Implementing Mathematical Investigations With Young Children

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Engaging children in mathematical investigations is advocated as a means of facilitating mathematical learning. However there is limited guidance for teachers on ways to support young children engaged in investigations. This study provides insights into the mathematical literacy required by seven-to-eight-year-old students undertaking investigations. Examples of difficulties are described in relation to problem solving, representation, manipulation, and reasoning. While mathematical investigations can enhance young children's learning, teachers need to provide guidance to address necessary skills and knowledge.

Advocates of an inquiry-based approach to learning argue that young children should engage in mathematical investigations (e.g., Baroody & Coslick, 1998). Mathematical investigations are contextualised problem solving tasks through which students can speculate, test ideas and argue with others to defend their solutions (Jaworski, 1986). Additionally, through investigations, children gain insight into cultural practices of mathematicians, and mathematics as a career (National Council of Teachers of Mathematics [NCTM], 2000). Investigations represent radically new practices (Klinman, Russell, Wright, & Mokros, 1998; Taber, 1998) and while research exists on ways teachers can support older students' investigations (e.g., Greenes, 1996; Oliveira, Segurado, da Ponte, & Cunha, 1997; Taber, 1998), research with young children appears to be limited to descriptions of individual children's learning (e.g., Whitin, 1993) and classroom mathematics programs (e.g., Skinner, 1999; Whitin, 1989). Given the importance attributed to investigations, research is urgently needed to establish what pedagogical content knowledge that early childhood teachers require in order to implement investigations in their classrooms. In this paper, we report on a study that explores those aspects of mathematical literacy that might impede young children undertaking mathematical investigations for the first time. Understanding children's difficulties is a key facet of pedagogical content knowledge (Carpenter, Fennema, & Franke, 1996).

Mathematical Literacy

In a technological world, mathematical literacy is of paramount importance to enable citizens to participate effectively in everyday life (Steen, 1997). Four interrelated thinking processes, namely problem solving, representing, manipulating and reasoning underpin mathematical literacy (Pugalee, 1999). Each of these is briefly described.

Problem solving is central to mathematics and requires the use of prior knowledge and skills to deal with novelty, to overcome obstacles, to reach and validate solutions, and to pose problems (English, 1998; Pugalee, 1999; Romberg, 1994). Accordingly, the Australian Education Council [AEC] (1991) argues that students need "considerable experience in dealing with non-routine mathematical problems and unfamiliar situations" (p. 12).

Representing is "the building block of mathematical inquiry" (Pugalee, 1999 p. 20) and involves the decoding and encoding of information presented in a variety of representational systems. These systems include pictures, symbols, models, written and spoken language (Lesh, Post, & Behr, 1987) and diagrams (Diezmann & English, 2001). Goldin (1998) argues that representational systems and their construction provide the foundation for a unified psychological model for mathematical learning and problem solving.

Manipulating encompasses the use of physical and technological tools and objects, and symbols to explore and understand mathematical situations (Clements, 1999). It may involve calculation, algorithms, procedures (Pugalee, 1999) or measurement (NCTM, 2000). In investigations, data collection and measurement are particularly important and include the use of a variety of tools and techniques (Klinman et al., 1998).

Reasoning uses facts, properties, and relationships to make and test conjectures and to follow and develop logical arguments. In the primary grades, mathematical reasoning provides insights into the discipline of mathematics by fostering generalisation from observation and experience, and by developing interconnected conceptual knowledge and supporting sense making with mathematics (Russell, 1999).

Mathematical Investigations

Advocates of mathematical investigations argue that they provide rich opportunities for mathematical learning. According to (Greenes, 1996):

> Investigations present curiosity provoking situations, problems, and questions that are intriguing and captivate students' interest and attention. At the outset, students are unable to solve the problem because they are complex, often necessitating the design of a plan or approach, and frequently require the completion of several tasks. Most investigations are interdisciplinary, requiring students to apply concepts from the various areas of mathematics, and, for some problems, from other disciplines as well ... Generally, there is more than one way to approach or solve each problem. Identifying different solution paths and evaluating them is often part of the solution process. Because of multiple tasks, investigations are often designed to be tackled by students working in pairs or teams and for long periods of time. (pp. 37-38)

