Understanding, Assessing, and Developing Young Children's Mathematical Thinking: Research as a Powerful Tool for Professional Growth

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This paper describes an ongoing research and professional development project involving 70 Victorian primary schools (1999-2001), seeking to identify processes for supporting and enhancing mathematics learning in the early years of school. The project involves three main components: the development and refinement of a set of research-based "growth points" in mathematical understanding in various mathematical domains; the creation and use of a one-to-one, task-based assessment interview with all children twice a year; and a multi-level professional development program. In this paper, the characteristics and effects of each of these three components is discussed. Data is presented on growth in student understanding across the mathematical domains and grade levels (K-4). A discussion of the professional growth of project teachers and the view of the teacher underpinning the research is also given.

The Early Numeracy Research Project (ENRP) was initiated in Victoria following the success of the Early Literacy Research Project. The Early Literacy Research Project (Hill & Crevola, 1998) worked with 27 disadvantaged Victorian primary schools to bring about substantial improvements in early literacy outcomes. Part of this research involved the development of models and guidelines for teaching, assessment and additional support for young children learning to read. As a result of the research, Hill and Crevola offered a "general design for improving learning outcomes" (p. 122), which they believed had application in literacy, numeracy, and other curriculum areas. The nine elements of the design are leadership and coordination; standards and targets, monitoring and assessment, classroom teaching programs; professional learning teams; school and class organisation; intervention and special assistance; home, school and community partnerships; and beliefs and understandings.

The Early Numeracy Research Project was established in 1999 by the (then) Victorian Department of Education, with similar aims to those of the Early Literacy Research Project, but with a P-2 mathematics focus. The ENRP is now a collaborative venture between Australian Catholic University, Monash University, the Victorian Department of Employment, Education and Training, the Catholic Education Office (Melbourne), and the Association of Independent Schools Victoria. The project is funded to early 2002 in 35 project ("trial") schools and 35 control ("reference") schools (for details, see Clarke, 1999; Clarke, 2000; Clarke & Cheeseman, 2000; Clarke, Sullivan, Cheeseman, & Clarke, 2000; Gervasoni, 2000).

Important differences from the literacy project included the need for development of a comprehensive and appropriate learning and assessment framework for early mathematics (such frameworks were well established for reading), and the need to address the personal confidence with, and understanding of mathematics of many primary teachers.

The Three Main Components of the Early Numeracy Research Project

The three main components of the ENRP are closely related to three terms used in the title of this paper. The framework of growth points provides a means for *understanding* young children’s mathematical thinking in general, the interview provides a tool for *assessing* this thinking for particular individuals and groups, and the professional development program is geared towards *developing* further such thinking. In the following sections, each of these three components is discussed in turn.

It should be noted that, for brevity’s sake, the term “understanding” is used in much of this paper. We are usually speaking of students’ knowledge, skills and understandings, but even this, as argued by Kilpatrick, Swafford and Findell (2001), is somewhat limited in terms of an appropriate focus. Their preferred term “mathematical proficiency” includes conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Of course, these terms in turn need appropriate definitions, but all five aspects are of interest in the ENRP.

The ENRP Learning and Assessment Framework

The impetus for the Early Numeracy Research Project was a desire to improve mathematics learning and so it was necessary to quantify such improvement. It would not have been adequate to describe, for example, the effectiveness of the professional development in terms of teachers’ professional growth, or the children’s engagement, or even to produce some success stories. It was decided to create a framework of key “growth points” in numeracy learning. Students’ movement through these growth points in trial schools could then be compared to that of students in the reference schools.

The project team first came across the term “growth points” in the work of O’Toole, Rubino, Parker, and Fitzpatrick (1998), and discussions with members of that team from the Catholic Education Office (Adelaide) were most helpful in considering aspects of the measurement domains of the framework. The earliest use of this term (to our knowledge) was by Pengelly (1985).

The project team studied available research on key “stages” or “levels” in young children’s mathematics learning (e.g., Bobis, 1996; Boulton-Lewis, 1996; Clements, Swaminathan, Hannibal, & Sarama, 1999; Fuson, 1992; Lehrer & Chazan, 1998; McIntosh, Bana, & Farrell, 1995; Mulligan & Mitchelmore, 1995, 1996; Owens & Gould, 1999; Pearn & Merrifield, 1992; Thomas, 1996, Wilson & Osborne, 1992; Wright, 1998; Young-Loveridge, 1997), as well as frameworks developed by other authors and groups to describe learning.

A major influence on the project design was the New South Wales Department of Education initiative *Count Me In Too* (Bobis & Gould, 1999; NSW Department of Education and Training, 1998) that developed a learning framework in number (Wright, 1998). It was soundly based on prior research and, in particular, on the stages in the construction of the number sequence (Steffe, Cobb, & von Glaserfeld, 1988; Steffe, von Glaserfeld, Richards, & Cobb, 1983), and it formed the basis of an individual interview designed to measure children’s learning against the framework.

