

Secondary Teachers' Attitudes toward Probability and their Teaching Strategies

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This exploratory qualitative research study uses observations and semi-structured interviews to examine secondary mathematics teachers' attitudes toward probability and their use of probabilistic reasoning in teaching two activities designed to extend students' models of exponential relationships. Student learning difficulties present problems for the teachers. Teachers' preparation in probability is typically weak and they are uncomfortable teaching probability. Thus, they emphasised the exponential aspects of the activities, concentrating on developing generalised deterministic equations that model growth and decay.

Many decisions faced in everyday life require probabilistic reasoning for effective evaluation of the alternatives. In addition to obviously probabilistic situations such as games and lotteries, many everyday events are the result of many complicated and interrelated causes that cannot be identified or measured satisfactorily. Media reports involving information explicitly or implicitly based on stochastic data inundate us. Using media information or misinformation, people form deterministic opinions about many topics that affect their actions, opinions, and positions on controversial issues. Without the ability to reason probabilistically about data and data analysis, otherwise well-informed citizens and consumers may make costly mistakes affecting their lives and the lives of others. A review of the literature suggests that student difficulties in learning probabilistic reasoning present unusually difficult problems for the teacher. Watson's (2001) recent study confirms that teachers are aware of these difficulties and it provides some insight into teachers' attitudes, expressions of feeling or behavior intention, toward probability. Many authors (including Borovcnik & Peard, 1996; Konold, 1989; Wilensky, 1997) maintain that teachers need to introduce situations and hold conversations that confront students' misconceptions about probability.

This study is set within the framework of a larger research project, led by the second author, on understanding the development of teachers' knowledge as students engage in technology enhanced modeling activities (Doerr & Zangor, 2000). Two of the activities were designed to elicit students' models about phenomena that involved probabilistic elements. In this study, we explore how teachers responded to these elements of uncertainty embedded within a sequence of deterministic modeling tasks. In this paper, we report on the teachers' attitudes toward probability and their general treatment of the probability aspects of the activities.

Background and Theoretical Approach

The well-known difficulties in student learning of probabilistic concepts has led to criticism of traditional approaches to teaching probability because they reduce uncertainty to a determined formula and fail to confront students' misconceptions (Batanero, Green & Serrano, 1998; Konold, 1991; Lajoie, Jacobs, & Lavigne, 1995). Recommended approaches rely on: teacher understanding of student thinking; teacher initiated conversations that confront student misconceptions; and, learner owned investigations that

allow students to refine their ideas of probability. For instance, Konold (1989) asserts that students must be forced to confront their probabilistic misconceptions by facing situations that demonstrate that their approach is invalid. He concluded that teachers' "understanding of how students are thinking about a topic ... puts the teacher in the position of being able to initiate conversations and design curriculum that can facilitate the development of more adequate concepts of probability" (1989, p. 147). He suggests that teacher initiated conversations should encourage students to evaluate their current probabilistic misconceptions against three criteria: how they fit the beliefs of others; how they fit their own related beliefs; and, how they fit their own observations. These three criteria provide an organisational background for teacher-student interaction in the modeling probability classroom.

After reviewing several possible approaches to teaching probability, Borovcnik and Peard conclude, "Teaching has to focus on establishing stable intuitive insights and thus cannot rely on the presentation of the mathematics" (1996, p. 244). In other words, students have to construct and reconstruct probability concepts themselves. The teacher cannot present probability theory to the students and expect them to internalise and use the concepts appropriately. Wilensky (1997) also criticises traditional teaching, which relies on formalism. He focuses on the student's epistemological anxiety, which he defines as a feeling that one does not comprehend the meanings, purposes, sources, or legitimacy of the mathematical objects being manipulated. A popular response to the difficulties in learning probability has been what he calls the accomodationist's view: a deference to the mechanical manipulations as a guard against unreliable intuitions. Wilensky argues that this avoidance not only exacerbates the anxiety but it also fails to confront the student's misconceptions, precluding the necessary construction of appropriate conceptions. His solution focuses on learner-owned investigations followed by reflections. This approach validates the importance of the student's personal views and relationship to personal experience. It seems that a models and modeling approach (described below), with the teacher actively identifying misconceptions and adapting the curriculum and activities to confront those misconceptions, might be able satisfy all of these teaching strategy recommendations

Are teachers initiating conversations and posing situations that challenge their students to refine their concepts of probability? The response patterns to Watson's (2001) survey seem to indicate that a telling teaching style is common among the secondary teachers in her sample. In the only reference specific to secondary teachers in her discussion section, she reports that senior secondary teachers "recognised the difficulty of the topics for students but have made little effort to introduce activity-based aspects, such as simulation or actual sampling, that would reinforce the theory" (p. 325).