Investigations provide children with opportunities to engage in the authentic practices of mathematicians as they discover, invent and use mathematics to understand the world (Lappan & Briars, 1995; Papert, 1972; NCTM, 2000; Wells, 1985). Such inquiry-based approaches have long been advocated in teaching mathematics (e.g., Baroody & Coslick, 1998; Borasi, 1992; Greenes, 1996; Jaworski, 1994; Papert, 1972; Roper, 1999; Wells, 1985). Inquiry-based approaches encourage children to engage in divergent or creative thinking processes which result in the proposition of multiple solution paths. In this situation, they find productive ways to adapt, modify, and build on prior knowledge, rather than just to apply learned techniques to overcome a lack of knowledge or understandings (Lesh & Doerr, 2000). Owing to the multitude of solution paths that may result, students need to evaluate their own solution paths, and to critique and provide feedback on their peers' solution paths.

The interest and complexity of a task is dependent on its cognitive challenge for individual learners (Henningsen & Stein, 1997). Solving novel problems should provide students with the opportunity to work mathematically on real-world problems and engage in high-level cognition by exploring, conjecturing, analysing, justifying, questioning, discussing, writing about, and applying mathematics (AEC, 1991; NCTM, 2000; Romberg, 1994). However, the opportunity to employ these processes is dependent on the individual challenge that the task provides. To facilitate high-level cognition, the teacher needs to "select and setup worthwhile mathematical tasks ... [and] proactively and consistently support students' cognitive activity without reducing the complexity and cognitive demand of the task" (Henningsen & Stein, 1997, p. 546).

Ideally, mathematical investigations are undertaken within a community of inquiry in which classrooms are "environments for collaborative mathematical thinking" (Cobb & Bowers, 1999; Stein, Grover, & Henningsen, 1996; Stein, Silver, & Smith, 1998). Such an environment provides children with opportunities to engage in open questioning, to seek evidence, to participate in constructive dialogue and debate, and to explain, clarify, and revise their mathematical ideas and problem constructions (e.g. Baroody & Coslick, 1998; Bowers, Cobb, & McClain, 1999; Turner et al., 1998).

Design and Methods

The methodology involved a teaching experiment within a case study design (Yin, 1994). One of the researchers (CMD) assumed the role of the teacher and engaged in reflective practice while the other researchers provided feedback as a non-participant observer (JJW) and "critical friend" (LDE). Children worked as a "class group" and received 90 minutes weekly of investigatory activities over a 14-week period.

Participants were 20 seven-to-eight-year-old students. They were selected from four class groups in the same school on the basis of their interest and strength in mathematics.

The research was undertaken in an inductive theory-building framework, which requires description and explanation (Krathwohl, 1993). Ideally, in this process, data are collected until a "saturation" point is reached in which new observations do not provide further insight into the phenomena. The data comprised class video recordings, field notes taken by the research team, and work samples collected from students. The team reviewed tapes and work samples at the conclusion of each lesson and developed conjectures to explore in subsequent sessions. At the conclusion of the program, the range of children's difficulties were identified and a pattern matching approach was used to explain these difficulties within a theoretical framework for mathematical literacy (i.e, Pugalee, 1999).

Owing to space limitations, the results reported here only pertain to the difficulties children experienced in the initial five weeks of the program, which was implemented in the early part of the school year. However, these preliminary results will contribute to an understanding of the breadth and nature of the difficulties that young children experience in undertaking investigations.

In the first phase of the program, the children worked on a series of mathematical investigations involving Smarties (Figure 1). The first three investigations were teacher-initiated, although questions posed by children during these investigations were followed up. The fourth investigation was a student-initiated task, which the children undertook with a partner. These investigations are described in detail elsewhere (Diezmann, Watters, & English, 2001).

Results and Discussion

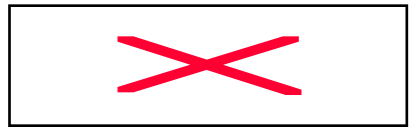
The investigations generated considerable excitement and fun. Although learning was evident, there were also difficulties with the processes of mathematical literacy, which impeded learning. Examples of difficulties will now be presented.

