

Factors in Primary School Teachers' Beliefs about Mathematics and Teaching and Learning Mathematics

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This paper is a report on the development of an instrument to identify and measure factors associated with primary teachers' purported beliefs about mathematics and teaching and assessing mathematics. Current literature relating to teachers' beliefs provided a theoretical framework for the development of the questionnaire. The sample consisted of 387 primary teachers. The results of a factor analysis identified seven factors that appear to represent teachers' purported beliefs – three relating to beliefs about assessment, two relating to beliefs about mathematics, and two relating to beliefs about teaching mathematics.

Introduction

This paper is a report on the ongoing development of an instrument to identify and measure factors associated with primary teachers' purported beliefs about mathematics and teaching and assessing mathematics. In a previous paper, the authors reported on the beliefs of primary teachers with respect to the uses of assessment in mathematics and the types of assessment they use. Those results indicated that primary teachers use a wide variety of assessment methods and that they believe that the predominant purpose of assessment is to provide feedback for the teacher rather than for the learner or parents. This paper is an analysis of the data on teachers' beliefs not only about assessment of mathematics but also about mathematics as a discipline, and approaches to the teaching of mathematics.

The study of teachers' beliefs is important because of the influences beliefs have on (a) teachers' classroom practice, and (b) their likelihood of implementing changes of practice in the classroom (Ball, 1990; Thompson, 1992). The relationship between teachers' beliefs and classroom practice is quite a complex one. On one hand, there is much research indicating that teachers' practices are shaped by their beliefs about the subject mathematics and the nature of teaching and learning (Hoyles, 1992; Fernandez, 1997; Putnam, 1992). In a study of primary teachers' beliefs about problem solving in mathematics, Ford (1994) concluded that although there existed a relationship between teachers' beliefs and classroom practices, the extent of the relationship was not found.

On the other hand, there is evidence to suggest that changes in teachers' beliefs about teaching and learning are derived largely from classroom practice (Clarke, 1994; Brosnan, 1994). Guskey's (1986) model of professional development proposes that teachers' beliefs and attitudes change only when they see changes in students' learning. The Clarke-Peter model of professional growth (Clarke & Peter, 1993) recognises that teachers' beliefs are influenced by production of valued outcomes from classroom experimentation. Foley (1993) claims that even traditional teachers are willing to change their teaching methods and beliefs when empowered to make decisions that impact on their classrooms.

The close association between beliefs and practices is supported by Borko (1997) who claims that when teachers' beliefs are compatible with the ideas which underlie a professional development program, these beliefs support the change. Hence it is important to attend to both beliefs and practices in staff development. Borko and Putnam (1996) note that

meaningful change in one requires change in the other. However, the order in which beliefs and practices are addressed in professional development may not be important.

Further evidence of the complexity of the relationship between beliefs and practice is research indicating that some teachers carry on with certain classroom practices in contradiction of their stated beliefs and intentions (Renne, 1992; Sullivan, 1989).

Many studies of mathematics teachers' beliefs have been conducted in specific contexts, namely the teaching of particular mathematics topics, the implementation of certain methodologies, special groups of students, or various professional development models. Hence, the instruments designed to identify and measure teachers' beliefs are themselves related to these contexts for example, implementing the NCTM Standards (Furner, 1996), teaching using the cognitively guided instruction approach (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996), teaching in a socio-constructivist environment (Van Zoest, Jones, & Thornton, 1994), teaching problem solving (Erickson, 1993; Ford, 1994; Lubinsky, 1993), low achievers in mathematics (Dwyer, 1993), and teaching mathematics with manipulatives (Sherman & Richardson, 1995).

The instrument devised for the current project is also related to a specific area, namely, assessment methods in mathematics, but it was thought that the instrument should be developed with a solid theoretical basis, to see whether the data generated from the instrument reflected the theoretical constructs. Therefore, in line with the research on beliefs relevant to classroom practice learning (Fernandez, 1997; Ford, 1994; Hoyles, 1992; Putnam, 1992), the authors decided to include beliefs about the nature of mathematics, and the teaching of mathematics, along with beliefs about the nature and purposes of assessment, and to construct items congruent with the literature in each of those areas.

In relation to the first area of beliefs, the nature of mathematics, Ernest (1989) identified three differing conceptions of mathematics, namely, (i) a dynamic problem-driven view of mathematics where mathematics is a continually expanding field of human creation and invention, (ii) a static unified body of knowledge where mathematics is viewed as interconnecting structures bound together by logic and meaning, and (iii) a bag of tools where mathematics is made up of an accumulation of facts, rules and skills. The items designed for the instrument reflected these three categories.