In developing the ENRP framework it was intended that the framework would

- reflect the findings of relevant research in mathematics education from Australia and overseas;
- emphasise important ideas in early mathematics understanding in a form and language readily understood and, in time, retained by teachers;
- reflect, where possible, the structure of mathematics;
- allow the description of the mathematical knowledge and understanding of individuals and groups;
- form the basis of planning and teaching;
- provide a basis for task construction for interviews, and the recording and coding process that would follow;
- allow the identification and description of improvement where it exists;
- enable a consideration of those students who may benefit from additional assistance;
- have sufficient “ceiling” to describe the knowledge and understanding of all children in the first three years of school; and
- build on the work of successful, similar projects such as *Count Me in Too.*

These principles informed the process of developing and refining the framework to the form it takes in 2001. It continues to be regarded as “work in progress”. Not all possible mathematical domains were included. The decision was taken to focus upon the strands of *Number* (incorporating the domains of Counting, Place value, Addition and subtraction strategies, and Multiplication and division strategies), *Measurement* (incorporating the domains of Length, Mass and Time), and *Space* (incorporating the domains of Properties of shape, and Visualisation and orientation).

Within each mathematical domain, growth points were stated with brief descriptors in each case. There are typically five or six growth points in each domain. To illustrate the notion of a growth point, consider the child who is asked to find the total of two collections of objects (with nine objects screened and another four objects). Many young children “count-all” to find the total (“1, 2, 3, . . . , 11, 12, 13”), even once they are aware that there are nine objects in one set and four in the other. Other children realise that by starting at 9 and counting on (“10, 11, 12, 13”), they can solve the problem in an easier way. *Counting All* and *Counting On* are therefore two important growth points in children’s developing understanding of Addition.

The six growth points for the domain of *Addition and subtraction strategies* are shown in Figure 1. These growth points informed the creation of assessment items, and the recording, scoring and subsequent analysis, as is discussed in later sections.

1. Count-all (two collections)
   Counts all to find the total of two collections.
2. Count-on
   Counts on from one number to find the total of two collections.
3. Count-back/count-down-to/count-up-from
   Given a subtraction situation, chooses appropriately from strategies including count-back, count-down-to and count-up-from.
4. Basic strategies (doubles, commutativity, adding 10, tens facts, other known facts)
   Given an addition or subtraction problem, strategies such as doubles, commutativity, adding 10, tens facts, and other known facts are evident.

5. Derived strategies (near doubles, adding 9, build to next ten, fact families, intuitive strategies)
Given an addition or subtraction problem, strategies such as near doubles, adding 9, build to next ten, fact families and intuitive strategies are evident.

6. Extending and applying addition and subtraction using basic, derived and intuitive strategies
Given a range of tasks (including multi-digit numbers), can solve them mentally, using the appropriate strategies and a clear understanding of key concepts.

Figure 1. ENRP growth points for the domain of addition and subtraction strategies.

In discussions with teachers, we have come to describe growth points as key "stepping stones" along paths to mathematical understanding. They provide a kind of conceptual landscape. However, we do not claim that all growth points are passed by every student along the way. For example, one of our growth points in Addition and Subtraction involves "count-back", "count-down-to" and "count-up-from" in subtraction situations, as appropriate. But there appears to be a number of children who view a subtraction situation (say, 12-9) as "what do I need to add to 9 to give 12?" and do not appear to use one of those three strategies in such contexts.

The interpretation of these growth points reflects the description by Owens and Gould (1999) in the *Count Me In Too* project: "the order is more or less the order in which strategies are likely to emerge and be used by children. ... Intuitive and incidental learning can influence these strategies in unexpected ways" (p. 4).

In discussing "higher" level growth points in a given domain, the comments of Clements, Swaminathan, Hannibal, and Sarama (1999) in a geometrical context are also helpful: "the adjective *higher* should be understood as a higher level of abstraction and generality, without implying either inherent superiority or the abandonment of lower levels as a consequence of the development of higher levels of thinking" (p. 208). Similarly, Konold, Khalil, Higgins, & Russell (2001), proposed five perspectives children take in reasoning about data, and described them as follows:

> These categories form a hierarchy of sorts, where a higher level subsumes or encapsulates lower ones. Different contexts may cue different views of data even within the same student. . . . Thus we see these not as levels or perspectives to graduate from, but rather to master. (p. 1)

Also, the growth points should not be regarded as necessarily discrete. As with Wright's (1998) framework, the extent of the overlap is likely to vary widely across young children, and "it is insufficient to think that all children's early arithmetical knowledge develops along a common developmental path" (p. 702).

The ENRP Task-Based Assessment Interview

A major feature of the project is a one-to-one interview with every child in trial schools and a random sample of around 40 children in each reference school at the beginning and end of the school year (February/March and November respectively), over a 30- to 40-minute period. The disadvantages of pen and paper tests have been well established by Clements (1995) and others, and these disadvantages are particularly evident with young children, where reading issues are of great significance. The face-to-face interview is an appropriate response to these concerns. The interviews are conducted by the regular classroom teacher in trial schools, and a trained team of interviewers in reference schools. A range of procedures has been developed to maximise consistency in the way in which the interview is administered across the 70 schools.

