tall people are. The teacher measured everyone's height. Then they began to talk about their brothers and sisters. Some were taller, and some were shorter.
> *(B1)* David said, "People grow taller as they get older".
> *Draw a graph to show what David is saying. Label the graph.*
> *(B2)* Mary agreed, "People do grow taller. But when you are 20 years old, you stop growing."
> *Draw a graph to show what Mary is saying. Label the graph.*
> *(M)* The teacher said, "For 10 year olds, girls and boys are about the same height.
> But men usually grow to be taller than women."
> *Draw a graph to show what the teacher is saying. Label the graph.*

Figure 1. Survey item to graph statistical associations ["*(B1)*", "*(B2)*", and "*(M)*" not on original survey].

Responses were gathered from 23 fourth-grade, 25 fifth-grade, and 26 sixth-grade students at a private school for males, and 18 fourth-grade, and 13 fifth-grade students at a private school for females. When requested by students, the researcher or classroom teacher offered assistance in reading the questions and encouraging students to do their best when they were uncertain about their responses. Of these 105 surveys, 93 students' complete responses were included in the analysis, and another 4 responses to part (B) were deemed complete and also included; 8 surveys were not analysed because of non-response to parts (B2) or (M), possibly due to time constraints and other class interventions.

Each student's responses to part (B) were assigned to one of the three levels, as were responses to part (M). The three levels were comparable for responses to parts (B) and to part (M). At the unsuccessful association level, responses failed to show an association in the data, often omitting one of the relevant variables. At the partial association level, responses indicated an association between some variables but not all critical features were displayed. Responses at the level of complete association represented all critical features. For part (B2), critical features included a curvilinear trend that showed increasing height with age and also constant height with changing age by age 20. For part (M), three critical features included (a) at age 10, females same height as males, (b) association of height with age (for females, males, or both), and (c) for some age greater than 10, males taller than females. The coding schemes were devised to assess at higher levels those responses indicating the student understood the numerical association of the variables and had some coherent system for representing this, and not to penalise the student for violating a graphing convention, such as interchanging axes or reversing the order of an axis scale. Despite the task requirement for labelling, it was acknowledged that some students might have assumed height and age were obvious variables since they were written on the page just above where their graph was to be drawn. Hence "indicative labelling" was credited: students were given the benefit of doubt if they provided some features of the graph or the data that indicated a distinction between height, age, and sex. Examples include a pictograph of people implying the vertical axis indicated height, or the number 20 indicating a significant age in response to part (B2).

An independent coder assigned responses to levels.[1] There was agreement for 80% of responses to part (B) and 70% to part (M). Many of the discrepancies were due to the coder misunderstanding the criteria. Following joint discussion, there were few remaining cases of disagreement: one for part (B) and four for part (M). For these cases, students had represented features of the associations in idiosyncratic unconventional ways that appeared to the author to represent relevant features, and the author's categorisation was preferred despite the coder remaining unconvinced after discussion. Compared to the author's initial coding, the final coded data had no differences for part (B), and one difference for part (M).

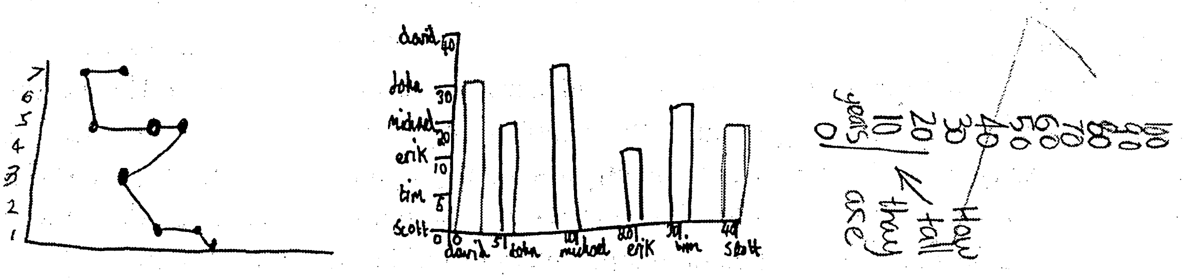
There were no significant sex differences, nor item order effects, except at grade 5 for part (B): of students for whom this was the 2nd or 4th item of the 4 in the survey, 69% (n = 26) and 25% (n = 12) respectively represented the curvilinear trend at the level of Complete Bivariate Association. This difference may be due to chance, to less time devoted by students when the last survey item, or to the other graphing item influencing responses.

