The Process of Introducing New Tasks Using Dynamic Geometry Into the Teaching of Mathematics

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Since 1997, there has been a very strong incentive from the French Ministry of Education for promoting the integration of information technologies in education at all levels. Among various actions, the Ministry provided a financial support for organizing pre and in service teacher education (IUFM) at national and local levels. The renewed curricula offer a real integration of technology into the teaching of mathematics and they claim the necessity of this integration: the use of dynamic geometry software programs and spreadsheets is compulsory.

It is quite difficult to estimate the proportion of teachers making a real use of technology. But it should be around 20%. One can be surprised by the gap between the institutional situation promoting technology and the real situation in classroom. One can seek the reasons for this reluctance by the changes of several kinds introduced by technology:

- changes in the concepts as mediated by the environment
- changes in the tasks given to students
- changes in the management of classroom, due to a growing autonomy of the student due to technology

Technology allows new kinds of tasks revealing the meaning of theoretical objects. But creating these new tasks is not easy and requires time. We will present below the learning tasks of a new type allowed by dynamic geometry software and try to explain why it takes time for teachers to design and use these tasks. Examples will be given in Cabri-geometry, existing both as a software program or application of the calculators TI 92, TI 89 and TI 83.

I - The Spatial and the Theoretical

Dynamic geometry software based on direct manipulation offer a microworld in which theoretical objects and relations (sometimes very complex from a conceptual point of view) can be visualized and physically manipulated. Such kind of environments offers the possibility for students of constructing knowledge in action and not only by having recourse to language. At first, some theoretical distinction will be made.

I.1 - The Distinction Between Spatial and Theoretical Properties

Physical space and geometry as a theory are two separate domains. Space is considered here as part of reality and geometry as a set of theories partly modelling space but also developing its own questions and solutions.

Diagrams in 2D geometry play an ambiguous role: one the one hand they refer to theoretical objects whereas on the other hand they offer *graphical - spatial* properties which can give rise to a perceptual activity from the individual. For example, the diagram in Figure 1 represents a parallelogram.

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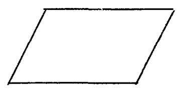


Figure 1. Diagram representing a parallelogram.

It presents several graphical-spatial properties like two sides are horizontal, the two other are oblique with a given direction (bottom left to top right), the opposite sides are parallel, the horizontal sides have a given length. Note that these properties are selected among a larger set of properties like the colour or the width of the sides. Some of these graphical-spatial properties can be interpreted in a geometrical way, some of them would not be considered as interesting from a geometrical point of view. For example, the position of the diagram in the sheet of paper is generally considered as not relevant in geometry. The slope of the side may also be not relevant, it depends on the problem in which the parallelogram occurs. Some graphical-spatial properties of the diagram are *incidental* with respect to the geometrical problem to be solved, some other are *necessary* like the parallelism properties. Furthermore some graphical-spatial properties necessarily follow from other ones. So there is a link of *necessity* between the parallelism of opposite sides and the fact that the intersecting point of the diagonals also is their midpoint. The teaching of geometry deals with these links of necessity between graphical-spatial properties. But one can understand the nature of these links if and only if one also can understand that some other links are incidental. Necessity makes sense as opposed to contingence. Geometry may appear as useful if it allows you to predict, to produce or to explain graphical-spatial properties of diagrams thanks to these necessity links. But it requires the awareness of the distinction between graphical-spatial properties and theoretical geometrical properties.

This ambiguous role of diagrams is completely implicit in the traditional teaching of geometry in which theoretical properties are assimilated into graphical ones. It is as if it is possible to abstract from the diagram the properties of the theoretical object that is represented by this diagram. One of the consequences is that pupils often draw the conclusion that it is possible to construct a geometrical diagram using only visual cues; or to deduce a property empirically by checking on the diagram. When pupils are asked by the teacher to construct a diagram, the teacher expects them to work at the level of geometry using theoretical knowledge whereas pupils very often stay at the graphical level and try to satisfy only visual constraints.

The task of drawing a tangent line to a circle passing through a given point is frequently viewed by pupils as the physical task of rotating a straight edge passing through the given point and adjusting it in order to "touch" the circle (Figure 2).

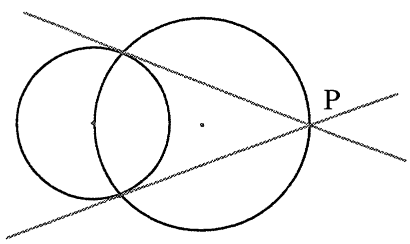


Figure 2. Tangents to circles task.

The teacher is expecting a drawing process based on geometrical relations – the tangent line is perpendicular to the radius and the locus of points from which it is possible to see a segment under a right angle is a circle.

The problem is that the final result may not be better from a visual point of view in this latter case than in the former one… Some traditional construction problems may fail to call for geometrical knowledge.

Therefore diagrams instead of helping students may become an obstacle to geometrical thinking in the sense that they avoid reasoning in theoretical terms (Fishbein, 1993; Mariotti, 1995; Salin, & Berthelot, 1994; Duval, 1998). Several investigations show that it is not easy for beginners in geometry to identify in a diagram the properties relevant from a geometrical point of view. Argaud (1998, pp. 290-297) could observe how it is difficult for children at the end of primary school to recognize a quadrilateral, parallel or perpendicular segments:

a- Is D a quadrilateral ?

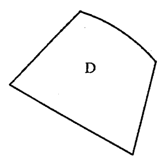


Figure 3.

14 children over 26 answer yes.

b – Is J a quadrilateral ?

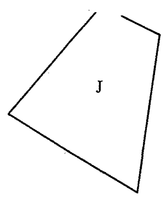


Figure 4.

10 children over 13 answer yes. The given arguments are: it has 4 sides even it is not finished (8 children), it has 4 sides (1 child), it has 4 edges (1 child).

c – Are the segments parallel ?