Investigation 1 (I-1): How many Smarties in the can?

Children were asked to investigate the numerical contents of small, white, translucent, sealed (film) canisters filled with Smarties. They were provided with a few Smarties, an empty can and a filled, sealed can. Children also had access to a range of common tools, such as kitchen scales, balance scales, rulers, calculators, and magnifying glasses.

Investigation 2 (I-2): Smartie Cans

Children were asked to predict the numerical contents of cans that varied in fullness and contained different sizes of Smarties.



Investigation 3 (I-3): Distribution of Smartie Colours

The children were each given a small packet of Smarties to explore the distribution of colours. This involved representing the number of each colour Smartie on a table and a graph, answering questions, and comparing their results with other students.

Investigation 4 (I-4): Independent Smartie Investigation

The children were given support to identify investigable questions about Smarties. Their findings were presented as pages for a class book about Smarties. Children had access to various common-place resource materials.

Figure 1. Overview of the Smartie Investigations.

Problem Solving

There were three problem-solving difficulties.

Solution method was inappropriate. Children had access to a range of classroom resources (e.g., scales, rulers) during the investigations. However, some children's use of these resources to achieve a solution was inappropriate. For example in I-1, when challenged to ascertain how many Smarties were in a sealed can, Eddie's choice and use of a magnifying glass was inappropriate. Eddie focussed on observing the enlarged Smarties rather than estimating the number of Smarties in the can.

A focus on surface features of the problem rather than its structure. In I-2, children had predicted the numerical contents of Cans A, B and C, which varied in the size of the Smarties they contained and their fullness. Before asking children to predict the contents of another can, Can D, the children were invited to ask questions about its contents. With the exception of Toby, the children's questions related to the size of the Smarties in Can D and its fullness. Toby's question was unrelated to the contents of Can D, but referred to the size of the Smarties in Can C: "How come the Smarties (are) big?"

Difficulty posing a problem. In I-4, children were given examples of problems that could be investigated and asked to pose their own problem. However, rather than identify their own problems, children typically selected one of the example problems to investigate.

While teachers may initially pose and guide children's investigations (Baroody & Coslick, 1998), children should ultimately initiate problems (Rowan & Bourne, 1994).

Representing

Children experienced three difficulties with representation.

Misinterpretation of a key term. During I-4, the word "popular" was used by the teacher and the children. However children differed in their interpretation of its meaning. Whereas some children like Gemma, correctly interpreted "popular" as a personal preference, other children including Melissa, incorrectly interpreted "popular" to mean the most frequently occurring item.

> Gemma: You could ask people what Smartie colour they like the most.
>
> Melissa: I think you would open all the Smartie jars you had and then, and then put the colours into groups say, purple, yellow pink and different colours and when you are finished putting them into groups well you count them up and (find) the colour that has the highest number.

Melissa was unconvinced by Gemma's explanation of how to find the most popular colour and argued "But that won't give you the answer." An understanding of key language, such as "popular" is necessary if children are to conceptualise a problem correctly and understand peer reports of investigations.

Inadequate explanation. Although the children were able to undertake investigations, they had difficulty explaining their ideas and actions orally. For example, in I-1, each child was asked to explain how their use of specific tools (e.g., scales) had helped them to determine the numerical contents of the Smartie Can. A typical response was "I know there were 24 (Smarties) because we weighed it". Children's inability to explain their ideas to their peers limits what children can learn from each other.

Difficulty reporting findings. Children's difficulty in communicating their ideas extended to written language. For example, although the children kept written and pictorial records of their independent investigation (i.e., I-4), they needed considerable guidance to synthesise this information into a short illustrated report to include in the class book.

Manipulating

Ineffective use of a measuring tool. Children had problems using tools in unfamiliar situations. For example in I-4, children had difficulty using the scales to determine how many regular Smarties were equivalent in mass to a giant Smartie. This difficulty occurred because neither the kitchen scales nor balance scales were sufficiently sensitive to detect the mass of one giant Smartie. With teacher support, the problem was reconceptualised and the children were able to establish how many regular-sized Smarties were of equivalent mass to a group of giant Smarties.

Reasoning

Five reasoning difficulties were identified.

