

Emergence of Mathematical (In)Competence and Identity

Pat Forster

Curtin University of Technology
<forster@iinet.net.au>

Peter Taylor

Curtin University of Technology
<itaylorp@info.curtin.edu.au>

This paper is an inquiry into the development of mathematical self-concept or how students perceive themselves in relationship to mathematics. The inquiry is centred on two classroom episodes which involve changes in the nature and direction of conversation between students and the teacher. We suggest that if the episodes were typical of patterns of interaction in a class, they might significantly impact on the students' perception of their own competence in mathematics.

Introduction

In this paper we explore the notion of mathematical identity or self-concept (Jungwirth, 1996). The inquiry is set in the context of a Year 11 Geometry and Trigonometry class who were studying a topic on vectors. Taking a situated view of learning, we interrogate classroom dialogue for potential influences on students' perceptions of self, drawing on excerpts of conversation that we see illustrate (a) the attribution in social interaction of (in)competence and (b) the demands of two students being responded to over the needs of another.

Our situated perspective draws on social constructivist theory, especially on the *emergent perspective* of Cobb (1998), Cobb, Boufi, McClain and Whitenack (1997) and Cobb, Gravemeijer, McClain and Whitenack (1997). Accordingly, we see learning, including how students' perceive they stand in relationship to each other and to mathematics, to involve both social and cognitive processes. However, our focus in this paper is on the social dimension of learning where, consistent with social constructivism, we assume that the social practices in the class were founded on the teacher's and students' individual experiences (and understanding of those experiences) prior to coming together as a group, and continually evolved in their ongoing classroom activities.

Our intentions in writing the paper are (a) to provoke, you, the reader into considering the pedagogical and instructional implications of the episodes that we present and (b) to contribute to the debate on how to improve mathematics teaching/learning--by considering an area which seems to be given relatively little attention in the literature: namely, the development of mathematical identity. While writing the paper, we were aware that, while students' mathematical identities are constructed through their doing mathematics, our own identities as researchers are shaped by the decisions we make and actions we take at every stage of the research act. Therefore, we make explicit our choices and the limitations, as we came to see them, of our methodology. We see this as a way of highlighting our concern with representing fairly the teachers and students who agree to participate in our research.

Research Method

The site of the research was a Year 11 mathematics class of 18 students in a private college for girls in Western Australia. Pat (first author) took the role of participant-observer (Atkinson & Hammersley, 1994), to observe whole-class discussion and serve as an assistant teacher during small-group work, over one month. The main source of data for this paper was audio-recorded classroom conversation. Three small audio-recorders captured, we assume, most of what was said by seven students in small-group work, as well as the voices of the teacher (Mr C.) and the students during whole-class discussion. Other data were Pat's field

notes, journal entries, a video-recording of each lesson from a corner of the classroom, photocopies of students' written work and students' responses to a questionnaire ascertaining perceptions of their own learning (Forster, 1999).

Our research methodology (like the perspective we took on students' learning) was based on social constructivist principles: the inquiry involved personal, reflective sense-making and critical dialogue (by email) between us--centred on our written interpretation and successive reinterpretations of events. Thus, our analysis was motivated by writing:

writing as a method of inquiry ... moves through successive stages of self-reflection ... the field-workers texts flow from the field experience, through intermediate works, to later work, and finally to the research text ... Thus do fieldwork and writing blur into one another. (Denzin and Lincoln, 1994, p. 10)

The classroom episodes presented in the paper were purposively selected (Cohen & Manion, 1994) on the basis that they illustrate students interacting in problematic ways with each other and the teacher: ways that potentially could impact on the formation of self-concept. With the text of the episodes in front of us, we chose narrative as the genre for the inquiry and decided to write in the first person 'my' and 'I' (who is Pat--as participant-observer in the Year 11 class and principal writer of our co-constructed analysis). These choices allowed the interpretative nature of inquiry to be brought to the fore and brought the inquiry into the realm of fifth generation qualitative research (Lincoln & Denzin, 1994) with its issues of representation and legitimation. We address these issues now, albeit briefly because of the restricted length of the paper.

In presenting classroom life through only two short episodes, I am sharply aware that they represent to a very limited extent how 'it really was' in the class. Individual students would at one time be supportive of others, at other times be dismissive, so the excerpts of conversation--plucked from their context of 'dialogue over time'--are not to be read as portraying regular classroom practices.