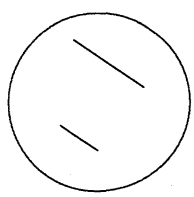


Figure 5.

16 children over 24 say that they are not parallel. The given arguments are: the segments are not opposed, or one is shifted, or they are not of same length.

I.2 - Diagrams in Computer Based Environments

Spatial-graphical and geometrical aspects are very much interrelated in the new kind of diagrams provided by geometry microworlds: these microworlds offer diagrams whose behaviour is controlled by theory. In dynamic geometry microworlds like Cabri-géomètre (Laborde & Straesser 1990), diagrams result from a sequence of primitives expressed in geometrical terms chosen by the user. When an element of such a diagram is dragged by means of the mouse , the diagram is modified preserving all geometric relations used in its construction. These artificial realities could be compared to entities of the real world: it is as if they react to the manipulations of the user by following the laws of geometry just like material objects react by following the laws of physics. A crucial feature of these realities is their quasi-independence of the user as soon as they have been created. When the user drags one element of the diagram, this latter is modified according to the geometrical way it has been constructed and not to the wishes of the user. This is not the case in paper and pencil diagrams which can be slightly distorted by the pupils in order to meet their expectations. Computers diagrams are external objects whose behaviour and feedback is no longer controlled by the user as soon as they have been created. Their behaviour requires the construction of an interpretation by the pupils. Geometry is a means, among others, of interpreting the behaviour of these computer diagrams.

II - Tasks in Dynamic Geometry Software Fostering the Learning of Geometry

The link between the spatial graphical and the geometrical properties of a diagram are reinforced in a dynamic geometry environment, as said above. This feature may be used to design tasks in which students will learn to link visual aspects to geometrical properties.

We propose to categorize the possible tasks to be offered in dynamic geometry environment in four categories of tasks with respect to visual phenomena produced by the environment

- tasks for interpreting visual phenomena
- tasks for producing or reproducing visual phenomena
- tasks for predicting visual phenomena
- tasks for explaining visual phenomena

In all these tasks, geometry provides a tool of solution.

II.1 - Tasks for Interpreting Visual Phenomena

This kind of tasks is adequate for beginners, especially at primary school. A dynamic Cabri- diagram is given at the screen of the computer and the children must describe it in terms of geometrical properties. An example is given in Figure 6, in which a quadrilateral is displayed on the screen. It looks like a square; if one of its vertices is dragged, it is visible that its sides are no longer equal but that it still has four right angles. If another vertex is dragged, it becomes visible that some of its angles are no longer right. The children must observe more precisely the behaviour of the diagram to identify the invariant properties in the drag mode: one of the pairs of opposite sides remains parallel in the drag mode.

In this kind of tasks, children learn to recognize a geometrical property from its various spatial-graphical representations.

At another school level, we also used this type of tasks for introducing new transformations. A point P and its image P' through the unknown transformation were given to the students (Jahn 1998). They could move P and observe the subsequent effect on P'. Students were asked to find the properties of the unknown transformation by means of this black box. In such a task students must themselves ask questions about the transformation :

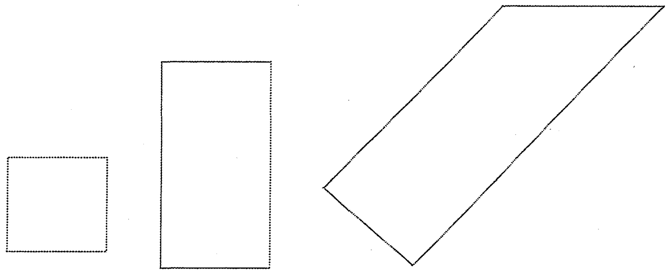


Figure 6.

Does it preserve collinearity ? Does it preserve distance ? Does it have invariant points ?

Cabri can be used to design experiments and get empirical answers. For example, one may redefine P as belonging to any given straight line and obtain the image of this line as the Locus of P' depending on the variable point P. Two specific tools of Cabri are used "Redefinition" and "Locus". It presupposes that the students not only master their use but also decide to use them. This decision actually involves mathematical knowledge: the fact that the image of a figure is the set of images of points of a figure; this is often completely implicit in our curricula but it presents a conceptual cut (even obstacle probably from both cognitive and didactical origin) with the view of a figure as an entity and not as a set of points.