[1] It is acknowledged that within each level, various categories of response might be observed, such as by Moritz and Watson (2000), however analyses of responses into categories are beyond the scope of this paper. The coder was in fact asked to assign responses to levels and also to categories, with two categories at the levels of Partial Bivariate Association and Partial Multivariate Association, and three categories at the level of Complete Multivariate Association. The coding scheme of levels and categories was presented with wording of criteria and stylized illustrative graphs. Inter-rater reliability for coding of categories was 69% for part (B) and 58% for part (M). Many of the discrepancies were due to the coder misunderstanding the criteria, such as using the illustrative graphs intended to assist coding; for example believing that form of graph (bar or line) was one of the criteria.

Student Responses to Represent Bivariate Association

Level 1. Unsuccessful Bivariate Association

Responses at this level indicated that students had difficulty coordinating the representation of two variables to show a bivariate association. Some students super-imposed the two variables on a single axis. If Cartesian axes were used, the data indicated either that there was only one unique measure, or there was no association between the two measures. Three examples are shown in Figure 2. In the first example, a grade 4 male represented a generic graph with no indication of the variables or of an association. The bar graph by a grade 5 male (second example) displayed six male names presented in reversed order on each axis, but there was no indication of the variables or of an association. The third example, by a grade 4 female, was labelled to indicate numbers denoted age, and position (length of the line) denoted height; position, however, was also used to denote age with the numbers on a linear scale, thus age and height were superimposed on a single axis.

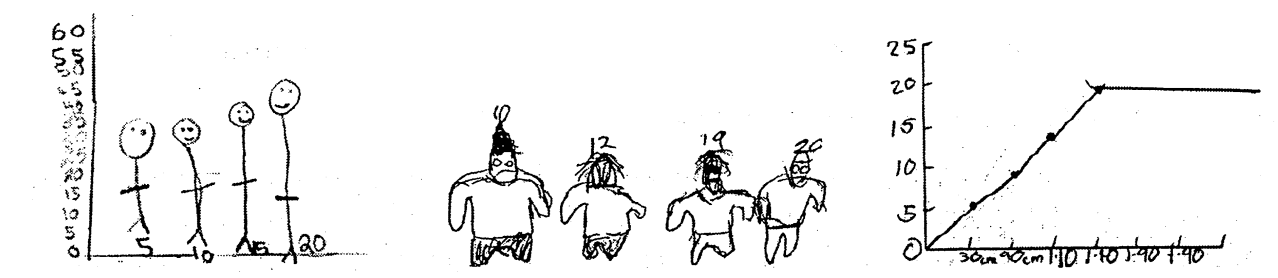


Part (B1), Grade 4 male — *Part (B1), Grade 5 male* — *Part (B2), Grade 4 female*

Figure 2. Three responses at the level of Unsuccessful Bivariate Association.

Level 2. Partial Bivariate Association

At the level of Partial Bivariate Association, responses to part (B) displayed some but not all of the features relevant to the task. Students represented the increasing trend of part (B1), but often had difficulties representing the curvilinear association for part (B2). These difficulties included errors or omissions of labels or scales, that resulted in failure to represent significant features of the association, such as failure to show constant height for changing age, or failure to show that this occurred by age 20. Three examples are shown in Figure 3. In the first example, a grade 4 female represented an increasing trend to part (B1) with height implied by the pictograph form and age scaled on the horizontal axis; for part (B2) she drew a single person at age 20. The second example is a pictograph that displayed a constant function for part (B2), but not the growth that occurred at younger ages, as had been represented for part (B1). In the third example, a grade 6 male reversed the axes, which posed no problems for representation until age 20, beyond which he represented the constant function and showed no change in vertical axis value. This error may indicate the student's confusion that height would naturally be on the vertical axis. His unsatisfactory solution, however, was to replicate constant height values on the horizontal axis.