In addition, I do not claim that my analysis accurately reflects actual effects on self-concept as experienced by students, immediately or in the long term. Even while an observer in the class, my understanding of what students were thinking and experiencing was, as for all estimations of 'the other', at best second order (Steffe, 1995)--an interpretation of their interpretation of events. To overcome this limitation, Guba and Lincoln (1989) recommend that analysis is member checked. However, I did not adopt this option for an important reason. I felt it would be unethical to present the students with analyses which they could take as criticism of the teacher. Besides, I do not claim that the inferences made are accurate but instead propose them as possibilities. Therefore the quality criterion which I pursued is that the analysis is *practically reasonable* (Fenstermacher, 1994)--in the opinion of others. To meet this criterion, interpretation is the consensual view of both authors; as well, analysis involved critical reinterpretation for progressive subjectivity, and I asked a teaching colleague (of English Literature) as well as the teacher of the Year 11 class (Mr C.) to read the paper. They did not disagree with the inferences drawn.

Theoretical Framework

I see students' learning largely in terms of social constructivist principles: that learning involves processes of active individual construction and, simultaneously, processes of enculturation (Cobb, Gravemeijer, et al., 1997). Social constructivism draws attention to cognitive development and to social processes: they are seen to literally not exist without each other, and to be reflexively, although indirectly related (ibid). Piaget's reflective abstraction, or *recognition* of regularities, for example, of what it means to add or multiply, links cognitive and social activity (Cobb, Boufi et al., 1997). So, in a social constructivist account, merely being 'carried along' by mathematical discourse does not automatically constitute individual understanding.

From a social constructivist perspective, mathematical discourse, as well as mediating students' conceptualisation of mathematical relationships, is characterised by what I think of as a second layer of regularities and patterns. Cobb (1998) refers to these regularities as *social norms* and *sociomathematical norms*. Bauserfeld (1995) describes how the genesis of these regularities is "covert and subconscious" (p. 155): they start with "routinized definition" of a situation, although the patterns emerge interactively and are influenced by all participants. Social norms can be subject to intervention so that the teacher might encourage students in particular types of behaviour, for example, to always answer with extended explanations. Gradually these behaviours, as individuals *recognise* them, can emerge to be accepted as normal among the class (e.g., Cobb, 1998; Cobb, Boufi, et al., 1997),

Thus, my reading of social constructivist theory led me to view mathematics learning as (a) conceptual development, or the student building on prior knowledge, reflectively, through social activity; and (b) as coming to know how to act in relationship to others, or the student building on previously known social practices, subconsciously and interactively in participation with others.

Self-concept

My inquiry was informed by a number of other studies that address students' self-concept, and these were based on a range of methodologies. Jungwirth (1996) uses discourse analysis to explore gender-related 'differences in confidence', in regard to learning mathematics. Klein (1999) draws on her observations of primary school classrooms to consider how constructivist teaching approaches might mediate against students' confidence and disposition towards mathematics. Roth (1998) uses phenomenological accounts by a female student and her male teacher to depict how the student felt about herself, and how she felt the teacher perceived her. Coulter (1999) uses stories of adults who were held back to repeat a grade as the basis for his inquiry into the long-term effects--on self-concept--of streaming. Murray & Whitehead (2000) provide an autobiographical account of Murray's struggle, as a person of mixed race, to establish his own identity.

The order I have chosen to list the above references is intentional and is according to the methodologies, which range from researchers commenting on self-concept from a viewpoint that is external and distanced from the students, to a student account of what identity is and meant to him. I see that each of the methodologies can alert us to different aspects of mathematical identity or self-concept. However, in offering the following analyses, I acknowledge again the limitations of my external, researcher's point of view.

The Attribution of Competence

March 2nd. It was the first lesson on vectors for the Year 11 class and my first day of observing the class. Mr C. started by introducing the terms vector, component, vector magnitude, and vector direction, and then specified new notations, including the graphics calculator function ABS for finding (vector) magnitude (see Figure 1).

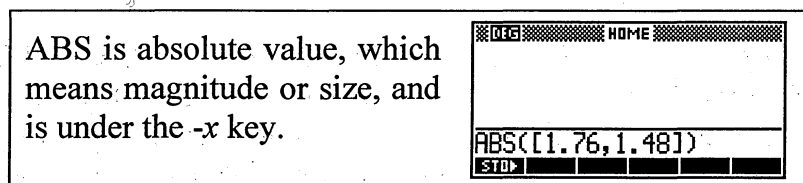


Figure 1. Extract from the worksheet showing instructions for finding magnitude.

I watched the action and then, when students started on questions from their text, I moved among them, looking at their work and talking with them. After a while, I heard some excitement and then a laugh--it was Katie and Jenny explaining something to the teacher. They had programmed their calculators to automate the calculation of absolute value (see Figure 2). Mr C. asked Jenny if she would explain the applet or program to the class at the end of the lesson. She said she would.