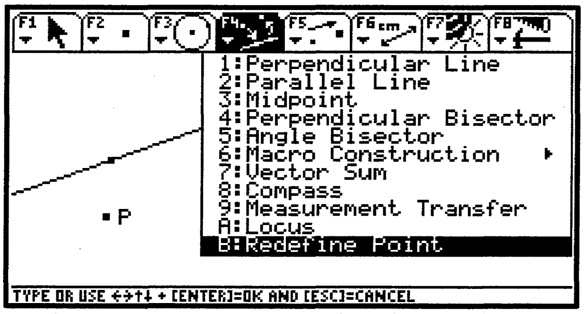


Figure 7. – Redefining the given point P…

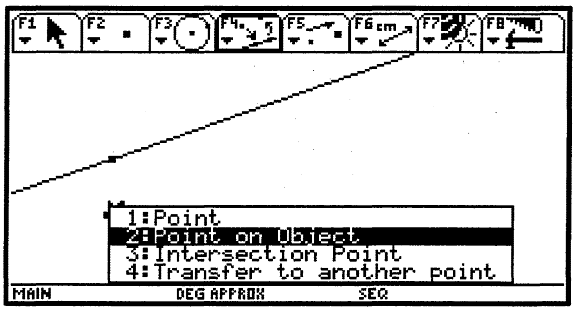


Figure 8. – as a point on …

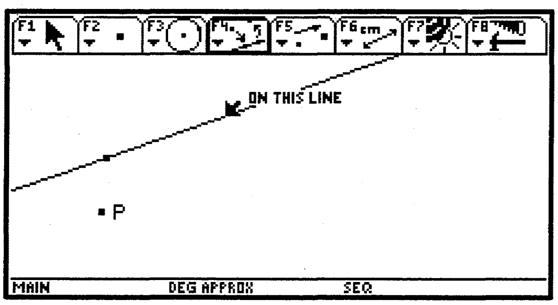


Figure 9. – a line

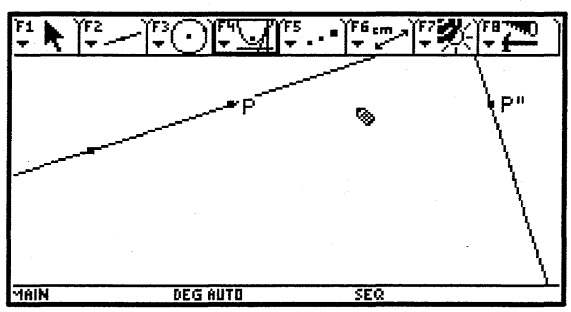


Figure 10. –Locus of P' when P moves on the line

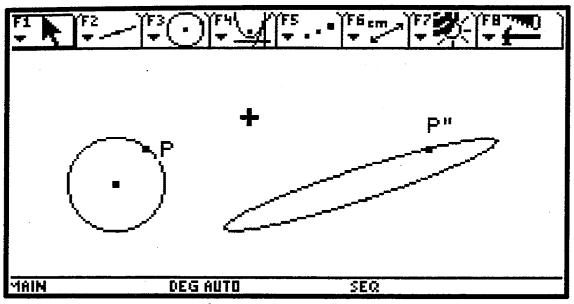


Figure 11. Locus of P' when P is moving on a circle.

Such a task offers a very different point of view on the notion of geometrical transformation. Instead of studying the effects of a known transformation, students are asked to characterize the transformation by means of its properties. Of course this may be an attractive task only if some exotic transformations and not only the usual ones are given to students. Theorems of invariance receive a new meaning in this kind of task: they are tools for identifying the category which the unknown transformation belongs to. An effect of this kind of task is that students may understand why to study all these theorems about invariant elements of transformations. The invariance properties become remarkable phenomena instead of being the routine.

II.2 - Tasks for Producing or Reproducing Visual Phenomena

The construction tasks in Cabri-geometry differ from construction tasks in paper and pencil environment in that

- the constructed diagram must keep all the expected geometrical properties in the drag mode
- to obtain drag mode proof properties, the construction must be done using the primitives of Cabri, given in form of geometrical terms : "parallel line", "perpendicular bisector", "reflection"...

Producing a diagram requires thus in Cabri-geometry explicit knowledge of geometry .

Cabri-geometry offers construction tools that are not existing in paper and pencil environment like vectors or transformations. Construction tasks may receive new efficient solving strategies.

Example with the tool vector:

Construct a triangle ABC from the given points A, B and M centroid of triangle ABC.

Point C is theoretically determined by the relation :

vector MA + vector MB + vector MC = vector 0. (1)

In paper and pencil environment, it is impossible to make direct use of this relation. The sum MS of vectors MA and MB must be constructed through the parallelogram construction and point C is then constructed so that M is midpoint of CS. This strategy is not simpler than a purely geometric strategy consisting in considering M as a point at two thirds of a median. Thus the vector relation is not efficient for the task in paper and pencil environment.

In Cabri, C can be constructed only by two operations as the symmetrical point with respect to M of the endpoint of the vector sum of the two vectors MA and MB (see Figure 12).

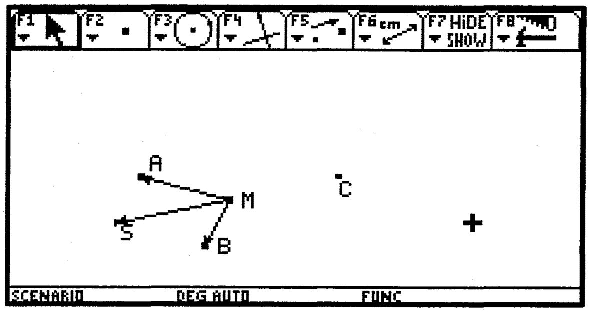


Figure12.

This solution requires to change relation (1) into
vector MC = -(vector MA + vector MB) (2)
and to identify a point symmetry in relation vector MC = - vector MS.

Cabri contributes thus to linking the algebraic aspects of vectors to the geometrical aspects. Relation (1) is restricted in paper and pencil environment to algebraic calculations, whilst in Cabri, it also receives a geometrical meaning since it is a tool of construction. It offers a new connection in the conceptual field of vectors (Vergnaud, 1991) or in the web of vectors (Noss & Hoyles, 1996).

In the same way, a geometrical transformation like point symmetry can be used as a tool for constructing a parallelogram. When students are given the task of constructing a parallelogram, they all do it by using parallel lines and the parallelogram collapses when moving a vertex until three vertices are collinear. The construction by point symmetry appears as more powerful because preserving the parallelogram even when it is flat. The environment Cabri-geometry reveals thus the power of point symmetry under two aspects: the operational aspect as a construction tool and the generality of this tool allowing a construction resisting to a limit case.

The property of symmetry of the parallelogram in a paper and pencil environment is mainly used for proving, dynamic geometry software allows the students to experience this property in action before using it at a more formal level (Laborde, 1995).

In paper and pencil environment, vectors and transformations are only used in reasoning and proofs on theoretical objects. It has been often observed that it is quite difficult for students to have recourse to them in their proofs. They prefer to use Euclidean arguments. We assume that Cabri allows tasks in which theoretical objects like vectors or transformations receive a kind of reification by being tools of construction.

Tasks for Reproducing Visual Phenomena

In this kind of tasks, the students are given a diagram on the screen of the computer and they have to reproduce it. These tasks are an extension of the first kind of tasks. They combine interpretation and construction.