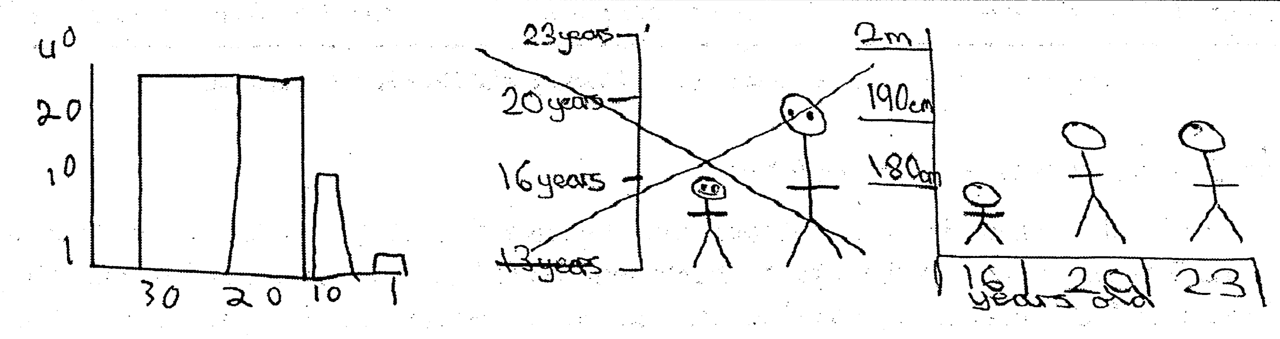


Part (B1), Grade 4 female *Part (B2), Grade 4 male* *Part (B2), Grade 6 male*

Figure 3. Three responses at the level of Partial Bivariate Association.

Level 3. Complete Bivariate Association

At this level, responses represented all relevant features of the task: a curvilinear trend that showed increasing height with age and showed constant height with changing age by age 20. Two examples are shown in Figure 4. In the first example, a grade 6 male reversed the horizontal scale and did not label the graph, but features of the graph data indicate for values 20 and 30 on the horizontal axis (age), there is a constant vertical value (height). In the second example, a grade 5 male learned in part (B2) about the need for independent axes to display repeated values of the same height for different ages. In response to part (B1), he drew a pictograph showing an increasing trend, with height implied in the pictures and age labelled with a vertical scale also. He then began drawing a similar graph arrangement to part (B2), but found difficulty drawing a 23 year old, since his system could not represent age differing when height remained constant, and he then developed an appropriate two-axis system.



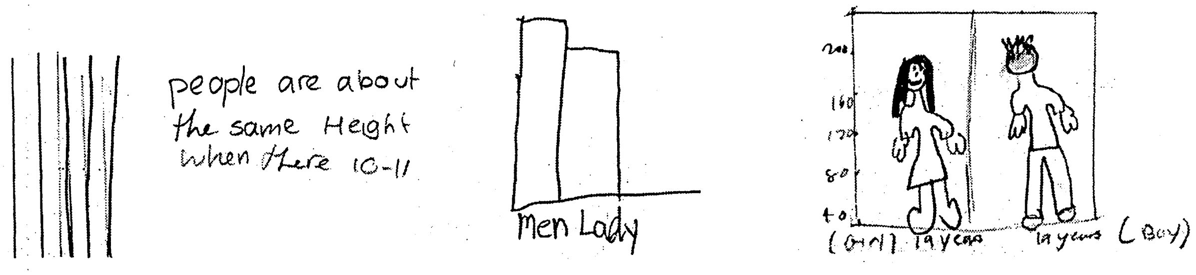
Part (B1), Grade 6 male *Part (B2), Grade 5 male*

Figure 4. Two responses at the level of Complete Bivariate Association.

Student Responses to Represent Multivariate Association

Level 1. Unsuccessful Multivariate Association

Responses that were unsuccessful at representing the multivariate association omitted one variable or super-imposed two variables to represent bivariate data. Three examples are shown in Figure 5. In the first example, a grade 4 female represented a constant function for part (M) just as she had for part (B2), however for part (B2) constant height was implied to be across varying ages, whereas for part (M), she annotated that the graph represented the same height across varying people (and possibly sexes) at the given age 10. The second and third examples in Figure 5 illustrate a method of omitting the variable of age to reduce complexity of the graph to a single comparison of the heights of a single female and male.