1. Mr C: ... Right, now we have stopped, Jenny has volunteered to tell us
2. something that she has discovered. So come up Jenny.
3. Katie: Yea. Go Jen.
4. Jenny: ... and I put in a little applet ... so the applet I got was, I put in ABS
5. brackets then 'I' and 'J', and I just said 'M' for magnitude ...
6. Mr C: Okay. Would everyone like to try it? Does everyone think it is a good
7. idea?
8. Ann: I think it's brilliant.

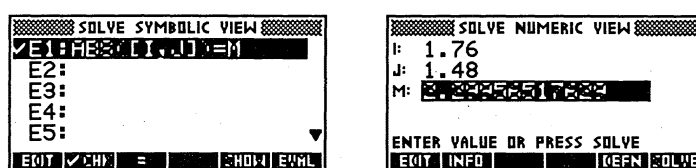


Figure 2. The symbolic and numeric screens for the ABS Applet.

So, Jenny had "discovered" (line 2) something, it was a "good idea" (line 6-7), in fact it was "brilliant" (line 8). And I was impressed. That evening I transcribed the tape to see how the discovery had happened. This is what was recorded:

9. Jenny: Katie, do you reckon you could put this into our G and T things, if you went
10. 'ABS brackets i j'?
- [pause]
11. Katie: You have to make an M.
12. Jenny: Yes. That would be okay. M could work. Then I could go '= M' [see Figure 2]

Later:

13. Mr C: How did you do those ones?
14. Jenny: We made an applet.
15. Mr C: Did you?
16. Jenny: Oh we used M because we couldn't use the little line bars [the || symbol
17. for magnitude]. We just did ABS, then I and J.
18. Mr C: Show me how it works.
19. Jenny: It works like this. You just enter I and J.
20. Mr C: Would you like to show everyone else?
21. Katie: [laughed]
22. Mr C: I think it is pretty clever
23. Katie: It's Jenny's idea.

Later:

24. Jenny: Yea. Back to my applet.

I am wondering if you, the reader, have the same sense of surprise that I experienced on reading the text. The first thing I noticed was that Jenny changed from using the inclusive "we" (line 14, 16-17) to the personal "my" (line 24), thus changing from expressing shared ownership to claim sole ownership of the applet. She continued using "I" (line 4) in explaining the applet to the class, even though it was Katie who had suggested "You have to make an 'M'" (line 11). ['=M' was needed for the program to run.] However, it was Katie herself who assigned all credit to Jenny with "It's Jenny's idea" (line 23). At this signal the teacher also

moved from using what I take was the inclusive "you" (line 13, 15) to promote the aplet to the class as Jenny's discovery (line 2).

This episode brought to life for me the observation of Erickson (1986) that "assessed ability is socially constructed ... rather than an attribute of the person" (p. 125). Jenny's ability was socially assessed as "pretty clever" (line 22), as up to making discoveries (line 2), "good" (line 6); and "brilliant" (line 8). Wouldn't such descriptors mediate students' perceptions of themselves (as mathematicians) in a favorable way? Conversely a repeated pattern of non-recognition might mediate against students' beliefs in their own capabilities.

In reading the episode, I recalled Davis's (1996) recommendation for teaching to involve a careful "hermeneutic listening" (p. 261): a listening to the text of students' conversations so that, for example, answers in terms of "we" (line 14, 16) are taken to imply joint effort. The episode also raises issues of what responsibilities students have to each other when working together, especially if shared work is brought into the public domain; and how might awareness of those responsibilities be fostered?

A Turn in Conversation

March 12th. It was the beginning of the lesson and a student, Amanda, asked Mr C:

1. Amanda: You know how you gave us an example where forces were going in the
2. opposite direction.
3. Mr C. Like that (drawing a diagram on the board--see Figure 3)?

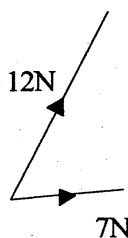


Figure 3. Diagram showing 12N and 7N forces.

4. Amanda: Yes. Sort of. Can you still solve it like that or do you have to move the vector (her
5. voice dropped away)?
6. Mr C: ... I think the question Amanda is asking, can we resolve the two vectors where
7. they are or do we have to shift one of the vectors and join it onto the end of the other
8. one. When you did the questions last night, what method did you use (to the class)?
9. [pause] Jenny?
10. Jenny: I put the 7 Newtons on the end of the 12 Newtons one.
11. Mr C: So you did the head to tail kind of thing?
12. Jenny: Yes.