Example

A triangle ABC and a point U depending on the triangle are given (Figure 13). Another triangle A'B'C' is given on the screen of the computer and the task of the students is to reconstruct point U' depending on A'B'C' as U depends on ABC. The students must identify the geometrical relationships between U and ABC in order to be able to construct U'. It can be done by dragging A, B or C and observing the behaviour of U (Fig.14). As students assume that it must be a remarkable point (effect of didactical contract), they can observe the limit cases and infer from them whether U is the orthocenter of ABC, the center of its circumscribed circle or its centroid. If U is the orthocenter, it must be coinciding with a vertex of ABC when the corresponding angle is right. If U is the center of the circumscribed circle, it must coincide with the midpoint of a side when ABC is a right angled triangle. Checking can be done by drawing lines AU, BU and CU and identifying their properties. Mathematical reasoning based on mathematical knowledge of the definition of the points is involved in this recognition task : "If it is this point, it should behave this way".

The construction of U' requires the use of the identified properties determining point U.

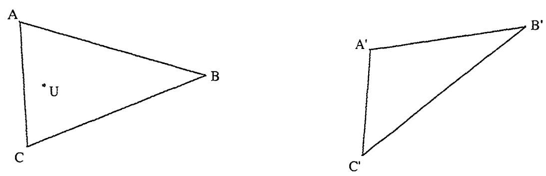


Figure13.

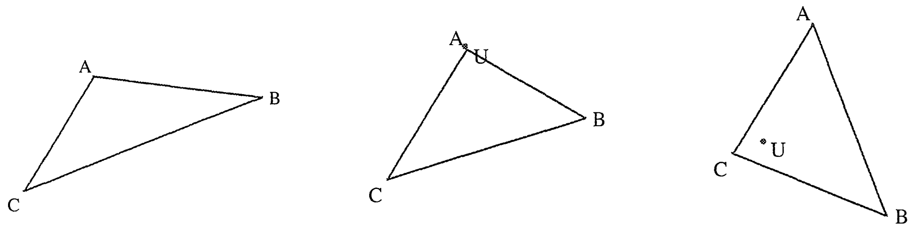


Figure 14.

The Possibility of Inverse Problems

All these tasks, interpretation, (re-)production, differ from usual tasks in which an object is defined and the definition is to be used for finding the properties of the object. Here the problem is inverse, an unknown mathematical object is given under the form of a dynamic "intelligent" diagram and one has to find its definition by experimenting mathematically on it.

Theoretical knowledge is a means of modelling the behaviour of spatial graphical objects, like in the LEGO experiment described in Isoda & al. (2001) in which students have to explain why a mechanism made in LEGO provided a rotation of 60°. This kind of activities requires the use of reasoning mathematically and thus is fostering learning since LEGO mechanisms or Cabri-geometry offers feedback invalidating wrong assumptions.

II.3 – Prediction tasks

Prediction activities are also interesting for the learning of geometry thanks to feedback provided by Cabri: the student may become aware of the inadequacy of his/her expectations when confronting his/her prediction with the observed result on the computer. Ex : what will happen to the image of a polygon through a translation if you move the vector of translation ? or how to modify the vector of translation so that a circle and its image become tangent ? We must stress that the facility of relaxing or modifying conditions (in Cabri "redefine an object") is a very good means of asking students to make predictions: "what will happen if this condition is no longer satisfied ?" or "what will happen if this condition changes into that one?"

II. 4 - Tasks for Explaining Visual Phenomena

Dynamic geometry software may be used for creating intriguing visual phenomena which are not expected by students. The only way of explaining those phenomena is the recourse to theory.

For example, students are asked to observe the behaviour of the sum of vectors MA and MB when M is dragged. They usually do not see anything remarkable. It they are asked to activate the trace of the sum when M is dragged, they immediately see that it is passing through a fixed point (Figure 15). This is not without surprising the students who do not expect this phenomenon even if they know that the sum of two vectors is the diagonal of the parallelogram constructed on the two vectors. It creates the opportunity for the teacher to ask why this intriguing phenomenon.

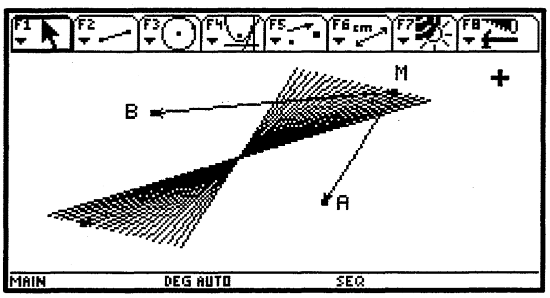


Figure 15.

In a computer environment, the need for proof cannot any longer be favoured by the uncertainty of the result. It may arise for intellectual motives because the student wants to know why a phenomenon takes place. As pointed out by the Piagetian perspective, a means of provoking this intellectual curiosity may be caused by conflict between what the learner believes or predicts and what actually happens. In terms of tasks, this may be done by asking the students to predict properties of the diagram before allowing them to check on the computer, as in the following example (Abd El All, 1996).

Students were given a rectangle ABCD and the quadrilateral IJKL of the midpoints of the sides of ABCD in a paper and pencil environment. They had to determine the nature of IJKL and to justify their answer. All students found that it is a rhombus. Then they had to predict whether IJKL would remain a rhombus in any movement of B which does not preserve ABCD as a rectangle. All students predicted that IJKL would not be any longer a rhombus. In the third step they were given in Cabri (Figure 16) a quadrilateral ABCD having two diagonals equal (B is moving on a circle with center D and radius AC, A, C and D being fixed).

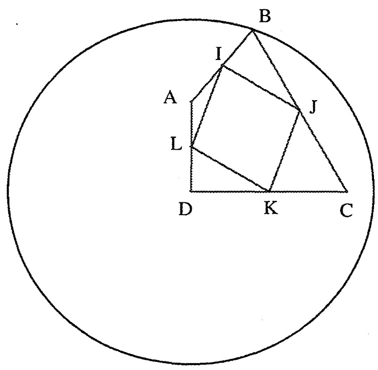


Figure16.