However, teachers must understand the concepts themselves and be comfortable teaching them. Watson's (2001) multi part survey instrument provides a model for those attempting to profile quality teaching and is our only current source for data on teachers' understanding and attitudes toward probability. She reported that at least some teachers did not like probability, but it was unclear whether any of these were secondary teachers. She also reported that there were no large differences between primary and secondary teachers' confidence in handling statistics in social settings, the need for statistics, and sampling. However, high school teachers did express more confidence in understanding statistical terms in the media and felt more confident teaching statistical topics. Teachers were less familiar with sampling than with average and provided less sophisticated responses. "This may have been due to either a lack of familiarity with the topic or a belief that it was

peripheral to the curriculum “ p. 318). She reported that the teachers' success rate on a particular item was 56%, higher than the 23% success rate reported for grade 11 students in a previous survey. These findings indicate that teachers have a more sophisticated understanding of probability and statistics topics than their students do and that secondary teachers know more probability and statistics than primary teachers do. However, the weakness in sampling indicates that these teachers may have trouble helping students develop sophisticated models of probability and statistics.

This paper provides additional insight into secondary teachers' attitudes and knowledge of probability and their use of the recommended teaching strategies by addressing these research questions.

- i. What are the participating teachers' attitudes toward probability and toward their knowledge of probability?
- ii. Do they think probability is important in the curriculum?
- iii. Do they initiate conversations to confront their students' misconceptions about the probabilistic aspects of two exponential modeling activities?

The models and modeling perspective of Lesh and Doerr (2001) provides a framework not only for understanding the learning process but also for developing curricula and teaching strategies. Student concepts are developed in a multiple, non-linear, cyclical, interactive modeling process that allows modelers to express, test, revise, refine, extend, and sort out their ways of thinking” (Lesh & Doerr, 2001). These models “consist of conceptual systems that are expressed using a variety of interacting media ... for constructing, describing, explaining, manipulating, predicting or controlling systems that occur in the world” (p. 5).

The models and modeling perspective seems uniquely suited to the difficulties of teaching probability because:

- i. The students' activities must uncover, test, sort, refine, and extend the students' pre-existing models developed in and out of school. This requires exposure to a variety of alternative models presented by other students.
- ii. The persistence of the misconceptions requires gradual and iterative re-construction using tasks and teaching strategies that requires conceptual not procedural understanding.
- iii. The student's misconceptions will not respond to traditional linear teacher oriented presentation of the material.
- iv. The complementarity of chance and probability requires iterative modeling cycles, alternation of activities and mathematisation with much reflection.

This study uses this framework to interpret the teachers' perspectives and experiences with the probability aspects of two modeling tasks.

Methodology

Participants

The six teachers, observed in their classrooms in large suburban districts are white females with at least 13 years experience teaching high school mathematics courses. Five teachers in one district were using the activities for the first time in their classrooms. The sixth teacher, who had participated in a pilot study the previous year, was using the activities for the second time. Before using the activities in their own classrooms, all of the

teachers attended a two week long summer workshop led by Doerr exploring and discussing the activities, the technology, and appropriate teaching strategies. Before joining this project, the teachers had limited experience in leading extended activities that engaged their students in creating their own solutions to mathematical problems. The students had a range of abilities, motivation, and confidence as mathematical learners.

Probabilistic Activities

Two activities were taught as part of a course designed as a fourth year in the secondary mathematics curriculum. The M&M task, was designed primarily to extend the students' understanding of exponential functions to include exponential decay and only secondarily to introduce an understanding of the role of uncertainty in their models. Students were given a small cup full of M&M's. Starting with a few M&M's in a second cup, students were asked to roll their M&M's out of their cup, add one M&M to their stock for each M&M with an M showing, and then roll again. They repeated this process until they ran out of M&M's. Then the students were told to put all of their M&M's in a cup and roll them out, removing all of the M&M's with an M showing before they rolled again. They continued rolling until they ran out of M&M's. As a third task, students were asked to repeat the first two experiments with hypothetical 4-sided M&M's. For each situation, they developed an equation modeling their results and were asked to predict results outside of their domain.