There was more discussion, then Mr C. asked a student to come up and draw the head tail diagram on the board. He specified an angle of 40° between the forces then asked students to solve the problem:

13. Mr C: Everyone tell me what the resultant is. In component form.
14. Katie: In component form?
15. Jenny: Do we have to use component form?
16. Mr C: Oh sorry. You can or not.
17. Jenny: Can't we just use the cosine rule?
18. Mr C: Who prefers to use cosine rule?
19. Jenny & Katie: Yes. It's so much quicker.
20. Amanda: But we want to learn how to do vectors [the component method].
21. Mr C: You want to learn how to do vectors? You want to learn the other way?
22. ... Just get the resultant. Use whatever way you choose. You choose.
23. Katie (to Jenny): I don't know how to do the vector way.
24. Jenny: Well, you would use the 12 in components and that one [the 7 Newton vector].

Later, using student feedback which relied on the cosine rule, Mr C. wrote the solution on the board. When everyone was working again, Amanda asked him on a one-to-one basis about the problem. He discussed it with her.

My first interpretation of the episode is that it was a 'war of words' (Lakoff & Johnson, 1980), but it isn't clear to me if anyone perceived others as the enemy. The first shot was fired with Katie's challenge (line 14). Jenny quickly followed with "Do we" (line 15), then "Can't we" (line 17), which could be seen to have consolidated their position. The strength of the onslaught meant it was a struggle for the mediator Mr C. to keep his neutral stance (line 16, 18). Amanda attempted to restate her case but no-one supported her, despite her plea with the inclusive "we" (line 20). However, that anyone was a winner was questionable (line 23), especially as the final outcome was that a problem-solving method was used which was already very familiar to students.

My second interpretation is that the dialogue was 'dance' (Lakoff and Johnson, 1980). The problem was that when Amanda tried to join the company she only 'sort of' knew the set routines and didn't know how to ask the right questions (line 4). As often happens in this situation, someone will step in and ask a question for you, but mightn't get it quite right (line 6-8). Then, all you can do is stand at the side and watch the choreography and it sometimes ends up that you can't remember the elements of the question that you originally wanted to ask (line 20).

My third interpretation is that the dialogue was building self-concept and I ask, did Jenny and Katie learn that two people can achieve more than one? Did Amanda learn that it can be lonely and frustrating in a mathematics classroom? Would she have felt more empowered if she'd been invited to restate at the beginning her original question (line 4)? Did the practical action of the teacher of asking a student who could be relied upon to answer (line 9) further constitute her own sense of competency and at the same time mediate against that of others?

I want to remind you that "metaphor provides a way of *partially* communicating unshared experiences" (Lakoff & Johnson, 1980, p. 225). So, to make it clear, I did not consider that the classroom was a battlefield, but I did see that some students joined the dance more often than others. So, if we say that a sense of agency in a mathematics classroom arises from construction of a learner as one with the right to speak and be heard (Klein, 1999), then which learners are asked or offer to speak needs to be in teachers' awareness. Also relevant to the situation is the importance of helping students develop linguistic skills so that participation is open to them (Cobb, 1998; Taylor, 1996), and that learning in co-participation with others implies a concern for others' learning, which needs to be brought into students' awareness (Tobin, 1998).

Conclusion

In re-viewing the two episodes a theme that emerged is how students work in relationship with each other. In the first, I cast as problematic Jenny and not Katie receiving public acclaim for the design of the applet, and suggested that students have responsibilities when they choose to work together. In the second, I questioned that Amanda's query was lost in the classroom debate, and highlighted that students also have responsibilities towards each other in the whole-class domain. The implication for teaching is that recognition of responsibilities needs nurturing, and this might be achieved for example, by making ways of acting the subject of discussion.

Both episodes display the crucial role a teacher plays in regards to who speaks and the direction that discussion takes. I identified the practices of the teacher filling in questions for students and choosing students who can be relied on to answer, and that these practices might need to be scrutinised, even though they are practical in so far as keeping the discourse moving. I take these practices as examples of social norms to which Cobb (1998) refers. They

become part of the classroom culture. The implication for teaching is that it is desirable for teachers to be critically aware of their ways of responding to students and that these ways might subtly impact quite significantly on students' perception of their own capabilities. Critical awareness might be achieved through systematic reflection on practice (Schön, 1987)-through which I uncovered some previously unrecognised regularities in my own teaching!

In the analysis I have been careful not to impute intention, by any of the students or the teacher. In other words I don't believe there was purposeful commendation or recognition of some students and not others, by the teacher and by students of each other. They were carried along by the discourse.

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