Almost all students recognized that IJKL is a rhombus and were very surprised by this fact; they expressed their desire to know why. This did not lead to a successful proof for all students because it turned out that some of them were unable to mobilize the property of the midpoints line in a triangle. As in a paper and pencil environment, the interventions of the teacher if they come at a right time may play a critical role not only in the solving process of the specific problem but also on the students themselves: after the students have clearly defined a question (here for example, how to prove that the sides of IJKL are equal) and have been confronted to several unsuccessful trials to solve it. In such conditions, even though the property of the midpoints line is brought by the teacher, this property may receive the meaning for the students to be a means of proving that a segment length is half of another one because it answers a question they could determine on their own and express.

Problems of inexistence of objects seem to foster the move from spatial-graphical level to the theoretical level. In absence of existence of any instance of an object, only the recourse to theory may justify the inexistence. However the observations of students in such a problem showed that the recourse to theory may not take place.

The question of existence of a triangle in which two perpendicular bisectors are perpendicular which was asked to 12-13 year old students in both environments (paper and pencil and Cabri-géomètre) seemed for example to raise more justifications in a Cabri environment than in a paper and pencil environment in which students only tried to solve by checking on diagrams. In a paper and pencil environment, they did not try to have recourse to a proof because they were uncertain of the inexistence whereas in Cabri they were visually convinced of the inexistence of such a triangle and try to find reasons to this surprising visual phenomenon (Abrougui, 1995). Finally the intriguing phenomenon led also in this situation to a need of proof. As stressed by De Villiers (1990, p. 18), "proof is not necessarily a prerequisite for conviction, conviction is far more frequently a prerequisite for proof".

For students, conviction certainly comes to a great part from the observation of spatial-graphical properties. This brings us back to the importance of the ability of recognizing these properties... We would like to conclude that learning geometry should be achieved by learning in interaction to deal with diagrams, to make experiments on them. The new generation of computer dynamic diagrams renews dramatically the situation.

III – The Process of Integration: Two Case Studies

III.1 – Design of Teaching Scenarios Based on Cabri by Teachers

Our experience of designing teaching learning scenarios with teachers based on Cabri-geometry and integrating them in the regular course of the teaching shows that it takes a long time before reaching the adequate tasks that take advantage of the technology environment and find the good time management in the classroom.
Over several years, our team of teachers designed successive versions of the same scenarios. The evolution of their features can be summarized as follows.

The first versions called often for immediate visual observations and generalization by inductive reasoning. One of the teacher who was a novice teacher used mainly technology as a provider of data and the solving of the tasks did not require the use of Cabri. These tasks gave a great role to measuring and did not use the animation facility of numbers, or even the continuous dragging possibility.

In the first versions by experienced teachers, Cabri was supposed to facilitate the mathematical task that was considered as unchanged: Cabri was used as a visual amplifier (Pea, 1985). For instance, in tasks of identifying properties, it was assumed that it was easier to observe that three lines always intersect in one point during the deformation of the diagram by the drag mode than in a static paper and pencil diagram. The need for proof was less important than in a paper and pencil environment.

A teacher designed a task about the composition of point symmetries and he wrote about it:

> It is just about grasping at an intuitive level a possible generalisation of results just observed in particular cases. It is necessary to remain modest at the level of Seconde (Grade 10) and to take into account the real possibilities of students. The computer environment turns out to be useful here as a means of exploration and help to conjecture: If the number of the polygon is even, it is the image of MNPQ through a translation. If it is odd, it is the image through a point symmetry.

He wrote about a task of conjecturing properties of dilations:

> The animation of the screen (which results from the use of the tool "Dilation" in conjunction with dragging) implies that the students acquire without too much investment a global visualisation of a dilation: the effect of Dilation on usual figures and their main properties which are conjectured after multiple trials that they can be achieved in a short period of time. [...] In paper and pencil environment, the properties are immediately required to construct the image of a figure.

The role of the software was to save time, to avoid complex constructions requiring the use of properties that are exactly the properties to be discovered, and to favour visualisation. The declared intention of the teacher was to keep the demands of the task at a modest level. The role of Cabri was mainly to facilitate conjecturing and not to cause a problem, as in construction tasks, where the solving strategies have to be constructed with the Cabri tools.

Only the later versions introduced two new kinds of tasks:

- tasks in which the environment allows efficient strategies which are not possible to perform in a paper and pencil environment, such as construction tasks in which tools offered by Cabri were used for constructions;
- tasks raised by the computer context, i.e. tasks which can be carried out only in the computer environment, such as reproducing visual phenomena or prediction tasks.

One of the teachers who introduced the task of constructing a triangle from its centroid and two vertices did it because he was aware of the change of strategy and of the efficiency of vectors and transformations as tools of construction. He commented on it in one of the research meetings, stressing that it was a completely new kind of task belonging to a new culture in which theoretically complex objects become construction tools. He supported the idea that this culture needs time to be developed and must be introduced early in the school year. It was clear from his comments, that this teacher was aware of the conceptual change involved in this kind of task. It is interesting to note the evolution of this teacher, who at the beginning was mainly using Cabri as provider of dynamic imagery for facilitating formulations on the theoretical objects evoked by the diagrams. In this case, the theoretical objects are tools *operating* on the diagrams.

The inverse problems of studying a transformation not from its definition but from the behaviour of a point and of its image on the screen was introduced very late by the teachers and mainly under influence of researchers.

III.2 – Epistemological Beliefs

The design of tasks is based on implicit assumptions about the way students learn mathematics and about the mathematical content itself. The reactions of the teachers when integrating technology could be used as a window on their own epistemology (Noss & Hoyles, 1996, ch. 8).

