# Mental Computation Competence Across Years 3 to 10

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Mental computation is an important aspect of numeracy. There is little information available, however, about appropriate sequences of learning, or what can be expected of students in different years. The responses of 3035 students in years 3 to 10 to linked tests of mental computation were placed on a single scale using Rasch modelling techniques. An eight level scale of mental computation development was identified, and student achievement across the years was mapped onto these levels. The findings and their implications for the classroom are discussed.

There is an increasing emphasis on the place of mental computation in today's classrooms as part of a suite of computational strategies that students should develop over the years of schooling (e.g. Australian Education Council, 1990). This emphasis implies that teachers will monitor students' progress in mental computation. There is little research base, however, to inform the assessment process. This leads to varying expectations as to what students should be expected to be able to do at different stages of education. At the end of year 2, for example, the Victorian Curriculum and Standards Framework (Board of Studies, 1994) expects automatic recall of addition and subtraction facts up to 20, whereas at the same stage of schooling the Tasmanian Key Intended Numeracy Outcomes (Department of Education, Community and Cultural Development, 1997) expect addition and subtraction of numbers to 10 and extensions based on place value, with only some facts committed to memory. The National Numeracy Benchmarks for year 5 require basic multiplication facts to 10 x 10 and some extensions of these facts (Curriculum Corporation, 2000) and these same outcomes are expected for most students by the end of year 4 in the New South Wales Outcomes and Indicators statements (Board of Studies, 1998). There is very little expectation for mental computation outside of the "basic facts" for whole numbers, and none at all for decimals, percentages and fractions.

The research base that exists is generally based on small interview-based studies that focus on strategy use rather than outcomes. In one larger scale study, Bana and Korbosky (1995) used sets of ten "basic facts" in each of the four operations with 390 students across years 3 to 7. Using raw scores as the basis they demonstrated an increased performance of students from years 3 to 4 but a much smaller increase between years 5 and 6. This did not, however, provide generalisable information about generic kinds of mental computational problems that students might be expected to be able to solve at different stages of schooling. McIntosh, Nohda, Reys and Reys (1995) used a test of mental computation in a study across three countries at four different year levels. Although there were some common items, they were unable to compare years directly since the test administered to each year was different. There was some evidence of growth across years from performance on the common items, and also of curriculum effects across the three countries. Callingham and McIntosh (2001) used Rasch modelling techniques (Rasch, 1980) to produce a developmental scale of mental computation based on the responses of nearly 1500 students across years 3 to 10. Although an initial comparison across years was undertaken, no attempt was made in this study to suggest reasonable expectations for

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students' outcomes in each year. The study did, however, establish Rasch modelling as an appropriate approach to the problem associated with using a large item pool across multiple years.

The aim of this current study was to confirm the developmental nature of the mental computation construct, and to establish reasonable expectations for student outcomes in each year from 3 to 10. To this end the research questions were:

1. What are the features of a hierarchy of mental computation development?
2. What happens to the performance of students as they move through school?
3. What are reasonable expectations of mental computation achievement for students in years 3 to 10?

## Methodology

Tests of mental computation were developed for adjacent years, years 3/4, years 5/6, years 7/8 and years 9/10, based on the tests used in a previous study (Callingham & McIntosh, 2001). Since increasing the wait time did not make the item easier, it was decided in this study to reduce the pause between questions from five seconds to three seconds for the first part of each test (*Short* items) and to maintain the 15-second pause for the second part of the test (*Long* items) to differentiate between "instant recall" and "working it out" approaches. In addition, nine items that were common across all year levels of the test were presented with a 5-second delay to enable linking to the previous study (*Link* items). Items were recorded and schools provided with audiotapes of the appropriate tests for each year that also contained all instructions for students. In this way the item presentation was consistent across all years. The Short and Link items comprised the first part of the test, with a short break before the Long items were administered. Students in years 3 and 4 were given slightly fewer items than older students to minimise test fatigue. Students were provided with an answer sheet and a pencil only, and instructed that they were not to write anything other than the answer. The test formats are summarised in Table 1.