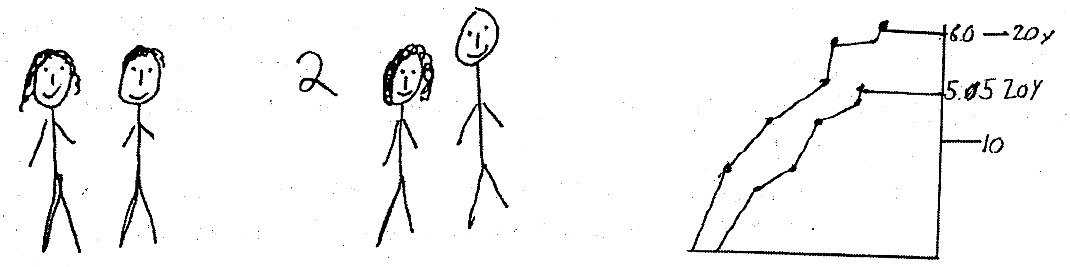


Part (M), Grade 4 female *Part (M), Grade 4 male* *Part (M), Grade 5 female*

Figure 5. Three responses at the level of Unsuccessful Multivariate Association.

Level 2. Partial Multivariate Association

At this level, responses indicated all three variables—height, age, and sex—and displayed some but not all of the features relevant to the task. Two examples are shown in Figure 6. In the first example, a grade 4 male represented a double comparison of female and male heights, indicating the same height at age "1", and males taller at age "2", but there was no indication of the critical feature of age 10. In the second example, a grade 6 male displayed an increasing trend for both females and males (unlabelled), but did not clearly represent the critical feature of same height at age 10.

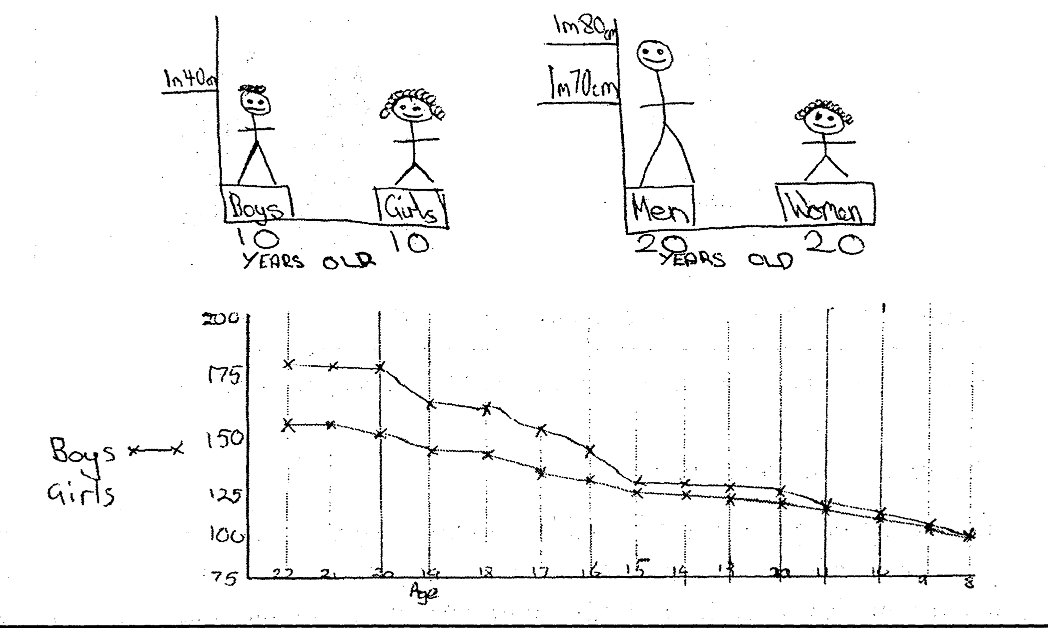


Part (M), Grade 4 male *Part (M), Grade 6 male*

Figure 6. Two responses at the level of Partial Multivariate Association.

Level 3. Complete Multivariate Association

At this level, responses represented all relevant features of the task: (a) at age 10, females same height as males; (b) association of height with age (for females, males, or both); and (c) for some age greater than 10, males taller than females. Two examples are shown in Figure 7. The first example is from the same grade 5 male who drew the pictograph in Figure 4 for part (B2). He adapted his two-axis system from Figure 4 to produce a double comparison, the horizontal axis denoting sex as a variable, and two graphs denoting two ages. In the second example, a grade 6 scaled the horizontal axis increasing right-to-left rather than left-to-right, but represented all relevant features of the task.



Part (M), Grade 5 male

Part (M), Grade 6 male

Figure 7. Two responses at the level of Complete Multivariate Association.