From these reactions some hypotheses could be made about their beliefs about mathematics and their conceptions of learning. Three types of reactions will be commented below in terms of possible beliefs:

- dichotomy between conjecture and explanation or proof
- repetition of the same tasks in Cabri and in paper and pencil environment
- complexification of tasks

Dichotomy Between Conjecture and Explanation or Proof

The most obvious contribution of Cabri is the possibility of dynamic visualisation of geometrical relations preserved by the drag mode. Teachers (even the novice in using technology) immediately exploited this possibility by asking students to conjecture properties from what they could see. However, when the students were asked to justify, the teachers did not mention the possibility of using Cabri to find a reason or to elaborate a proof. It is as if there was no interaction between visualisation and proving. Technology was used in these tasks, as facilitating the formulation of conjectures but its role did not go beyond that. Quoting the formulation of Hölzl (2001, p. 65), we would say that the drag mode was "used only in a verifying manner" and that "learners are just supposed to vary geometric configurations and confirm empirically more or less explicitly stated facts". A heuristic context was not really created in which the dynamic geometry environment supported a solving process of proving based on experimentation in the environment (trials of particular cases with the drag mode, change of conditions on the givens by using redefinition).

It was as if the process of elaborating a proof should deal with theoretical objects unrelated to their representations, not modified by actions on these representations. Bosch and Chevallard (1998) argue that mathematicians have always considered their work as dealing with non-ostensive objects and that the treatment of ostensive objects (expressions, diagrams, formulas, graphical representations) plays just an auxiliary role for them. This conception, according to which mathematical concepts exist independently of their representations, and which does not take into account interactions and mutual controls between non-ostensive and ostensive objects, seems to underpin this dichotomy.

It is not without relation to another conception: the intrinsic link of geometry with paper and pencil that is presented below.

Repetition of the Same Tasks in Cabri and in Paper and Pencil Environment

One of the teachers who was novice in using technology did not rely on technology based activities for learning geometry and, in addition to technology based activities, proposed similar paper and pencil tasks seemingly unaware that a paper and pencil task may be less demanding in terms of knowledge, by allowing perceptive strategies instead of strategies based on theoretical properties. It seems that she had an epistemological view of geometry as intrinsically linked to paper and pencil. This belief of the canonical form of mathematics linked to paper and pencil environment is widely shared. Povey and Ransom (2000) report about an inquiry carried out among undergraduate students in mathematics in UK. Each of them seemed to refer to a single specific mode of understanding mathematics, - the paper and pencil mode (pp.52-3). “Technology can help if you have a paper and pencil understanding” told one of the students and this formulation could be taken as summarizing the philosophy of the scenarios written by this teacher. As expressed by Povey and Ransom, the underlying learning assumption is that “doing maths by hand indicates that one understands it”. This is exactly the type of claim made by the teacher when she explained to us that, without the material action of drawing the image of a straight line through dilation with a straight edge, students would not appropriate this invariant of dilation.

This point of view is often linked with the conception of a paper and pencil environment as ‘not a context’. Knowing how to carry out a construction in paper and pencil environment would be the warrant of de-contextualised knowledge. Noss and Hoyles (1996, p. 48) propose an alternative view of abstraction as not necessarily linked to de-contextualisation and “as a process of connection rather than ascension”. They add that the “situated, the activity based, the experiential can contain within it the seeds for something more general” (*ibid,* p. 49). In the interaction with the computer, learners may construct what Noss and Hoyles call *situated abstractions*. Situated abstractions are invariants that are shaped by the specific situation in which they are forged by the learner. Although those invariants are situated, they simultaneously contain the seed of the general that could be valid in other contexts:

"Within a computational environment, some at least of these objects and relationships become real for the learner (we are using 'real' here to mean something other than simply ontologically existent-perhaps meaningful or broadly connected are better descriptions): learners web their own knowledge and understandings by action within the microworld, and simultaneously articulate fragments of that knowledge encapsulated in computational objects and relationships-abstracting *within*, not *away from*, the situation. In computational environments, there can be an explicit appreciation of the form of generalized relations within them (the relational invariants) while the functionality and semantics of these invariants-their meanings- is preserved and extended by the learner" (p. 125).

Such linkage between understanding and paper and pencil may also be explained by the institutional context. Even if all kinds of calculators are allowed in our French national examination, all examination tasks are given in a paper and pencil environment. The teacher thus prioritises this context to be sure that students are able to perform the tasks in the examination environment.

Complexification of Tasks

Experienced teachers involved in teacher education, such as the teachers we worked with, very often have a constructivist view of learning based on two assumptions:

- Students learn when they are faced with tasks for which mathematics notions are efficient tools of solutions;
- Feedback coming from the situation may favour an evolution of solving strategies more than a judgement coming from the teacher.

Feedback coming from dynamic geometry software may from this point of view be very rich in that it allows an interaction between the visual and the theoretical aspects of geometry. If a constructed diagram in the drag mode does not keep the shape that was expected, it means that the construction process must be wrong. The drag mode can also invalidate a conjectured property and thus lead the students to abandon it.

The teacher may rely too much on feedback from the calculator/computer and propose tasks of a greater complexity than corresponding paper and pencil tasks. The teacher underestimates the complexity of the task, and the time needed for the student to solve the task because he has little reference in his experience. He overestimates the possibility of interpretations by the student of feedback given by the software.

We observed this in the first version of a scenario on vectors in which students were asked to construct all diagrams for the tasks in Cabri. Instead of teaching vectors for two weeks, it took two months! It is also a common phenomenon that any kind of teaching innovation provokes time inflation. Schneider (1999) repórted on teaching about logarithms and exponentials based on the use of the TI 92 which took 40 hours of teaching instead of the usual nine hours.

Returning to the project, after the first year, the teachers attempted to find an optimal balance

- between what is prepared and demonstrated by the teacher on the LCD display and what is done by students,
- between what is ready made and given on the calculators to students and what has to be done by the students with the software.