Table 1
*Test Formats for Mental Computation Tests*

| Year | 3/4 | 5/6 | 7/8 | 9/10 |
|---|---|---|---|---|
| Short items | 24 | 24 | 24 | 24 |
| Link items | 5 | 5 | 5 | 5 |
| Long items | 21 | 36 | 36 | 36 |
| Total | 50 | 65 | 65 | 65 |

Each year level test had four forms linked within years and between years by common items. This allowed a large pool of items to be presented. In all 244 items were used, 87 Short, 9 Link and 148 Long items. These addressed whole number operations, fractions, decimals and percentages. Each year level test contained mainly items that were considered to be appropriate to the particular year level and the tests were organised by item topic, addition, subtraction, multiplication and division, rather than using a mixed operation format, following methodology from previous research studies (McIntosh, Bana and Farrell, 1995).

The tests were administered to 3035 students in 8 primary, 2 high and 2 combined primary/high schools in the Tasmanian Government and Catholic sectors and the ACT Government sector. The distribution of students across years is shown in Table 2.

Table 2
*Number of Students Across Years*

| Gr3 | Gr4 | Gr5 | Gr6 | Gr7 | Gr8 | Gr9 | Gr10 |
|---|---|---|---|---|---|---|---|
| 436 | 441 | 420 | 445 | 403 | 344 | 343 | 203 |

The data were scored as correct or incorrect; items presented to a student but omitted were scored as incorrect. Rasch analysis of the data was undertaken using Quest v2.1 software (Adams & Khoo, 1996). Initially, the Link items were calibrated as an anchor file. In the subsequent analyses, the tests were simultaneously calibrated and equated, anchored to the Link items. This placed all items and students on a single scale; the same metric, the logit, is used to describe the difficulty level of the item and the ability of the student (Callingham & McIntosh, 2001). Ability measures for each student were obtained as a basis for further analysis.

A variable map produced by the analysis showed the distribution of the items and students along the mental computation variable. Item clusters along the variable were interpreted in terms of their content and skill demand. Points along the scale where there was a qualitative change in the demands of the item were identified as thresholds between levels of different mental computation competence. In this way eight levels of competence could be described.

## Results

### *Hierarchy of Mental Computation Development*

A content and skills analysis of the item clusters suggested eight levels of mental computation competence. Levels 1 to 3 address only whole number computation, and, apart from "one half add one half", Level 4 also addresses whole number. These first four levels generally reflect curriculum emphases in the lower primary years, except for the early emergence of doubling and multiplication by 10. Simple decimals and percentages begin to appear in Level 5, together with addition and subtraction of halves and quarters. Levels 5 and 6 are where all basic multiplication facts and inverses become secure. In Level 6 the whole number emphasis is on more complex 2-digit computations. Addition and subtraction of decimals with one decimal place appears at this level, as does addition of halves and quarters, including simple equivalences. Unlike whole numbers where addition is typically one or two levels earlier than related subtraction computations, addition and subtraction of decimals and fractions begin to appear at the same level. Level 7 includes a range of complex computations, such as multiplication of 2-digit by single digit whole numbers, multiplication and division of unit fractions and whole numbers by one-half and quarters, and calculation of key percentages. Level 8 predominantly comprises decimal, fraction and percentage computations, and particularly division calculations. These levels are summarised in Table 3.