The Rumor Mill activity was primarily designed to extend the students' exponential model by introducing the logistics function and secondarily to reinforce the role of uncertainty in their model, first seen in the M&M activity. All of the students in a class were given small pieces of paper with the numbers one through ten written in a column. Next to the number one, one student had an R, meaning that that student knew the rumor. Each student was told to show his/her paper to one other student during each round. If the student saw an R on any of the papers that s/he saw, s/he then knew the rumor and wrote an R next to the number of that round. The rounds continued until all of the students knew the rumor. Then the students were asked to plan and execute a simulation of the spread of a rumor. To support this activity, they were given a chart with a grid of the numbers from one to a hundred and an instruction sheet for generating random numbers on their calculators. They were not told that they had to use these items or what to do with them.

Data Sources and Analysis

The qualitative data collected for this study included audiotape transcriptions and field notes of each class session, teacher interviews before and after each activity, teacher workshops before and after the school year, and monthly teacher meetings. The workshops and meetings were used to review and discuss all of a larger set of tasks, not just the two activities of interest in this project, so only parts of these sessions were of interest to this study. We coded the transcripts and field notes using grounded theory (Strauss & Corbin, 1998), which allowed the codes to emerge from the data in the first pass. Subsequent passes refined and modified the coding. Copies of student work from each activity were coded for validity and misconceptions and this information was used in the post interviews. The issues from the M&M observations and interviews were compared with the issues that emerged for the Rumor Mill observations and interviews. The results were then related to the research questions and the literature.

Results

The results of this study include three major findings related to teacher knowledge and teaching strategies. First, this study confirmed the finding of Watson (2001) indicating that some teachers dislike probability and feel unprepared to teach it. Second, these teachers recognise the importance of teaching probability. Third, the teachers emphasised only the exponential aspects of the activity.

In this study, we examine the teachers' responses to two of these activities primarily designed to extend the students' models of exponential functions. Since the secondary purpose of these tasks was to refine students' models of probability, we thought that we would be able to observe the teachers' general probabilistic reasoning and attitudes toward probability without the specific impetus of a probability agenda. We were keenly interested in how the teachers responded to the probability misconceptions of their students and how they handled those misconceptions because of the importance of confronting those misconceptions with situations that would lead their students to re-construct their concepts.

Some Teachers Dislike Probability

Three of the teachers do not like probability and four feel that they are not prepared to teach the subject. Penny addressed this issue:

I know most math teachers are confused. If you ask the entire floor (of math teachers) what they like to teach the least, what they feel the least comfortable with, it's gonna be probability. Anytime you walk into the faculty room, people talk about probability and everybody saying they hate it. The first time I had to teach it, I always had the answer key out in front of me. And I work ahead to make sure that I have it all down. Teachers would routinely say they did this and they did it wrong and they had to go back and figure out why, and the kids lose confidence.

Only one teacher had taken more than one probability or statistics course. Three had never taken any. None of the teachers remembers any discussion of probability and statistics in any mathematics education courses. Joyce thinks that her "students know as much as she does". She believes that any group of three of her honors students will solve any problem at least as well as she can and their reports seem to confirm this. The other teachers are sure that their students have limited procedural knowledge and that few have a conceptual understanding of what they learned to pass the standardised state exams. The teachers do not voluntarily make connections to probability units taught to the students in prior courses, encourage discussions of questions raised by the students, or raise issues that the students have missed. Even the two teachers who feel comfortable with high school level probability do not often encourage their students to connect the concepts in these units to previously learned material or provoke students to re-conceptualise their probabilistic misconceptions.

Teachers Recognise the Importance of Probability

The teachers think that the basic conceptual knowledge of probability required to explore the issues raised by these activities should have been taught in earlier courses, as early as the fifth or sixth grade. In the interviews, all of the teachers agreed that probability education was important and that there was not enough probability in the curriculum. They agreed with the NCTM (2000) position that probability should be a part of the mathematics curriculum at all levels. They resent the pressure of the standardised state examinations in the earlier high school courses that forces teachers to expose their students only to the procedural aspects of probability that are tested. There is not enough time to cover

everything on the test. As Joyce said, “Probability is almost always left out except for the test questions”, and so teachers tend to present solution strategies only for likely test questions. They suggested that the activities should be rewritten to specifically address the probability issues involved. However, they were worried about the time this would take away from their current mathematical agenda.