For example, after one year the teachers preferred to give the macro-construction of the multiplication of a vector by a number for students to explore and interpret rather than for the students to construct themselves. Even apparently minor aspects may slow down the construction of a diagram. In the scenario "Vectors", the first task was about polygons that students had to draw. To this end, they had to designate the successive vertices of the polygon and, at the end of the sequence, again the first vertex. Actually it turned out that students tried to do polygons with a large number of sides, and sometimes had difficulties in designating at the end exactly the first vertex and not a close point. If they had used a double click on the last vertex of the polygon, it would have avoided difficulties. But the teacher did not anticipate the long time spent on drawing the polygons and mentioned this shortcut only orally during the activity. This meant that only some students paid attention to his remark.

Evaluating the complexity of a task with technology requires taking into account not only the conceptual difficulties but also the use of the technology by the students. This is not easy and the wrong a priori evaluation by the teacher of the complexity of the task came from an absence of reference about students' behaviour in the tasks. The complexification of tasks may also come from the uncertainty of the teacher about what students would learn from the tasks. They tended to presuppose that technology would facilitate the solving process, so that in order to be sure that students learned something from the new kind of tasks, the teachers increased the level of complexity. In all cases, it is clear that a deep and precise knowledge of students' behaviour and strategies in the Cabri environment is essential for evaluating a priori the degree of difficulty of a task.

III.3 The Design of Tasks by Prospective Teachers[1]

In order to evaluate the effect of teacher education to the use of Cabri-geometry in the teaching, we asked prospective teachers to design tasks allowing students of grade 8 or 9 to overcome difficulties in proof[2] tasks.

The prospective teachers were in their last year of professional education. They were in charge of a class (4 to 6 hours teaching per week) and followed two days of professional development sessions per week at the university institute for teacher education. They receive education in pedagogy of mathematics and "didactique" as well as help for practical problems in teaching. They also have to carry out a small research project about a teaching problem they define themselves.

They followed general sessions about the use of Cabri during six hours introducing them to the features of Cabri and helping them to solve mathematics problems with Cabri. These sessions were purely devoted to the learning of how to use Cabri. After these sessions, four prospective teachers were given fictive proofs to problems supposed to be written by students of grade 8-9. They were asked

- to analyse the errors occurring in the solutions
- and to design tasks based on Cabri and meant for helping students overcome the difficulties they encounter when writing proofs.

Then these four students attended sessions (altogether 12 hours) on the way to integrate Cabri into the mathematics teaching in which they had to reflect on the changes brought by Cabri on the notion of figure as well as on new tasks made possible by Cabri.

Two months after the sessions, they were given the same type of tasks as in the first experiment with the only change that after they proposed some tasks, they were requested by the experimenter to design tasks

- using the ambiguity, or the replay of the Cabri construction
- and tasks of reproduction of a dynamic Cabri-diagram.

Examples of tasks and of their fictive solutions given to the prospective teachers

Task 1:

The midsegment theorem says: "In a triangle, if a line is passing by the midpoint of a side and is parallel to the third side, then it cuts the second side in its midpoint."

Its reciprocal says: "In a triangle, if a line is passing by the midpoints of two sides, then it is parallel to the third one."

Explain all differences that you see between the two statements.

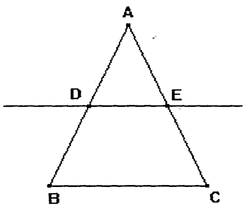
Solution 1

For both statements, there is a triangle and there is a straight line which cuts the two sides in their midpoint. For the first statement, this line cuts the second side in its midpoint and for the second statement, this line is parallel to the third side.

Solution 2

[1] This work is a master thesis (called in French DEA) of Seden Tapan supervised by H. Chaachoua and myself that will be defended in June 2002 (Tapan 2002)

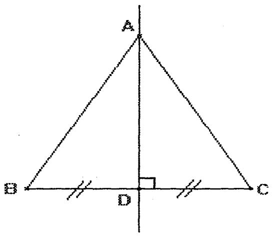
[2] Proof is introduced in the curriculum in France at the beginning of secondary school (grade 6) and becomes an usual task at grade 8.



These both statements say almost the same thing but they only turn the sentence the other way. Because in the two statements, there is a triangle ABC and there is a line (DE) that is passing through the midpoints of segments AB and AC and which is parallel to line (BC).
Hence I can say that if the one is true then the other one is also true.

Solution 2 does not distinguish statements in function of their status in a proof, hypothesis or conclusion of a step and considers only the content of each statement. Solution 1 distinguishes statements trough their position in the statement (in the first part or in the second part after "then") but no more than that. As solution 2, solution 1 does not express the status of each part of a deductive step.

Task 2:



In the above diagram, BD = DC and (AD) ⊥ (BC.)
Prove that ABC is an isocèles triangle.

Solution 1
In an isosceles triangle, the height is also the perpendicular bisector. As line (AD) is height and perpendicular bisector, ABC is isosceles.
Solution 2
As ABC is an isosceles triangle, line (AD) is height and perpendicular bisector. Thus ABC is isosceles since its height is also perpendicular bisector.

Solution 1 seems to express the status of a statement in a deductive step but mentions the reciprocal theorem instead of the adequate one. Solution 2 may be interpreted in that the student is unable to recognize the status of a statement and to distinguish between hypothesis and conclusion, he/she just formulates statements by using external signs of deduction.

To students who do not consider the status of statements but only their content, Cabri offers several ways of externalising this difference:

- Students who do not see the difference between hypotheses and conclusions may infer that properties are true just because they seem true on the diagram (confusion between spatial and theoretical relations, cf. §I). The drag mode allows to eliminate properties purely taken from the diagram that seem to be true on the particular case of the diagram
- The drag mode can also be used to show by relaxing a property P that as soon as P is satisfied, property Q is also satisfied. When dragging the point on the first side, as soon as it is the midpoint, the line parallel to the third side cuts the second side in its midpoint (task 1). The tool Redefinition can also be used in this way.