Table 3
*Levels of Mental Computation*

| Level | Content/Skills | Example |
|---|---|---|
| 1 | *Whole Number*: Add single digits; Add/subtract 0; Double single digit; Multiply single digit by 10; Double small multiples of 10 | 5+3(S); 7+6 (L); 6+7 (L); 8+0; 4x10; 8x10; Double 30 |
| 2 | *Whole Number*: Add larger single digit numbers; Add/subtract single digit to/from 2-digit (no exchanges); Add small single digit to 2 digit over next 10; Small multiples of 3, 4 & 5 | 9+8; 8-3; 3+28; 72+6; 36-5; 4x3; 6x5 |
| 3 | *Whole number*: Basic addition facts to 20/related subtraction facts; Add single digit to 2-digits; Add/subtract multiples of 10 (total <100); 3, 4, 5 tables; Double simple multiples of 10 and 5; Simple halves <20 | 14-8; 11-4; 4+39; 16+9; 20+70; 80-30; 3x6; 8x4; 15x2; 40x2; Half of 18 |
| 4 | *Whole number*: Add two 2-digit numbers (total < 100); Subtract single from 2-digits; Subtract simple 2-digit numbers (no exchanges); Related division 3, 4, 5 tables; *Fractions*: 1/2 + 1/2 | 42+16; 27+25; 36-8; 43-12; 30÷5; 12÷4; 23x3; 12x4; 1/2 + 1/2 |
| 5 | *Whole number*: Table facts/inverses; Add/subtract two 2-digit numbers; Multiply 2-digit number by 10 and 2; *Decimals , fractions & %*: Add/subtract halves & quarters (Total<1); 50%/100% of even 2-digit number | 9x8; 72÷8; 79+26; 65-35; 27x10; 40x5; 1/2 + 1/4; 0.25+0.25; 2/7 + 3/7; 50% of 24 |
| 6 | *Whole number*: Table facts/inverses; Add 2-digits to a 2- or 3-digits; Subtract 2-digits from 2-digits; Multiply 2-digit by small single digit; Halve even 2-digit number; *Decimals, fractions, & %*: Add/Subtract decimals (1 place); Add halves & quarters beyond 1; Subtract familiar unit fraction from 1; Half/quarter of some 3-digit numbers; 25% of some 2-digit numbers | 54÷9; 68+19; 92-34; 105-97; 60x7; 5x24; Half of 76; 0.3+0.7; 1-0.6; 1/2 + 4/8; 1/2 + 3/4; 1-1/3; 1/4 of 120; 25% of 80 |
| 7 | *Whole number*: Subtract 2-digits from 2 & 3-digits; Multiply 2-digit by single digit; Divide 2-digits by single digit; Multiply some 2-digit numbers by multiple of 10; *Decimals, fractions, & %*: Add/subtract decimals (1 or 2 places); Multiply 2-digit number by 0.1/0.5; Multiply decimals (2 places) by 100; Multiply/divide familiar unit fractions, single digits by half and quarters; Key % of simple 2- and 3-digit numbers | 111-67; 49x3; 150÷6; 92÷4; 40x25; 12x20; 6.2+1.9; 0.19+0.1; 1.25-0.5; 0.6x10; 0.37x100; 0.5x48; 0.1÷0.1; 1/2 of 1/3; 3÷1/2; 20% of 15; 75% of 200 |
| 8 | *Whole number*: Divide 2- or 3-digits by single digit; *Decimals, fractions, & %*: Divide single digit by 0.1; Divide decimal (1 place) by 5; Divide simple whole number by 0.5; Add familiar unit fractions; Straightforward % of simple whole numbers | 343÷7; 0.1x0.1; 2÷0.1; 0.2÷5; 90÷0.5; 1/2+1/3; 1/2x3/4; 331/3% of 600; 12.5% of 24 |

### *Student Performance Across Years*

Box and whisker plots of student ability (in logits) across years are shown in Figure 1. The box shows 50% of the ability range and the heavy line in the box is the median value. The ends of the whiskers are at the 10th and 90th percentiles.

There is a general increase in ability across the years. The growth is more marked in the lower years and plateaus through the middle years of schooling. In year 7, although the median value and 50% box increases slightly over the values for year 6, at both the 90$^{th}$ and 10$^{th}$ percentiles the achievement appears to drop compared with year 6.