Although the full text of the ENRP interview involves around 60 tasks (with several sub-tasks in many cases), no child moves through all of these. The interview is of the form of a "choose your own adventure" story, in that the interviewer makes one of three decisions after each task, as instructed in the interview schedule. Given success with the task, the interviewer continues with the next task in the given mathematical domain as far as the child can go with success. Given difficulty with the task, the interviewer either abandons that section of the interview and moves on to the next domain or moves into a detour, designed to elaborate more clearly the difficulty a child might be having with a particular content area. In 2001, in response to concerns about how long the interview was taking in some cases, some sampling of domains occurred for children in Prep and Grade 1, in *Measurement* and *Space*.

All tasks were piloted with children of ages five to eight in non-project schools, in order to gain a sense of their clarity and their capacity to reveal a wide range of levels of understanding in children. This was followed by a process of refining tasks, further piloting and refinement, and where necessary, adjusting the framework.

The form and wording of the tasks are influenced by the growth points for which they are intended to provide evidence, while at the same time the consideration of the data provided by a given task can lead to a refining of the wording of a given growth point.

The interview provides information about those growth points achieved by a child in each of the nine domains. Figure 2 shows three questions from the interview, from the section on *Addition and subtraction strategies.* Words in italics are instructions to the interviewer; normal type are the words the interviewer uses with the child.

(18) Counting On

- (a) Please get four green teddies for me. (*Place 9 green teddies on the table.*)
- (b) I have nine green teddies here. (*Show the child the nine teddies, and then screen the nine teddies with the ice-cream lid*).
  That's nine teddies hiding here and four teddies here. (*Point to the groups*).
- (c) Tell me how many teddies we have altogether. . . . Please explain how you worked it out.
- (d) (*If unsuccessful, remove the lid.*) Please tell me how many there are altogether.

(19) Counting Back

For this question you need to listen to a story.

- (a) Imagine you have 8 little biscuits in your play lunch and you eat 3. How many do you have left? … How did you work that out?
  If incorrect answer, ask part (b):
- (b) Could you use your fingers to help you to work it out? *It's fine to repeat the question, but no further prompts please).*

(20) Counting Down To / Counting Up From

I have 12 strawberries and I eat 9. How many are left? . . . Please explain.

Figure 2. An excerpt from the addition and subtraction interview questions.

For clarity, some instructions to the interviewer have been removed here. For example, lack of success with question 19 (in both parts *a* and *b*) would lead to the interviewer to skip question 20 and the remainder of the *Addition and subtraction strategies* section.

Question 18 provides information on whether the child is able to count-on or use a known fact, needs to count-all, or is unable to find the total by any means. Our aim in the interview is to gather information on the most powerful strategies that a child accesses in a particular domain. However, depending upon the context and the complexity of the numbers in a given task, a child (or an adult) may use a less powerful strategy than they actually possess, as the simpler strategy may do the job adequately in that situation. Questions 18-20 illustrate this well. Question 19 is often solved by children modelling the eight biscuits with their fingers and then counting back. By the nature of the numbers involved in Question 20, neither modelling the 12 objects nor counting back 9 is easy. Children are therefore given the opportunity to use a more sophisticated strategy (if they possess it), such as count-down-to 9 (11, 10, 9) or count-up-from 9 (10, 11, 12).

Wright (1998) highlights the challenge of determining the actual strategy used by a child in solving a problem, as "a child may unwittingly or intentionally describe a strategy different from the one used" (p. 703). Not surprisingly, teachers' facility with determining the strategy used increases over time.

Two of our more interesting responses to the question, "how did you work that out?", are "my brain told me" and "God told me".

A professional development footnote at this point is that in making decisions about the strategies used by children in solving these problems, the teachers are themselves becoming increasingly comfortable with the distinction between the various strategies and their various levels of sophistication. This is an important step in being able to facilitate the movement of their children to higher level thinking during classroom teaching.

A teacher and coordinator in her third year of involvement in the project wrote "I found the data much more valuable this year as I have a greater understanding of the growth points and the direction that my children need to take".

As well as moving carefully through the 20-page interview schedule, the interviewer completes a four-page Student Record Sheet. The information on this record sheet is then used by a trained team of coders together with a scoring algorithm to assign "achieved growth points" to each child for each domain. The rating of an individual child at a particular growth point is based on his or her responses to a number of different interview tasks. The raters demonstrated extremely high levels (all greater than 90%) of inter-rater reliability (Rowley & Horne, 2000).

Surprise at what many children were able to do.

- Some children did better than expected from my first impressions of them during the normal maths program. The one-to-one situation and wait time allowed them the opportunity to show what they knew.
- Working with a gifted Prep who actually worked out the answers quicker than I did was a highlight. Reading 24,746,154 on the calculator. Amazing!

Surprise at some difficulties children had

- A child of great potential, completed nearly all the interview, but couldn't tell the time.
- To discover that some children who you thought had particular concepts couldn't use these/didn't have them—they were good at 'hiding' within the group.

The emergence of the quiet achievers

- In every class there is that quiet child you feel that you never really 'know'—the one that some days you're never really sure that you have spoken to. To interact one-to-one and really 'talk' to them showed great insight into what kind of child they are and how they think.

