Evaluating Pre-service Teachers' Understanding of Middle School Mathematics

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This paper reports on an attempt to measure aspects of pre-service teachers' understanding of middle school mathematics together with their perceptions of its difficulties by using a simple questionnaire. Pre-service primary teachers did not, in general, perform as well as their secondary counterparts, and they rated the items as more difficult. While there were differences between perceptions of conceptual difficulty and computational difficulty, it is not certain that these distinctions are clear in the minds of pre-service teachers. Suggestions for improving the questionnaire are discussed.

In recent years there has been a growing interest in the multifaceted nature of teachers' knowledge, and how this affects students' learning. Shulman (1986) and others have identified several key components of teacher knowledge. These include (i) *content knowledge*, which involves both mastery and deep understanding of the content and structures of the subject matter; (ii) subject-specific *pedagogical content knowledge*, which incorporates the way subject matter knowledge is used in teaching, and includes knowing ways of formulating explanations and useful representations of concepts, etc.; (iii) *knowledge of students' thinking*, which incorporates knowing what makes topics easy or hard for students to learn, misconceptions and common difficulties, and the capabilities of students at different ages; and (iv) *curricular knowledge* of curriculum programs and instructional materials.

Ball (2000) notes that one distinguishing feature of knowledge for teaching is to be able to deconstruct it so as to be able to see it from the learner's perspective. Ball writes that "knowing for teaching requires a transcendence of tacit understanding" (2000, p. 245), so that the critical components central to but often invisible in one's own compressed mature knowledge are revealed. There is more to content understanding for teaching than the type of mastery that would be exhibited by an excellent student. In an influential book, Ma (1999) investigates primary school teachers' "Profound Understanding of Fundamental Mathematics" and contrasts the deep conceptual knowledge of Chinese teachers with the shallow procedural understandings of their U.S. counterparts, even in basic mathematics. In her research she used interviews to probe teachers' understanding of subtraction, multiplication, division by fractions, and area and perimeter. Results from other research (e.g., Stacey, Helme, Steinle, Baturo, Irwin, & Bana, 2001; Behr, Harel, Post, & Lesh, 1992) suggest that areas such as decimal and place value understanding, and ratio are also particularly problematic and worthy of further study.

Given the importance of teachers' knowledge of fundamental mathematics content, the focus of the current study is to investigate Australian pre-service teachers' understanding of aspects of elementary mathematics, both Shulman's content knowledge and some aspects of his pedagogical content knowledge. Since there has been a growing awareness of critical needs in the middle school it seems appropriate to consider both primary and secondary pre-service teachers, and investigate understanding of aspects of mathematics important in

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Grades 5 to 9. A comparison of the content knowledge of pre-service primary and secondary teachers is also of interest, since one group has more mathematical training and the other more training in pedagogical content knowledge. In order to investigate this for large numbers of teachers it would be useful to develop a questionnaire that could replace interview methodology, at least for some aspects of a future study.

There is also concern about the effects of teachers' judgements about the difficulty of items, because such judgements may influence what they decide to teach and how. It is possible that items judged very difficult may not be taught, or that teachers who do not appreciate the conceptual demand of an idea may treat it perfunctorily. This concern led to the inclusion of an investigation of teachers' beliefs about the difficulty of mathematical questions. Since it is common in our teacher education courses to distinguish informally between the conceptual and computational difficulty of items, students were asked to rate both these aspects. The outcomes reported here, then, investigate mathematical understanding and beliefs about difficulty, and are preliminary results based on a trial questionnaire that we hope to improve.

Method

Participants

Two groups of pre-service teachers from the author's Australian university were involved in the study. Participants from the BEd cohort (N = 39) were in the final year of a four-year Bachelor of Education program, preparing to become primary teachers. These students had done a mathematics education subject in each year of study, covering both elementary mathematical content and pedagogical issues. For admission to the BEd course they needed to have passed at least Year 11 mathematics. About half of the whole cohort (although not necessarily this sample) also passed a Year 12 mathematics subject. The second group, the DipEd cohort (N = 29), comprised students in the Diploma of Education program who were training to become secondary mathematics teachers. These students had at least a minor in mathematics from a previous degree. Participation in the study was voluntary, which may imply that the study is biased in favour of participants with greater mathematical self-confidence.