There is also a wide spread of abilities across all years, with considerable overlap among years. The best year 3 students appear to have an ability above the median of those in year 10, for example. Conversely, the weakest year 10 students have an ability level below the year 3 median. The implications of these findings are discussed further below.

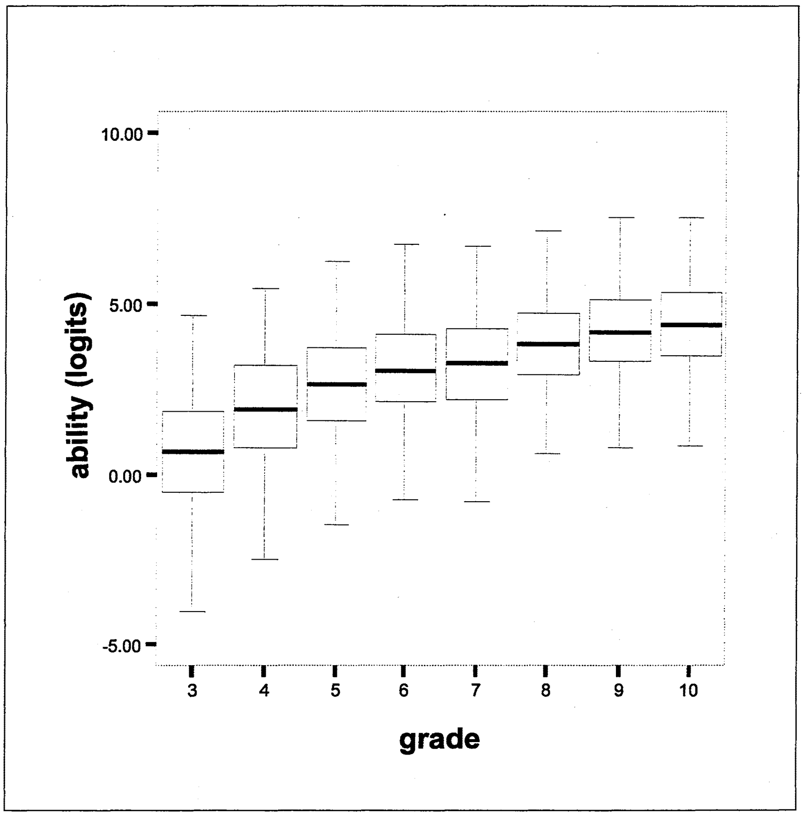


*Figure 1.* Student ability across years

The growth in competence across years is more clearly seen if the difference in mean ability across each pair of years is calculated. This provides a measure of growth rate across the years of schooling, shown in Figure 2.

The greatest period of growth in mental computation competence is between years 3 and 4. Growth is similar between years 4 and 5 and years 5 and 6 but drops sharply between years 6 and 7. It then increases again between years 7 and 8 but there is a gradual decline in growth as students enter the later years of school.