- Quiet achievers (especially girls).

The power of the interview data in informing teaching

- My greatest surprise was the wealth of assessment information gained from the assessment interview. . . . and how I've been able to adapt some of the ideas into my classroom practice.
- The one-to-one contact enabled me to focus on what I have to work on to enrich their learning.

The level of enjoyment and confidence displayed by the children during the interview

- The greatest highlight was that no matter at what level the children were operating mathematically, all children displayed a huge amount of confidence in what they were doing. They absolutely relished the individual time they had with you; the personal feel, and the chance to have you to themselves. They loved to show what they can do.

Figure 3. Interview themes and examples.

It is important to stress that the growth points are big mathematical ideas or concepts, and that much learning takes place between them. As a result, a child may have learned several important ideas or skills *necessary* for moving towards the next growth point, but perhaps not of themselves *sufficient* to do so. Also, to achieve many of the growth points requires success on several tasks, not just one or some. This enables us to know that a child uses a more powerful strategy consistently and appropriately.

Of course, decisions on assigning particular growth points to children for the purpose of this research project are based on a *single* interview on a *single* day. A teacher's knowledge of a child's learning is informed by a wider range of information, including observations during everyday interactions in classrooms. However, teachers agree that the data from the interviews reveal student mathematical understanding and development, in a way that would not be possible without that special opportunity for one-to-one extended interaction.

Each year after the initial interview, teachers have been invited to comment on "highlights, surprises or patterns" that emerged from the interviews and the data. Common themes are given in Figure 3, with an illustrative example in each case.

Student performance data from the first two years of the project will be presented later in this paper.

The Professional Development Program

The professional development program occurs (formally) at three levels. The 250 or so teachers from trial schools meet with the research team each year for around five full days, spread across the year. The focus of these days is on understanding the framework and interview, and on appropriate classroom strategies, content and activities for meeting identified needs of their students. Many teachers comment that their own mathematical knowledge has been enhanced considerably as they have focused on children's mathematical thinking. Readings are provided, as are follow-up tasks, for later sharing. In conjunction with these meetings, Early Numeracy Coordinators from the trial schools usually meet for an additional three days each year, and the principals for two days. The focus of these days is on finer grained data analysis and the development of school leadership roles within the ENRP.

On four or five occasions each year, the teachers meet in regional cluster groups for two hours, usually after school. Each cluster contains from three to five school teams. One member of the university research team is responsible for each cluster group. The focus of these meetings is to complement the statewide professional development. There is usually a time of sharing, during which teachers discuss readings or particular activities or approaches that they have tried since last meeting together. This is followed by the content focus for the day, and further tasks are set that need to be completed before the groups meet again.

The third level of professional development takes place at the school and classroom level. The cluster coordinator visits each school approximately three times per year, spending time in classrooms team teaching or observing, participating in planing meetings, jointly leading parent evenings, and acting as a "sounding board" for teachers, coordinators and principals. In addition, the Early Numeracy Coordinator at each school conducts weekly or fortnightly meetings of the "professional learning team", to maintain continuity, communication, team cohesion and purpose.

The Student Data

As has been described earlier, all children in trial schools and a sample of approximately 40 children in each reference school are interviewed early in the school year (in a three-week period in February/March) and late in the school year (in a three-week period in November), using the ENRP task-based interview.

In the discussion below, the association between the growth points and the interview tasks must be kept in mind. As already mentioned, achievement of a particular growth point in the context of the following discussion means the capacity to successfully answer a series of particular questions on an interview, on a particular day. Changes in the interview arrangements or even subtle changes to questions are likely to yield different results.

An example illustrates this. One of our growth points in *Counting* refers to counting forwards and backwards from given starting points, and knowing the number before and after a given number. Success on six different interview tasks is necessary for a student to be assigned this growth point (at least). As part of a small sub-project, we modified one task. Instead of asking the children to start counting from 84 until we stopped them at 102 (the version used in 1999 and 2000), we encouraged the children to keep going, stopping them now at 113. This slight change meant that approximately 4.2% of those children in reference schools who could count to 102 were not able to continue successfully to 113 (often saying 109, 200, 201, 202, . . .). The growth points and the interview therefore form a kind of package when considering the data, because questions such as "what do you mean by this particular growth point?" are often answered in part by a description of the related interview tasks.

Some Data From the Domain of Addition and Subtraction Strategies

The ENRP has provided a unique opportunity to gather data on what large numbers of young children know and can do in various mathematical domains. To this point, 11,384 children have been interviewed using the ENRP interview (8802 in trial schools and 2582 in reference schools). Of these, 1324 have been interviewed on five occasions (beginning and end of Prep, beginning and end of Grade 1, and beginning of Grade 2).

The trial schools were chosen to represent a diversity of school sizes, geographical locations, socio-economic levels, and English-speaking backgrounds across Victoria. The reference schools were chosen to carefully match the trial schools on all these variables. Given that the teachers in those schools have been "uncontaminated" by contact with the research team and the professional development program, the children in their classes provide a useful measure of what "typical children" can do. In order to give an accurate picture of typical children therefore, in much of the following discussion, the data used will be from reference schools.