The Fundamental Mathematical Understanding Questionnaire

Data for the study came from participants' responses to a written questionnaire. There were two versions of the questionnaire, although most questions were common to both. The items were designed to cover a variety of topics in middle school mathematics, including decimals, ratios, simple geometry, fractions, and probability. These are topics treated in Grades 5 to 9; none involved algebra. Some items required only numerical answers, while others (items 3, 6B, 9, and part of 10) required some explanation. The items are described in Table 1. Many items had been used in or were based on the research literature (e.g., Greer, 1992; Hart, 1981; Henningsen & Stein, 1997; Stacey et al., 2001).

Participants were asked to rate the difficulty of many of the items, separately identifying computational difficulty and conceptual difficulty. This was done on a three-point scale of Easy, Medium, and Hard. In some cases, one response applied to two or more parts of an item. The questionnaire was administered during class time, allowing about 45 minutes, and the two versions of the questionnaire were distributed randomly. Calculators were not permitted.

Data Analysis

For the purposes of this analysis, items requiring elaborated responses were not analysed (item 6B, item 9, and two parts of item 10). Responses to the remaining item parts were scored as either correct (1) or incorrect (0); a partial score was given to some responses to item 1d. Omitted questions were marked as incorrect. The two difficulty ratings (computational and conceptual) were each assigned numerical values of 1 (Easy), 2 (Medium), or 3 (Hard). The data were analysed to compare the performances of the two groups, identify common difficulties in the understanding of fundamental mathematics, examine differences in perceptions of the conceptual and computational difficulty of questions, and investigate the relationship between performance and perceived difficulty.

Results

Comparisons Between BEd and DipEd Cohorts

As might be expected the DipEd cohort performed significantly better overall than the BEd cohort, averaging 88% correct compared with 71%. This overall result was reflected in the individual questions, with the DipEd students performing better on nearly all items (see Table 1). The differences between the groups were greatest on the part of question 10 requiring students to identify which of the two spaceships travelled further (BEd:DipEd = 28%:90%), although the wording may have caused some students (particularly those less confident) to be deflected from the correct answer that the spaceships travel the same distance. Other items proving particularly difficult for the BEd students were question 1d, which asked how many hundredths of a litre are there altogether in 0.485 litres (28%:83%); questions 7 and 9, requiring estimates of the product and quotient of two decimals (see additional discussion later in this report); and question 11, where clearly showing 0.725 of the 8×10 rectangle was essential for obtaining full marks.

The two groups performed comparably on the decimal comparison item (96%:100%; this was part of question 1), which was pleasing given the emphasis that had been placed on this in one of the BEd mathematics subjects, since earlier research (Stacey et al., 2001) had shown this topic had been problematic. They also performed comparably on the easiest of the ratio comparison items in question 3 (87%:90%). Both versions of question 4 required students to compare side lengths and areas of triangles determined by the diagonals of either a parallelogram or a rhombus. The parallelogram version proved more difficult, and although the BEd students did worse than the DipEd students for both versions of this geometry item, the difference in performance was not as great as for the remaining items. This may well be because the BEd students had done some work on geometry as part of their course (18 months prior to the questionnaire), whereas the DipEd students may not have encountered geometry since high school.

Table 1
Description of Questionnaire Items and Average Scores for the Two Cohorts.

Item	Topic	No. of parts	BEd score	DipEd score
1	Comparison of size of pairs of decimals, placement of decimals on a number line, place value understanding (e.g., how many hundredths in 0.485)	5	75%	92%
2	Simple ratio. Mr Short is 6 paperclips or 4 matchsticks tall. Ms Tall is 6 matchsticks tall. How tall is Ms Tall?	1	59%	86%
3	Comparison of pairs of ratios (e.g., which is sweeter: 5 spoons of sugar mixed with 15 spoons of lemon juice or 3 spoons of sugar mixed with 12 spoons of lemon juice?). Item had 3 parts, increasing in difficulty.	3	76%	93%
4A	Comparison of sizes and shapes of triangles in a parallelogram	5	70%	78%
4B	Comparison of sizes and shapes of triangles in a rhombus	5	81%	90%
5	Probabilities using a spinner	1	72%	84%
6A	Design a spinner with regions of probability 1/2, 1/3 and 1/6.	1	69%	95%
6B	Give three explanations for why 3/8 is the same as 37.5%	3	NA	NA
7	Multiplication: Choose value closest to 19.7×3.52	1	54%	90%
8	Division: Choose value closest to 5.67÷0.032	1	44%	83%
9	Explain why you "add a zero" when you multiply by 10	1	NA	NA
10	Describe and graph the movement of two spaceships, one moving at 1.88km/hr for 0.32 hours, and the other moving at 0.32 km/hr for 1.88 hours (values for 1.88÷0.32, 0.32÷1.88 and 0.32×1.88 were supplied)	5	71%	91%
11	Shade 0.725 of an 8×10 rectangle and give the fraction and decimal equivalents.	3	32%	71%