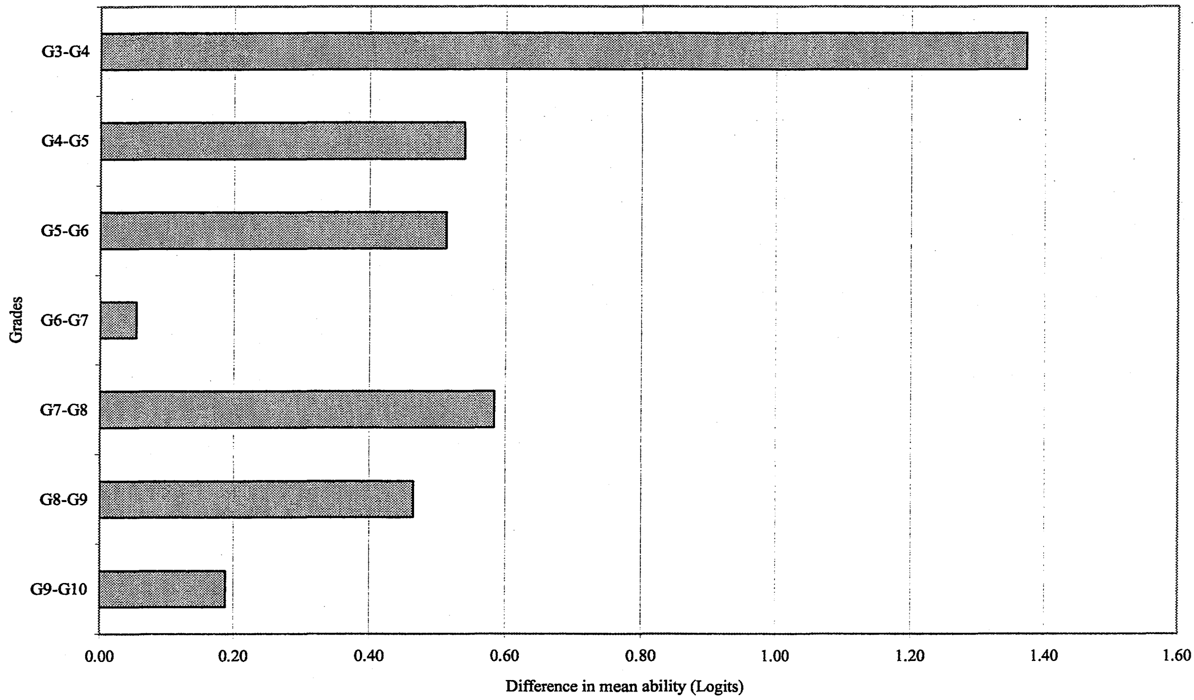


*Figure 2.* Growth in mental computation competence across years.

## *Expectations of Students in Years 3 to 10*

The percentage of students in each year that were in the different levels of competence identified was calculated to determine what was a reasonable expectation of students in the different years. These results are summarised in Table 4. The shaded cells show levels in which 20% or over of the students in any given year appeared, and show a general increase in competence across the years.

Table 4
*Percentage of Students in Each Level by Year*

| Year | L1 | L2 | L3 | L4 | L5 | L6 | L7 | L8 |
|---|---|---|---|---|---|---|---|---|
| 3 | 8.03 | 13.99 | 26.61 | 20.64 | 20.18 | 6.42 | 2.29 | 1.83 |
| 4 | 1.36 | 6.12 | 16.10 | 18.37 | 27.21 | 15.42 | 11.56 | 3.85 |
| 5 | 1.43 | 2.62 | 5.48 | 15.71 | 28.10 | 23.10 | 18.10 | 5.48 |
| 6 | 0.45 | 0.67 | 3.60 | 10.56 | 27.64 | 25.39 | 21.35 | 10.34 |
| 7 | 0 | 1.74 | 2.73 | 9.93 | 25.81 | 22.83 | 26.55 | 10.42 |
| 8 | 0.29 | 1.16 | 1.16 | 5.23 | 15.70 | 24.71 | 33.14 | 18.60 |
| 9 | 0 | 0.29 | 0 | 1.75 | 10.79 | 23.32 | 38.19 | 25.66 |
| 10 | 0 | 0 | 0.49 | 1.97 | 10.84 | 18.72 | 38.92 | 29.06 |

In all years there is considerable spread across levels. In year 3, for example, although the largest group of students was at Level 3 (116/436 or 27%), over 50% of year 3 students were in Level 4 or above. This suggests that year 3 students have greater facility with mental computation than might be expected. Although nearly 30% of all year 10 students appeared at Level 8 the largest group of students in all high school years was in Level 7. Given the content and skills required at the level, this may be a reasonable level to which to aspire by the end of year 10. Similarly, Level 5 would seem to be a reasonable level of expectation for students at the end of primary school. Over 50% of year 6 students are above this level, but the largest single group of years 4, 5 and 6 students appears in Level 5.