Guessing without accounting for evidence. Children were often observed to guess answers during their investigations and overlook available information. For example in I-1, children predicted and counted the number of Smarties in their Smartie Can. After a number line was created that showed that the number of Smarties in the 20 cans ranged from 19 to 24, they were asked to predict the contents of a similar can (see I-2, Can A). All students, except Eddie, predicted that it would be between 19 and 24. Eddie predicted it would be less than 19 but was unable to justify his reasoning.

Inability to account for discrepancies. During the investigations, there were occasions when children were confronted with unexpected findings. However, the children failed to question these findings or seek explanations for discrepancies. For example in I-2, the children were surprised that Can A (full can) contained more Smarties than Can B (partially-filled). However, they failed to detect that the “fullness of the can” was a critical variable. None of the children spontaneously examined the cans, but after prompting, one child reported, “Well this one here (Can B) is not as full as this one (Can A).

Not using common units in a measurement situation. During their independent investigation in I-4, Caroline and Gemma attempted to use kitchen scales to weigh a few giant Smarties. However, owing to the lack of sensitivity of the scales, they continued to add more Smarties in order to obtain a reading. In doing so, they added regular Smarties instead of giant Smarties indicating a faulty assumption about measuring and common units (i.e., giant and regular Smarties).

Difficulty comparing two sets of objects. In her independent investigation in I-4, Melissa attempted to compare the mass of giant and regular Smarties on a balance scale. She put two giant Smarties on one side and also placed giant Smarties in the other bucket. In doing, so she failed to appreciate the need for giant Smarties in one bucket and regular Smarties in the other bucket to make a comparison.

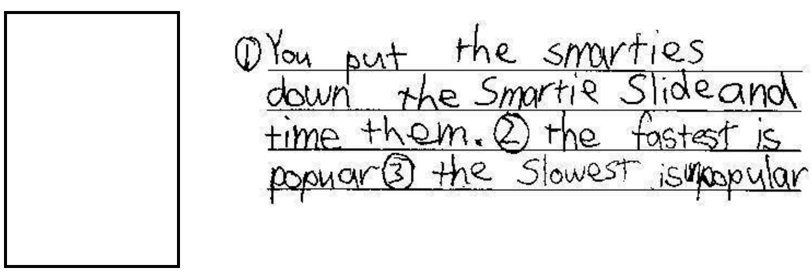


Figure 1. Smartie slide and Tate’s findings.

Making unfounded assumptions. In I-4, Tate assumed that the fastest Smartie to travel down the Smartie slide would be the most popular (see Figure 1). The Smartie slide was a cardboard construction that was used for measuring the speed of Smarties as they travelled down the slide. While Tate’s assumption might ultimately be correct, there was no evidence to support his claim.

Conclusions and Implications

This study has highlighted those aspects of mathematical literacy that inhibit young children’s success in learning to undertake investigations. In doing so, we draw attention to specific areas that need attention in instruction. Importantly, we have shown how failure to understand certain aspects of language inhibits the child's capacity to identify the key issues in a problem. The outcomes of this study also reinforce the importance of representation and problem solving as crucial processes of mathematical literacy. Engagement with complex problems thus affords opportunities to create and interpret representations in context. Similarly, manipulating, too often seen as the endpoint of mathematics, is an important process of mathematical literacy that enables children to investigate meaningful problems. Investigations provide children with opportunities to perform calculations, and use mathematical tools in context. They also provide a context for children to reason, explain thinking, to justify conclusions and to analyse situations, all indicators of mathematical literacy.

This study has implications for teacher's pedagogical content knowledge. The role of the teacher is to optimise conditions for learning and to introduce children to the culture of mathematics by teaching them how to think like "experts." In particular, mathematical investigations are genuine "thought-revealing" activities (Lesh, Hoover, Hole, Kelly, & Post, 2000) that provide teachers with an insight into children's mathematical literacy as they work in unfamiliar situations. Being aware of the mathematical literacy necessary to undertake investigations will enable teachers to plan and implement mathematically rich tasks that develop children's investigatory abilities. While investigations place demands on mathematical literacy, they also provide a powerful context for the development of mathematical literacy.

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