Addition and Subtraction is the domain for which data are provided in Table 1. For particular grade levels, the percentage of children achieving each growth point (or better) is given. Data are given for children commencing school, and then at the end of the first, second, third, fourth and fifth years of school.

As an additional minor project, a stratified random sample of Grade 3 and 4 children was interviewed in November 2000, to explore the usefulness of the interview and framework for describing the understanding of eight- and nine-year olds. The exercise was most useful, however it needs to be stressed that the ENRP interview was created for use with P-2 children, and there are key content domains appropriate for older students not assessed (e.g., fractions) that would need to be given attention if the interview was extended to Grade 4.

Table 1
Percentage of Reference School Children in 2000 at Each Addition and Subtraction Growth Point or Above, by Grade Level (%)

	Prep (Feb) *n* = 506	Prep (Nov) *n* = 506	Grade 1 (Nov) *n* = 488	Grade 2 (Nov) *n* = 446	Grade 3 (Nov) *n* = 187	Grade 4 (Nov) *n* = 172
1. Count-all	44	80	94	99	99	98
2. Count-on	6	28	69	88	96	96
3. Count-back/down-to/from	0.2	4	22	55	86	94
4. Basic strategies	0	1	10	38	77	92
5. Derived strategies	0	0	3	10	35	51
6. Extending and applying	0	0	0	0.6	3	13

The table shows the progress that young children make over time in using increasingly sophisticated strategies in addition and subtraction situations. A rough summary is that most children are able to count-all by the end of the first year of school (80%) and develop counting on by the end of Grade 1 (69%). By the end of Grade 4, just over half are able to use both basic and derived strategies (51%).

One of the more interesting aspects of these data is the percentage of children who successfully used what we have termed basic and derived strategies in addition and subtraction situations. These include the use of commutativity, doubles, near doubles, combinations that add to 10, adding 10, and fact families, as well as the use of known facts. Given that only 10% of children at the end of Grade 2 have proficient use of these strategies, it adds further weight to the argument (see, e.g., Kamii & Dominick, 1998, Narode, Board, & Davenport, 1993; Plunkett, 1979) that teaching children two column addition and subtraction *written* algorithms in the early years of school is inappropriate.

Growth Over Time

The involvement of the Reference Schools in the Early Numeracy Research Project serves two important purposes. First, it provides a sense of what typical children know and can do at various times in their schooling. Second, given the careful match of Trial and Reference Schools, aggregated data provide a measure of the contribution that the full involvement in the research and professional development program by trial school teachers can make to student growth over time.

Over time, the research team has explored different images for displaying growth over time. The ENRP growth points without a transformation don't form an interval scale. Rowley and Horne (2000) developed a procedure to accomplish such a transformation, enabling the calculation and use of mean growth points for cohorts of students.

The calculation of means is straightforward for a given group of students, and enables the kinds of comparisons shown in Figure 4. The graph (developed by Marj Horne) shows the various cohorts of students over the first four interview periods of the ENRP, for *Addition and subtraction*. Trial school data are marked with rectangles at each assessment point, while reference school data are marked with crosses. The data can be viewed as four sets of matching graphs.