## Discussion

The scale of mental computation broadly confirms the scale developed by Callingham and McIntosh (2001) in scope and sequence. Some lower difficulty items were omitted from the tests used in this study and more difficult items included. This has improved the description of the upper levels but has perhaps removed some detail from Levels 1 and 2. The general structure of the scale, however, remains. By Level 5, when decimals, fractions and percentages appear, students would seem to have a good understanding of the principles of the four operations, allowing addition and subtraction, and, in Level 6, multiplication and division to appear together, rather than in adjacent levels as is typically the case with whole numbers. Basic multiplication facts and inverses are not completely secure until Level 6, although multiplication and division by 3, 4 and 5 appear by Level 4. The pattern of multiplication of 2-digit numbers mirrors that of single digit numbers: multiplication by 10 and 2 at Level 5, followed in the next level by multiplication by 3, 4 and 5, and, in Level 7 by any single digit number. The recurrence of these patterns suggests an appropriate teaching sequence for multiplication and related division facts.

When the competence of students in different years is considered the range of ability is a notable factor. The high growth rate between Years 3 and 4 is similar to that described by Bana and Korbosky (1995), and the overall pattern of growth is similar to findings in other research studies that have described a dip in the transition years between primary and high school (Hill, Rowe, Holmes-Smith & Russell, 1996). These findings may reflect curriculum change as well as changed contexts. There is a greater emphasis on knowledge of addition and subtraction facts, and basic multiplication facts in the early years of primary school. This is consolidated in upper primary classrooms, while Year 7 in Tasmania marks the transfer to high school and, often, a difference in the approach to mathematics and the introduction of harder content such as decimals and percentage. The smaller growth in the upper years of high school may be related to the test structure not allowing the best students to demonstrate their competency at the highest levels. However, Figure 1, the box-and-whisker graph, also shows a levelling out at the 10$^{th}$ percentile and this would not be expected if the growth pattern were due entirely to a test ceiling effect. An alternative explanation for the slowdown in growth could be that curriculum documents have no expectations for mental computation of fractions, decimals and percentages, and thus this area receives little emphasis in high schools.

In terms of reasonable expectations of students in different years, it seems that some of the current expectations are a little unrealistic. If the National Numeracy Benchmarks are considered as the current standard, the Year 3 benchmark is about Level 3 on this scale, Year 5 lies at about Level 6 and Year 7 at about Level 7. The National Numeracy Benchmarks are expected to be achieved by about 80% of the population. From the results of this study, the standard set would seem to be reasonable for Year 3 benchmarks, with 78% of Year 3 students at Level 3 and above. For the later years, however, this is not the case. Less than 50% of Year 5 students are in Level 6 or above, and this figure drops to less than 40% of Year 7 students meeting the equivalent benchmark standard of Level 7. More reasonable benchmark expectations, using the current definitions, would seem to be Level 5 for Year 5 and Level 6 for Year 7.

The scale as presented here does not show differences between Short and Long items. There was some evidence that contracting wait time to three seconds did make items slightly harder at the lower levels. This is an area that requires further research into students' strategies, and how these are applied.

Finally, the finding that small numbers of students are achieving at much higher levels than might be reasonably expected for their year level does not necessarily imply that these students can deal with all the content associated with the high level. It is probably unreasonable, for example, to expect a Year 3 student apparently in Level 8 to be able to do all the decimal calculations. To some extent the placement of students in these high levels is an artifact of the form of analysis. Year 3 students were not asked any decimal questions, for example, and the fractions questions were limited to those appearing in Level 5. There is, however, a small group of students who can be identified as very competent at mental computation and these students need to be provided for in the classroom. Similar comments could be made about those students at the other end of the scale.

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