The literature on teaching mathematics delineates essentially two main classification schemes for classifying beliefs on how to teach mathematics. One scheme (Burton, 1993) identifies two basic approaches to teaching mathematics - a transmission approach where the teacher simply transmits information and rules to the students who are expected to absorb and regurgitate it, and a constructivist approach in which teachers are seen as facilitators of learning and students construct their own mathematical knowledge through interaction with the physical and social environment. Perry, Howard, and Tracey (1999), who called these categories transmission and child-centredness respectively, reflect this second dichotomy. Another classification scheme (Kuhs & Ball, 1986) posits four dominant views on how mathematics can be taught: content-focused with an emphasis on performance (procedural); content-focused with an emphasis on understanding (conceptual); classroom-focused where the focus is on mathematical content through classroom activity; and learner-focused where mathematics teaching focuses on the learner's personal construction of knowledge. The items written for this questionnaire reflected the classification scheme suggested by Kuhs and Ball (1986).

There is a paucity of research on teacher's beliefs about the uses of assessment. Clarke, Clarke and Lovitt (1990) claim that the major uses of assessment focus on three areas, the teacher, the student, and the parent. First, the teacher uses assessment to improve instruction by using students' responses to help identify (i) instructional strategies that are most successful; and, (ii) student learning behaviours that need to be "encouraged and developed or discouraged and replaced". Second, assessment informs students of their identified strengths and weaknesses and informs subsequent teachers of students' competencies. Finally, parents are informed of their child's behaviour so that they can give more effective support. The NCTM Assessment Standards (1995) delineates four purposes for assessment. These are promoting student growth, improving instruction, recognising accomplishments, and modifying programs. These purposes are teacher-learner focused. The items developed for the questionnaire classified assessment as informing teaching, assessment as informing learning, and assessment as a tool for reporting to external parties such as parents, other teachers and the general public.

The specific aim of this study was to identify factors in the beliefs of mathematics teachers that affect the teaching and assessment of mathematics.

Method

Many investigations of the relationship between teachers' beliefs and practices have been qualitative studies of a small number of teachers, however a number have been conducted using quantitative methods with large samples of teachers. This study falls into the latter category.

Instrument

A 56-item Likert-scale survey was designed for a large comprehensive study. The questionnaire consisted of five sections relating to beliefs about mathematics, beliefs about teaching mathematics, beliefs about the purposes of assessment, beliefs about the nature of assessment, and assessment practices. The questions used in the study were based on current literature on beliefs about mathematics, teaching mathematics, the purposes of assessment and the nature of assessment. This paper reports on the first four sections of the survey (46 items). Teacher were also asked to indicate their gender, year level at which they taught, level of qualification in mathematics (Year 10, Year 12, teacher education course in mathematics, University mathematics) and years of experience in teaching (0-5 years, 6-10 years, 11-15 years and >15 years).

Participants

Fifteen hundred forms were sent to a random selection of primary teachers representing different school year levels (one through to seven), school systems (government, catholic), socio-economic areas (high and low), and geographic locations (metropolitan, provincial, and rural). Although the return rate was low, (27%, $n = 398$) the resulting sample was representative of the different systems, socio-economic areas, and geographic locations. An analysis of the sample indicated that the schools served a wide range of communities. The two groups of teachers (State schools, $n = 179$; Catholic Schools, $n = 195$) did not differ significantly on the level of mathematics training they reported ($\text{Chi-square}_{(3)} = 7.19$, $N = 374$; $p = 0.07$). Nor did the sub-samples differ in their reported years of teaching ($\text{Chi-square}_{(3)} = 0.63$, $N = 387$; $p = 0.89$) or gender ($\text{Chi-square}_{(1)} = 0.05$, $N=383$; $p = 0.83$). Sample sizes in

the State school teachers ranged from 17 (Year 6) to 35 (Year7) and in the Catholic school group the range was from 26 (Year 6) to 31 (Year 7). No statistically significant difference was found between the two School system subgroups in the representation of Grades taught. The returned surveys also reflected a well-balanced distribution of school year level as summarised in Table 1.