Numerical Analysis of Student Responses

Table 1 shows the percentages of responses levels to part (B) and to part (M) for each sex and grade. In response to part (B), more than 90% of students represented partial or complete association. In response to part (M), about a third of grade 4 students were unsuccessful in representing the multivariate association, but by grade 6 most students represented complete association.

There was an association of response level to part (M) according to response level to part (B) ($\chi^2_2 = 27.31, p < 0.001$)[1]. Four students' responses were level 1 both to part (B) and part (M). Of 38 students who responded at level 2 to part (B), 12, 19, and 7 responded to part (M) at levels 1, 2, and 3 respectively. Of 51 students who responded at level 3 to part (B), 3, 15, and 33 responded to part (M) at levels 1, 2, and 3 respectively.

Table 1
Percentage Responses of Level by Sex and Grade

	Bivariate Association (N=97)					Multivariate Association (N=93)				
	Female Grade		Male Grade			Female Grade		Male Grade		
Level	4	5	4	5	6	4	5	4	5	6
1. Unsuccessful	12	0	5	4	0	33	8	37	16	10
2. Partial	53	31	32	48	35	33	23	37	48	33
3. Complete	35	69	63	48	65	33	69	26	36	57
n	17	13	19	25	22	15	13	19	25	21

[1] To ensure χ^2 test reliability, for parts (a, b), Levels 1 and 2 were merged to avoid low expected cell counts.

Discussion

Three levels of representing bivariate associations and of representing multivariate associations were described, corresponding to levels observed by Moritz and Watson (2000). Over 90% of students from grade 4 represented an increasing trend at the level of Partial Bivariate Association, and at least 35% represented a curvilinear trend at the level of Complete Bivariate Association. These were high success rates in comparison to the grade 8 students in the study by Mevarech and Kramarsky (1997), of whom 55% and 38% respectively represented an increasing function and a curvilinear function. Their criteria, however, included labelling of axes. Indicative labelling was considered a strength of the current coding scheme, in acknowledging student understanding implicit in representations, for example, the height label being implicit in a pictograph of a person.

The task to represent a curvilinear association (part B2) posed difficulties of representation for over 30% students, who responded at the level of Partial Bivariate Association. Some of these students did not have a consolidated sense of a Cartesian coordinate system, which is perhaps not surprising at these grade levels. This did not mean that the task was inappropriate for these students; on the contrary, for some students, the difficulties posed in part (B2) provided an important learning exercise to revise their representational system to display two independent axes, one each for height and age.

The task to represent multiple variables (part M) involving comparison of bivariate associations posed difficulties for many students. The solution for over 30% grade 4 students was to represent a comparison of heights of one female and one male, thus eliminating age as a variable. This strategy to reduce representational complexity has been observed by other researchers as "one-point" graphs (Mevarech & Kramarsky, 1997) and "single comparison" graphs (Moritz & Watson, 2000). Responses at the levels of Partial and Complete Multivariate Association were either in the form of double comparisons (first examples in Figures 6 and 7), or in the form of series comparison graphs (second examples in Figures 6 and 7). These two forms are similar to those observed by Moritz and Watson (2000) in their use of a comparison schema, though for that task, the two forms were at the Partial and Complete levels of success, respectively. Given students' success in representing bivariate associations, sequences of graphing tasks that are carefully designed by teachers and researchers for students in upper primary years may prove fruitful in exploring and promoting these students' learning in representing statistical associations.[1]

References

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs and graphing: Tasks, learning and teaching. *Review of Educational Research, 60*, 1-64.

Mevarech, Z. R., & Kramarsky, B. (1997). From verbal descriptions to graphic representations: Stability and change in students' alternative conceptions. *Educational Studies in Mathematics*, *32*, 229-263.

Moritz, J. B. (1999). Graphing data: Relating representation and interpretation. In K. Baldwin & J. Roberts (Eds.), *Mathematics: the next millennium (Proceedings of the Seventeenth Biennial Conference of the Australian Association of Mathematics Teachers [AAMT])*, (pp. 90-99). Adelaide: AAMT.

Moritz, J. B., & Watson, J. M. (2000). *Representing and questioning statistical associations*. Manuscript submitted for publication.

Watson, J. M. (2000). Statistics in context. *Mathematics Teacher, 93*, 54-58.

[1] *Acknowledgment.* This paper is part of work by the author towards the qualification of PhD. Thanks to Dr. Jane Watson for many helpful comments in writing this paper.