- In the construction process are used only the given of the problem whilst the conclusions appear as staying true in the drag mode although not used for the construction of the diagram.
- The tool ambiguity shows that two geometrical objects have the same spatial position. This is exactly what a theorem expresses. In task 1, in the statement "In a triangle, if a line is passing by the midpoint of a side and is parallel to the third side, then it cuts the second side in its midpoint", the intersecting point of the side and the parallel line (hypotheses) is also the midpoint of the second side (conclusion). It results that in some cases, ambiguity reveals the difference between the given used in a construction and the conclusion.
- The replay of a ready made construction allows the user to see the objects used in the construction, in particular the objects that have been hidden by the designer of the construction. It also may be a means of externalising the difference between hypotheses and conclusions.
- Finally when trying to reproduce Cabri diagrams, the student must be able to identify those which characterize the construction and then imply the other properties he could observe.

From the observations the two pairs of prospective teachers designing tasks, it appears that before the specific teaching on the pedagogical use of Cabri, they were only able to use the construction process and the drag mode to show the difference between properties just been taken as granted from the diagram and proved properties. Only one pair could also use construction to show the difference between hypotheses and conclusion. After this teaching, both pairs could use construction and drag mode to make visible the difference between hypothesis and conclusion. They could use ambiguity only when requested to do this and about a specific task which was formulated to favour this use (The task consisted of proving that two points were coinciding). The drag mode was not used by them to make students aware of the sufficiency of a property with respect to another one. It was difficult for these teachers to use successfully the replay of a construction and situations of reproduction of a Cabri-diagram. This shows clearly the gap between the use of a tool for solving tasks and the use for designing tasks. The possible uses of Cabri described above for overcoming difficulties in proving were not taught in an explicit manner and had to be constructed by the teachers.

IV. Conclusion

In our introduction we claimed that the process of integrating technology into mathematics teaching is a long and complex process. In analysing the types of tasks developed by teachers over the three years of the project and their evolution, we can formulate tentative explanations for the length of this process.

As the didactic system as a complex system, technology is not just an additional element in the system since it interacts with all the components of the system, which are subject to change. This point of view is based on two theoretical approaches, the notion of instrument as developed by Rabardel and Vérillon and the mediating function of a computational learning environment (Noss & Hoyles, 1996). Vérillon and Rabardel (1995) stressed how an artefact is not taken as such by the learner but reconstructed by him/her. The learner constructs both a representation of the artefact (the instrument) and the structures that allow him/her to perform activities with the artefact (schemes of utilisation of the artefact). Both types of constructs depend on previous knowledge of the learner and affect this knowledge. According to a Vygotskian perspective, Rabardel claims that the "instruments" constructed by the learner constitute forms which structure the relationships with situations and knowledge and thus may have a considerable influence on the construction of knowledge. Noss and Hoyles (1996) investigated many years how learners construct situated abstractions dependent on the means of action and expression offered by the environment. Students construct an instrument in function of the tasks they have to solve, i.e. mathematical tasks. We assume that the tasks of the teachers differ from those of students. The teachers must be able to use Cabri for creating tasks to be solved by students. They must be able not only to consider the Cabri tools as tools for solving problems but also to consider them as mediating mathematical knowledge: construction combined with drag mode may mediate the status of a statement with respect to deduction; the locus or trace may mediate the notion of a geometrical object as a set of points.

We interpret the behaviour of novice teachers in the design of scenarios as resulting from their perspective that technology is an additional component of the teaching system but external to the learning processes. Technology was facilitating material aspects of the actions of the students (teacher novice in teaching), technology was used in observation and construction tasks but activities in paper and pencil environment were given in addition by the teacher who was a novice in the use of technology. It is interesting to note that this latter teacher planned what might be interpreted as a more verifying or test way of using the drag mode than search way (in Hölzl's terms). In the observation tasks that she gave, all steps of the conjectures were given explicitly.

A second interesting feature of the design process of the sçenarios by the experienced teachers can also be interpreted in terms of instrumentation and mediating function of the environment. These teachers offered more open exploration activities involving more a search use of the drag mode; they did it in two kinds of circumstances: at the beginning of sessions in observation tasks to introduce new properties and at the end of sessions in open problems to be solved. But the comments they added, expressed clearly that the drag mode was for them more facilitating visualisation than acting in the solving process, even for the open-ended problem. It took one or two years for them to accept that investigating the invariants of an unknown transformation under the form of a black box situation through the drag mode and the tool "Redefinition" could be part of a scenario. The difference between a reproduction of a Cabri diagram and an observation situation for conjecturing must be stressed here. A situation of reproducing a Cabri diagram is a problem situation and the invariants are the tools of solution of this problem. In an observation situation, in which students are asked to conjecture properties, the question is more to satisfy a contract of finding properties relevant from the perspective of the teacher.

We assume that really integrating technology into teaching takes time for teachers because it takes time for them to accept that learning might occur in computer-based situations without reference to paper and pencil environment and to be able to create appropriate learning situations. But it also takes time for them to accept that they might lose part of their control over what students do. Povey and Ransom (*op.cit.*) concluded from their inquiry among undergraduate students (already cited above) that the plea for learning by doing 'by hand' could be related to a "desire to feel in control"(p.56). As they mentioned, speaking about technology as "taking over" and depriving the human of control is usual in a wider social context. The situation is far more complex for a teacher who must not only understand what the computer does but also what the students do with the computer.

Cabri covers a broad domain of knowledge and action. It is a microworld allowing multiple ways of exploring, experimenting and solving a problem. If the basic use of Cabri can be learned rapidly because of its friendly interface, constructing a global and structured representation of all of its possibilities requires time. It requires even more time to analyse the possible uses of Cabri in terms of mediation of knowledge, and to construct correspondings tasks.

In the same way as teachers do not have to reconstruct all exercises and problems that they give to students, it is not expected that teachers should on their own find the adequate situations to use technology. Research and investigation should be carried out in order to have a better knowledge of students learning with technology. The results and data of these investigations could then be transferred to teacher education. This is why we consider that research on the integration of technology into maths teaching is important.

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