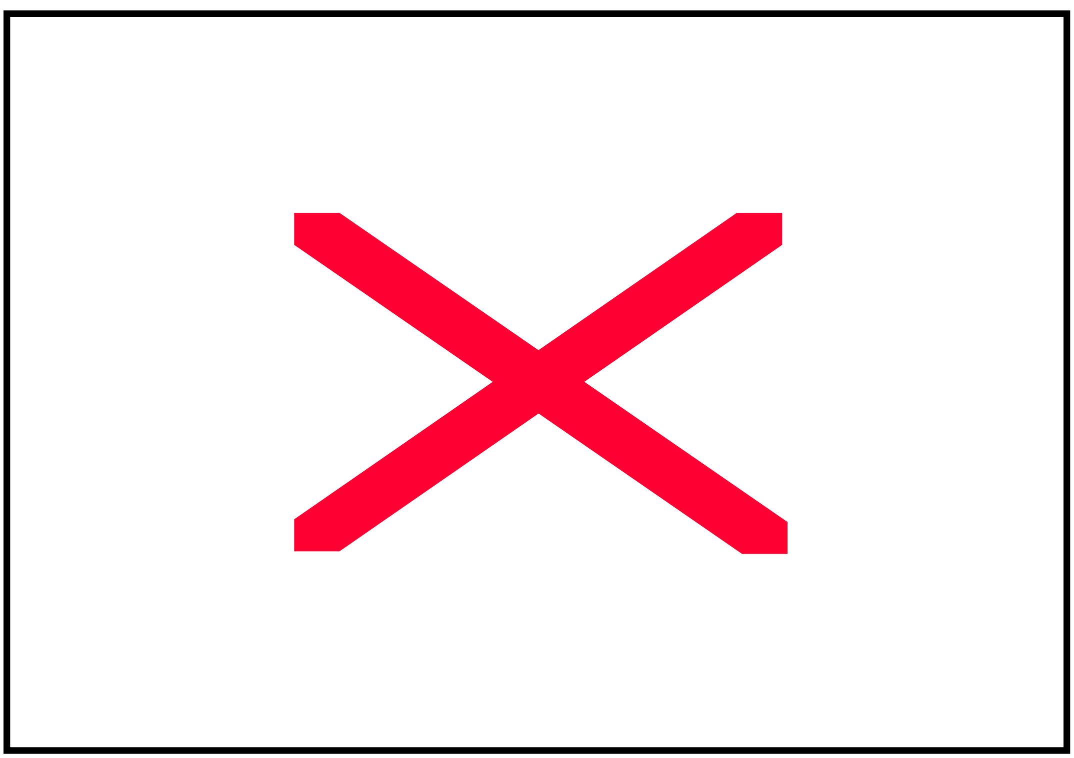


Figure 4. Mean growth points achieved for various cohorts of students for addition and subtraction domains, 1999-2000.

The various sets of lines show trial and reference means over time for four cohorts: the children who were in Grade 2 in 1999 and then "left" the project (the top pair of short graphs), the children who were in Grade 1 in 1999 and then in Grade 2 in 2000 (the pair of longer graphs below these), the children who were in Prep in 1999 and then in Grade 1 in 2000 (the next pair of longer graphs below), and the children who joined the project as new Preps in 2000 (the lowest pair of short graphs).

These graphs provide a large amount of information about particular cohorts and their comparison with others. Mean growth points can be read directly from the graph.

It is clear that there is steady growth over time, that the differences between trial and reference school students increase over time, and that the growth over the summer break, though positive, is at a relatively lower rate than during the school year.

Similar patterns are evident in all assessed domains, although it should be noted that in the case of the two *Space* domains (Properties of shape and Visualisation and orientation), the differences between trial school and reference school growth are much larger, with trial school Preps almost reaching the means of Grade 2 reference school children after one year.

One of the underpinning ideas of the framework, as stated earlier, was that it would "have sufficient 'ceiling' to describe the knowledge and understanding of all children in the first three years of school". It was hoped that all children would be challenged by the interview tasks across the various domains. In 2000, after around 9000 children had been interviewed (including 564 children at Grades 3 and 4), a Grade 2 boy became the first (and only to this point) student to successfully complete all interview tasks. The aim of challenging all students would appear therefore to have been achieved.

Other Interesting Findings from the Data

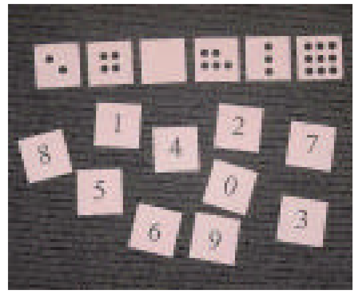
Some of the findings mentioned briefly here will be discussed in other papers in this volume or elsewhere, but some comments are appropriate here.

Data on entry to school. During the first two years of the project, Prep teachers commented that, not surprisingly, many children in the first weeks of school were unable to make much progress on tasks in the various domains during the interview. For assessment in 2001, it was decided to extend and hopefully enhance the Prep Detour. The Prep Detour was intended for all children who were unsuccessful in counting a collection of just over 20 plastic teddy bears in the first task in the interview, and originally involved tasks focusing on "more" and "less", conservation of number, pattern, and so on. In 2001, it was extended and Prep teachers were asked to use this part of the interview for *all* Prep children. A small example of what children were able to do is given in Table 2, which shows the performance on a task in which children were asked to match numeral cards to particular patterns of dots. The data given in Table 2 are those for trial school students.

Table 2
Performance of Trial School Children on Entry to School in February 2001 on Selected Tasks (%) (n = 1437), Showing Arrangement of the Task

Task	Success rate	
Match numeral to 2 dots	86%	

Match numeral to 4 dots	77%
Match numeral to 0 dots	63%
Match numeral to 5 dots	67%
Match numeral to 3 dots	79%
Match numeral to 9 dots	41%



Two comments are worth making here. First, the level of success in matching a blank card to the numeral "0" was quite high. Second, the recognition of the 3x3 array representation as a match for the numeral "9" was relatively low. This was somewhat surprising given its frequent use on television. One factor in this may be the occasional confusion between 6 and 9. Also, the previous task showed the dot cards for two seconds before removing them. Many children initially identified this pattern as 8 or 10, and seemed determined to match the numerals accordingly in the following task, even though 8 and 10 were not options. However, students did have the chance to count the dots should they wish, so the results remain somewhat surprising.

Additional Assistance. In conjunction with the ENRP, Ann Gervasoni studied the implementation of a program for children likely to benefit from additional assistance outside the classroom. Among the issues of interest were the nature of the program, its success in developing key underpinning mathematics ideas, the grade level or levels at which such intervention might best occur, and the relative advantages of small group and individual format. Further information can be found in Gervasoni (2000) and Gervasoni (this volume).