Teachers Emphasised only the Exponential Aspects of the Activities

At several points in the activity, there was an opportunity to help the students refine their model of probability by revisiting the relationship between experimental and classical probability. In the M&M activity, for example, students typically averaged the successive ratios of the total number of M&M’s at each roll to find a base for an exponential equation that fit their data. Only one teacher encouraged students to think about the relationship between the average of the successive ratios and the theoretical probability of rolling out an M&M with the M showing. In the interviews, the five teachers who did not explore this opportunity said that they “did not think about it” at the time. Their focus on the primary purpose of the activity and their concern over the amount of time that the activity was taking precluded the exploration of issues arising from the situation. One of the first time teachers said, “We don’t have time to discuss ... (all the) details and have them sit and think.” Although the teachers value student think-time, they feel pressured to save time. Dealing with student frustration was a recurrent theme at the workshop given the summer after they taught the units. They recognised that students needed to struggle with the issues raised by the activities; however, exactly how and when to rescue a student in distress was problematic. According to the teachers, their students had always been taught mathematics traditionally. Students were used to group work in other content areas, but considerably less so in mathematics. The students were generally proficient at applying procedures that they had been shown, but they had little, if any, experience in solving problems that they had not seen before. Using modeling activities to revise and extend mathematical knowledge was a new experience for the students and the five of the teachers.

After a lengthy discussion with Joyce about why she and the other teachers had not discussed distributions with the students, the first author summarised what he had heard:

First author: Number one, you didn’t think about it. Number two, it is not the purpose of the lesson, so why would you think of it? And number three, some teachers have said that they don’t like probability and don’t want to mess with it because they might make mistakes because they are uncomfortable with it. The kids will ask questions that they can’t answer and they are not comfortable with that.

Joyce: That is scary, but I’m sure that is very truthful ... And that is why the kids are where they are. Nobody feels comfortable with it...I would just add that the other critical thing here is time. We are crunching and if you are in the middle of an exponential unit, how much extra stuff can you fit in...As soon as you start with this stuff, we are in never never land now because you don’t know what in the devil is going to come up.

Most of the teachers felt unprepared to teach probability and all of the teachers felt pressed for time. They believe that there is too much in the curriculum to treat all of the possible topics and issues. Given the challenges that the teachers faced with the basic process of teaching using a modeling approach, it is not surprising that they did not emphasise the secondary, probabilistic aspects of the activities that we observed. After all of these teachers had worked through the activity themselves, discussed the activity several times, and taught the activity at least once, they wrote a teacher’s guide to the M&M

activity at the second summer workshop. All of the teachers agreed that probability is one of the key ideas of the activity. However, they did not emphasise probability in the sections of the guide dedicated to questions to provoke student thinking and issues to bring up in whole group discussion. As Bea put it, "Why do probability unless you are asked?"

Discussion

Teachers have a clear agenda; they know what they want to accomplish during a course. They are not going to jeopardise reaching their goals by spending time on all of the secondary issues inherent in a given activity. They have to select what they think is most important for their students to learn. The idea of probability was an important concept in the activity, but exploration and refinement of probability conceptions was not. This is understandable because probability was not the main point of the activity and this study was constructed to explore how the teachers thought about and used probability in this context.

In respect to my first research question, we found that most of the teachers in this study do not like probability and have a weak understanding of the topic. This extends the findings of Watson's (2001) study, indicating that many American teachers may not like probability or understand the topic well enough to be comfortable teaching beyond basic procedural elements. Many do not like to teach it. It typically does not occur to them to think probabilistically when faced with a probabilistic element in an activity, at least when the probability aspect is not central to the teacher's agenda. Sometimes, they did not recognise the problems or misconceptions implicit in their students' work. They tend to think about probability only when teaching a probability unit. They seemed to view the stochastic element as an impediment to getting to the deterministic problem, which was the focus of the activity.

The answer to the second research question is that the teachers did want their students to realise that probability was an essential element in the activities. . However, the teachers focused their attention on the exponential aspects of the activities and did not address or try to resolve many of the probabilistic issues that they noticed. They felt they did not have the time to deal with all of the issues that the activities generated and placed low priority on the probabilistic issues.

Teachers did not often initiate conversations to confront student misconceptions, the third research question. Understanding and confronting student misconceptions is essential for teachers, if students are to refine and extend their models explaining probabilistic situations. In order to do this, teachers must be comfortable with the probability issues and believe that they are important enough to spend the necessary time on them. Revising and extending the activities to include more explicit explorations of the underlying distributions could help teachers manage this difficult process. However, this would extend the time spent on the activity and may not be acceptable to some of the teachers. The teachers continue to struggle with changing their traditional, linear model of teaching. More experience with activities based on the models and modeling perspective may give teachers and students the tools required to address misconceptions and non-linear problems. Extended time on meaningful investigations allows and encourages the testing, revision, and extension of their mathematical and teaching/learning models.

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