Table 1
Percentage Distribution of sample by school year level

	School Year Level						
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Percentage of teachers	13.6	14.7	15.7	12.3	12.1	11.1	17.0

Results and Discussion

The responses to items relating to beliefs about mathematics, beliefs about teaching mathematics, and beliefs about the purposes of assessment (Sections 1, 2, & 3 of the survey) were subjected to a factor analysis. Initially, there were 15 factors with eigenvalues > 1 . Given the exploratory nature of the study and guided by the interpretability of the factors, a seven-factor orthogonal solution after the extraction of principal components and a Varimax rotation was accepted. The seven-factor solution accounted for forty-one percent of the variance. Thirty-one of the 46 items were used to delineate the factors. The solution is presented in Table 1. The naming of factors was guided by the content of the items identified with the factor. The factors were tentatively named as follows:

- Factor 1: Assessment is used to evaluate teaching
- Factor 2: Assessment is used to evaluate learning
- Factor 3: Assessment is used for accountability purposes
- Factor 4: A static view of mathematics
- Factor 5: Mechanistic view of mathematics
- Factor 6: A traditional view of teaching mathematics
- Factor 7: A contemporary view of teaching mathematics

The results for Factor 1, Factor 2 and Factor 3 (the three factors relating to beliefs about the uses of assessment) are summarised in Table 2.

Table 2
Factors related to assessment

Item	Item description	Loading
Factor 1: Assessment is used to evaluate teaching		
<i>I use assessment to:</i>		
36j	help me evaluate how effective my teaching has been	0.75
36f	judge how well the class is progressing	0.72
36e	help me identify students with problems	0.69
36g	help me plan the next phase in teaching	0.68
36i	inform me about the ability levels of the students	0.65
Factor 2: Assessment is used to evaluate learning		

<i>I use assessment to:</i>		
36a	inform the students about what they do not know	0.75
36c	encourage students to learn their work	0.75
36d	provide information for successful students	0.70
36h	to inform me about which students are working	0.56
36b	give students feedback on their strengths and abilities	0.55
Factor 3: Assessment is used for accountability purposes		
28	The main reason why I assess student performances is to inform parents	0.76
35	The main reason why I assess is to meet the school's requirements	0.68
37	I tend more to test rules and facts than have students solve problems	0.35

The uses of assessment seemed to fall into three main categories, assessment used to evaluate teaching, assessment used to evaluate learning, and assessment for accountability. The first factor seems to reflect a feedback component and a planning component, supporting two of the purposes of assessment delineated in the NCTM Standards, namely, improving instruction and modifying programs.

The second factor seems to support the notion of assessment promoting student growth and recognising accomplishment. There also seems to be a component that views learning as closely linked to working, "I use assessment to inform me about which students are working".

Of interest is the inclusion of the item 'I tend more to test rules and facts than have students solve problems' in the third factor. Does this imply that if teachers are assessing for an outside audience it is easier to quantify and justify rules and facts than it is to report students' problem-solving ability? What influence does parental expectations have on teachers' choice of assessment techniques? These points are worthy of further investigation.

Table 3 summarises the results for Factors 4, 5, 6, 7, the factors pertaining to beliefs about mathematics and the teaching of mathematics.

Table 3

Factors related to views of mathematics and teaching mathematics

Item	Item description	Loading
Factor 4: A static view of mathematics		
2	The good thing about mathematics is that it is an unchanging subject	0.83
3	The advantage of mathematics is that things are either right or wrong	0.70
15	Today's mathematics is no different from mathematics of long ago	0.41
1*	Mathematics is dynamic. It is a searching for patterns in the environment	-0.35
26	Students learn best by doing lots of exercises and practice	0.34
Factor 5: Mechanistic view of mathematics		
19	Mathematics is essentially about computation	0.62
20	The problem with low achieving students is that they don't learn the rules	0.57
11	I tend more to test rules and facts than have students solve problems	0.50

Factor 6: A traditional view of teaching mathematics		
14	The main tools I use for gathering data are timed tests rather than projects and investigations	0.75
34	It is difficult to plan for hands-on experiences in mathematics lessons	0.68
9*	In mathematics I teach interesting things about the world outside school	-0.56
Factor 7: A contemporary view of teaching mathematics		
17	It is important for student to use concrete materials in mathematics	0.69
16*	What I teach in maths should have little to do with students' out of school life	-0.67
5*	It is not worth spending time on collecting observational data on students	-0.55
10	Students learn better through solving problems in real world contexts	0.52
6	Students who do well on standard exercises don't necessarily do well on tests	0.45
7*	Students are encouraged to explain their ideas and teach each other	-0.36
33*	It is very important that children get the right answers to exercises	-0.32

Note. *Items with negative loadings

Primary teachers' beliefs about mathematics seemed to fall into two broad categories, a static view of mathematics and a mechanistic view of mathematics. These factors reflect Ernest's (1989) categories of mathematics as a static unified body of knowledge and mathematics as a bag of tools made up of an accumulation of facts, rules and skills. Missing is the category reflecting a dynamic problem-driven view of mathematics even though there were items in the questionnaire that were believed to reflect this view. This is a concern for all mathematics educators as this is the view strongly reflected in many curricular documents and *The National Statement on Mathematics for Australian Schools* (AEC, 1990).