Time Allocation. Teachers in trial and reference schools completed a questionnaire giving information on the amount of time given to mathematics teaching and the breakdown of this time into various routines. The majority of teachers (65.7%% in trial schools [*n* = 169] and 56.1% in reference schools [*n* = 157]) formally allocated one hour per day to the teaching of mathematics. Mean times were 297 minutes per week for trial schools and 280 minutes per week for reference schools per day. The difference of three or four minutes per day does not seem great, but the cumulative effect over the school year may be.

Teachers' Stated Professional Growth

One purpose of the ENRP framework is to provide a means of quantifying young children's numeracy learning. However, we are at least as interested in identifying factors that may contribute to such learning. To complement the data on children's learning, a range of other data are being collected, including detailed questionnaires on teachers' beliefs and understandings about numeracy learning, regular journals kept by Early Numeracy Coordinators (the leaders of the professional learning teams in each school), as well as teacher and principal data on the effect of the project on teaching practice, student attitudes to mathematics, and home/school community links.

Given the clearly successful efforts of trial school teachers in developing children's mathematical skills and understandings in 1999 and 2000, it becomes increasingly important for the research project to study successful teachers’ practice to try to discern those aspects of “what the teacher does” that make a difference. At a statewide professional development day in 2000, teachers were asked to identify changes in their teaching practice (if any). There were several common themes:

- more focused teaching (in relation to growth points);
- greater use of open-ended questions;
- provision of more time to explore concepts;
- greater opportunities for children to share strategies used in solving problems;
- provision of greater challenges to children, as a consequence of higher expectations;
- greater emphasis on “pulling it together” at the end of a lesson, as part of a whole-small-whole approach;
- more emphasis on links and connections between mathematical ideas and between classroom mathematics and “real life mathematics”; and
- less emphasis on formal recording and algorithms; allowing a variety of recording styles.

Several of these themes are evident in the following quote from a teacher:

> The assessment interview has given focus to my teaching. Constantly at the back of my mind I have the growth points there and I have a clear idea of where I’m heading and can match activities to the needs of the children. But I also try to make it challenging enough to make them stretch.

As Barbara Clarke (2000) noted, “teachers in the Early Numeracy Research Project have a clearer picture of the typical trajectories of student learning and can recognise landmarks of understanding in individuals” (p. 13).

In 2000, teachers were also asked to comment on aspects of children’s growth that they had observed which were not necessarily reflected in movement through the growth points. Although the research team has a great interest in cognitive growth as demonstrated by the response to interview tasks, growth can take other forms (e. g., productive disposition, as identified by Kilpatrick, Swafford, & Findell, 2001). It is important to document these other forms of growth.

Common themes were the following.

- Children are better at explaining their reasoning and strategies.
- Children enjoy maths more, look forward to maths time, and expect to be challenged.
- The development of a “give it a go” mentality is evident, with greater overall persistence.
- Children are thinking more about what they have learned and are learning.
- All children are experiencing a level of success.

One teacher commented on her children’s positive attitudes to mathematics:

> Children seem to be more enthusiastic, take more risks and have more confidence in their abilities. They can’t wait to participate. They’re excited about maths. For example, we brainstormed the combination of green or red lollies to make 10 and when the children opened their bag, they exploded with excitement! “I’ve got 3 and 7!” “I’ve got 2 and 8!” All this over adding to 10!!

In the third year of the project, teachers are reminded of how far students have come when they have the opportunity to interview children who have newly arrived at the school at the same time as children who are in the third year of the project. One teacher wrote, "new children to the school thought very differently and had difficulty expressing the strategies they used to solve problems. 'Project kids' knew how to express their ideas and strategies".

It is important to note here that in the third year of the project, the professional development has given much greater emphasis to problem solving and investigations, with a particular focus on the use of contexts of interest to children. During the first two years, much of the discussion focused on "pulling the maths apart", to enable a careful consideration of children's mathematical growth in specific domains. In the third year, we have been in a position to "put it back together", and explore the kind of classroom experiences that use the children's thinking across a range of domains in interesting contexts. Teachers report that they are able to see the way in which mathematics from a variety of domains can come together in a single task.

All teachers have facilitated growth in student learning over time, but the data for some teachers is particularly impressive. In 2001, the research team will conduct detailed case studies of some of these teachers, as well as those school professional learning teams whose overall data is impressive. It should be emphasised that *growth in student understanding* is the main measure of success, not achievement at a given time. Although leadership and other school factors are of interest, the major focus of these studies will be what the teacher does in the classroom. We know a lot already about successful schools and teams (see, e.g., Fullan & Stiegelbauer, 1991; Scull & Johnson, 1998), but the ENRP will contribute considerably to the literature if it can encapsulate those classroom approaches that excellent teachers use.

The Early Numeracy Research Project's View of Teachers

From the first professional development session, the research team attempted to make it clear that they regarded project teachers as *co-researchers*. It was explained that there was much to be learned, and that a collaborative approach was the desired one.