The two factors related to beliefs about teaching mathematics reflect a traditional view and a contemporary view of teaching mathematics. The traditional view of teaching mathematics implies not using hands-on materials, not teaching about interesting things about the world outside schools and using timed tests to assess teaching and learning. It appears that for some teachers, teaching mathematics occurs in a fairly sterile environment. This style of teaching could be related to a transmission model of teaching, although these particular items do not necessary reflect a teacher-centred classroom. Of interest would be what these particular teachers do rather than what they do not do. It is difficult to imagine how such teachers could believe in a constructivist approach to teaching mathematics. But do they focus their teaching on developing procedural understanding (an emphasis on performance) or are they concerned with developing conceptual understanding (an emphasis on mathematical content through classroom activity); are they 'show-and-tell' teachers or 'explaining' teachers? They certainly do not seem to be too concerned about making mathematics real-world related.

The contemporary view of teaching mathematics seems to reflect some of the elements of a constructivist approach to mathematics (Burton, 1993) or a child-centredness approach to mathematics (Perry, Howard, & Tracey, 1999). Of interest is the negative loading on 'Students are encouraged to explain their ideas and teach each other'. This may simply reflect the wording of the question and the fact that it consists of two components, explaining ideas and teaching each other. Teachers may believe that students need to explain their ideas but do not believe that students should teach each other. This issue needs further investigation.

Conclusion

This research begins to delineate some of the beliefs held by primary school teachers with regard to mathematics, teaching mathematics and the purposes of assessment. It was found that the factors identified with these data did reflect the theoretical constructs proposed initially. The groups of factors were associated with beliefs about (i) mathematics, (ii) teaching mathematics and (iii) purposes of assessment. Furthermore, within these groups, the separate factors were closely aligned with the theoretical components.

First, in relation to the nature of mathematics, two of the views proposed by Ernest (1989) – a static unified body of knowledge and a bag of tools - were evident in Factors 4 and 5 – a static view and a mechanistic view, respectively. It is of concern however that the third view – a dynamic problem-driven view – was not evident in the results of the factor analysis. It seems that primary teachers hold a fairly limited view of what mathematics is. This result has implications for professional development. Mathematics as a dynamic problem-driven ever-expanding field of human creation and invention seems more aligned with the constructivist model of learning, and if more teachers are to adopt teaching in the light of a constructivist view of learning, then their views of mathematics will need to be addressed along with issues of pedagogy, as noted by Borko (1997).

Second, the factors relating to teaching mathematics closely reflect the teaching styles defined by Burton (1993) and Perry, Howard and Tracey (1999), but only two categories defined by Kuhs and Ball (1986). These factors seem to reflect the types of environments that mathematics teaching tends to occur in - a traditional environment with few materials and unrelated to the real world, and a contemporary environment where hands on materials are valued, mathematics is related to out-of-school experiences, and problem solving is encouraged. The beliefs that teachers hold about teaching in these two differing environments are not clearly delineated in the responses. Perhaps further refinement of the items in the survey may be worthwhile in teasing out the different styles of teaching. If not, there may be reason to suspect that a two-category system is more appropriate than a four-category system.

Third, with regard to assessment, the three factors identified were closely aligned with the three purposes proposed initially – to evaluate teaching, to evaluate learning, and for accountability purposes. One question which arises from the results relates to the last factor: What role do outside bodies such as parents play in beliefs about assessment practices and the choices teachers make about how to assess learning? This warrants further study. Given the close relationship between beliefs and effective professional development (Borko & Putman, 1996), such issues need to be addressed if one wishes to change classroom practice.

This paper reports on the ongoing development of an instrument to identify and measure factors associated with primary teachers' purported beliefs about mathematics and teaching and assessing mathematics. While the questionnaire has resulted in clearly defined and interesting factors, as indicated in the results, the items need further refinement. The limitations of this type of research are well documented in the literature, and include the fact that what teachers believe does not necessarily reflect their classroom practice. Past research has indicated that this tension usually occurs between more liberal beliefs and more conservative practice rather than vice versa. Thus the factors delineated in this paper possibly reflect a "best scenario" situation.

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