Of the paradigms or themes that Doyle (1990) identified as underlying proposals for teacher education (the good employee, the junior professor, the fully functioning person, the innovator, and the reflective professional), the Reflective Professional provides the best "fit" for this desired approach. According to this paradigm, teacher education should

> foster capacities of observation, analysis, interpretation, and decision making. . . . Within this framework, research and theory do not produce rules or prescriptions for classroom application but rather knowledge of methods of inquiry useful in deliberating about teaching problems and practices. (Doyle, 1990, p. 6)

In the initial stages of the ENRP, many teachers commented that they would prefer that the research team "tell them what to do and they would just do it". This notion relates closely to Reddy's (1979) "conduit metaphor", in which the teacher is regarded as "worker" (Connell, 1985), "machine" (Grobman, 1970) or "manager" (Clegg, 1973), rather than "curriculum maker" (Clandinin & Connelly, 1992)—our preferred view. The notion of "mutual adaptation" (Berman, Greenwood, McLaughlin, & Pincus, 1975), where both the project design and institutional settings change in response to an innovation is most relevant here, as this kind of accommodation has been most evident in the ENRP over time.

It is interesting that in the third year of the project, a number of teachers have recalled their earlier requests for strong direction with some amusement, and there appears to be fairly general agreement that the co-researcher model was a powerful and appropriate approach. Teachers from other schools are now visiting project schools to gain the “good oil”, and are sometimes disappointed to find that project teachers are unable or unwilling to present a simple recipe for success. As the project findings are shared with a wider audience, it will be important to resist the temptation to tell teachers what to do, given that more is known now.

Research as a Powerful Professional Development Strategy

Clarke (1994) has argued that the research literature provides key principles for effective professional development. These are that professional development should:

1. Address issues of concern and interest, largely (but not exclusively) identified by the teachers themselves, and involve a degree of choice for participants.
2. Involve groups of teachers rather than individuals from a number of schools, and enlist the support of the school and district administration, students, parents, and the broader school community.
3. Recognise and address the many impediments to teachers’ growth at the individual, school and district level.
4. Use teachers as participants in classroom activities or students in real situations, modelling desired classroom approaches during in-service sessions to project a clearer vision of the proposed changes.
5. Solicit teachers’ conscious commitment to participate actively in the professional development sessions and to undertake required readings and classroom tasks, appropriately adapted for their own classroom.
6. Recognise that changes in teachers’ beliefs about teaching and learning are derived largely from classroom practice; as a result, such changes will follow the opportunity to validate, through observing positive student learning, information supplied by professional development programs.
7. Allow time and opportunities for planning, reflection and feedback in order to report successes and failures to the group, to share “the wisdom of practice”, and to discuss problems and solutions regarding individual students and new teaching approaches.
8. Enable participating teachers to gain a substantial degree of ownership by their involvement in decision making and by being regarded as true partners in the change process.
9. Recognise that change is a gradual, difficult, and often painful process, and afford opportunities for ongoing support from peers and critical friends.
10. Encourage participants to set further goals for their professional growth. (p. 38)

In considering the various features of the ENRP, we believe that all these key principles have been met. Although the project teachers were not part of the design process originally, every endeavour has been made to accommodate their input along the way.

During the last ten years, there has been much emphasis given to the power of the “assessment tail in wagging the instruction dog”. One argument is that if high stakes assessment requires certain forms of performance, this is likely to result in teachers preparing students for this assessment, by aligning instruction with assessment. Clarke Stephens, and Wallbridge (1993) referred to this as the "ripple effect". Depending upon the context and your point of view, this can be a good or a bad thing.

In the case of the ENRP, the requirement of teachers to participate in the assessment interviews has meant that they have been involved deeply in researching the understanding of their children, as individuals and as a group. Having access to data from a much larger group of students has also enabled them to consider patterns or trends and to start to consider reasons for these. Ongoing assessment and interviews in the latter part of each year have provided an opportunity to evaluate the effectiveness of their teaching across different domains. This process has proved very powerful in teachers' own professional development. They have increased their knowledge of how children learn mathematics in general, they have a much clearer picture of their own children's understanding, and they have a repertoire of teaching approaches to enhance this understanding. The role of the co-researcher has therefore been a powerful professional development tool.

The research team has noted with considerable pleasure, particularly in the third year of the project, the increasing fluency of trial school teachers with mathematics education research terminology, and the willingness to engage in complex ideas over extended periods. It appears that the "shared language" about young children's learning, so evident among teachers in the context of literacy, is becoming a reality in mathematics as well.

The Framework as a Lens for Teachers

When the ENRP learning and assessment framework was first developed, a major purpose for its creation was to enable a measure of the effectiveness of the professional development aspect of the project, by monitoring student movement through the growth points. However, the framework has proved powerful in a variety of other ways.

Teachers are increasingly "owning" the framework, and using it to enhance their own understanding of children's mathematical learning. Teachers' understanding of the framework is enhanced by their familiarity with the interview. As the framework becomes better known, teachers view student responses during the interview in the light of their understanding of the growth points. Most importantly, the growth points provide a kind of "lens" through which children's mathematical thinking can be viewed, in all individual, small group and whole class interactions.

In summary, the Early Numeracy Research Project offers:

- A research-based, readily understood framework for understanding children's mathematical thinking.
- A powerful, one-to-one interview to gain a picture of individual and group understanding of big mathematical ideas.
- A professional development program and approach designed to support teachers to build upon what children know and can do.

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