Note: Scores are based on all parts of the whole item, except for item 10 where two parts were not analysed.

A large number of students (12 of the 68, mostly BEd students), did not respond to question 2, which was surprising given that a distinctive large diagram accompanied the question. Questions 11b and 11c were also omitted by some of the BEd students, but since these were the last questions time constraints may have led to their omission.

Common Errors

Although the wording of question 1d—which asked how many hundredths of a litre there are in 0.485 litres—probably contributed to some justifiable variants of the intended answer of 48.5 (such as 48 or 8), there were a large number of answers showing a poor understanding of place value. For example, 36% of the BEd students responded with 485, while another 13% gave other values reflecting inadequate place value understanding, such as 5, 80 or 4850. In contrast, only 4 of the DipEd students (14%) gave such answers. The difficulty of the primary pre-service teachers with such a fundamental aspect of place value understanding in the final year of their course is of great concern.

Question 7 asked students to select which of five possibilities was the closest to the correct answer for 19.7×3.52. Most DipEd students selected the correct value of 69.34 whereas many BEd students, 15 of the 39, and two of the DipEd students selected 75.264. These students may have been distracted by there being 3 digits after the decimal point, as expected if calculating the answer using the conventional multiplication algorithm. Only three students selected answers that involved incorrect powers of ten. In future, an improved question will have the three decimal point answer further from the simplest estimated answer (which here is 70 = 20 x 3.5).

In contrast, the ability to estimate according to place value proved more difficult for the division problem, 5.67÷0.032, perhaps exacerbated by the fact that it involved calculation with a decimal less than 1 (e.g., Greer, 1992). Again, the distractor of a precise answer (17.71875) attracted some (6 BEd and 2 DipEd students), while other students selected 2000 (4 students), 20 (3), 0.2 (3), and 0.02 (2). Whereas experts may have rated this primarily a conceptual question, for some students it was heavily computational, as their scripts showed that they calculated the answer, rather than estimating it.

The first part of question 11 was the worst done of all the questions (18%:55%). It required students to shade an area of 0.725 in an 8×10 rectangle that was marked to show 80 squares. Of those who got it incorrect, most successfully shaded 0.7 by using 7 of the 10 columns of 8 squares, but then had difficulty representing the remaining 0.025. The most common mistake, by 8 of the 38 students who were unsuccessful overall, was to shade 2.5 of the remaining squares, as if each square was a hundredth.

Perceived Computational and Conceptual Difficulty

The students distinguished between questions when rating them for difficulty. The spinner question and the decimal comparison questions were rated as relatively easy both computationally and conceptually (average rating over both cohorts less than 1.5 for computational difficulty, and less than 2 for conceptual difficulty). Question 6B, which required three explanations for why 3/8 is the same as 37.5% and which has been excluded from most of the analysis here, was perceived as the most difficult item. Both groups rated both categories greater than 2.5 on average, and the BEd students were almost unanimous in rating it as Hard conceptually. Other "hard" questions included the parallelogram item (conceptually), the division estimation question, the spaceship distances question (especially for the BEd group), and the item requiring an area of 0.725 to be shaded.

The DipEd students generally rated the questions as easier than BEd students. This is partly apparent in Figure 1. There were some exceptions, with similar ratings being given by both cohorts for the computational difficulty of the first ratio comparison question (question 3a), the multiplication estimation question (question 7), and representing 0.725 as an area, a fraction and a decimal (question 11); and for the conceptual difficulty of the parallelogram item (question 4A), the spinner (question 5), and question 11.

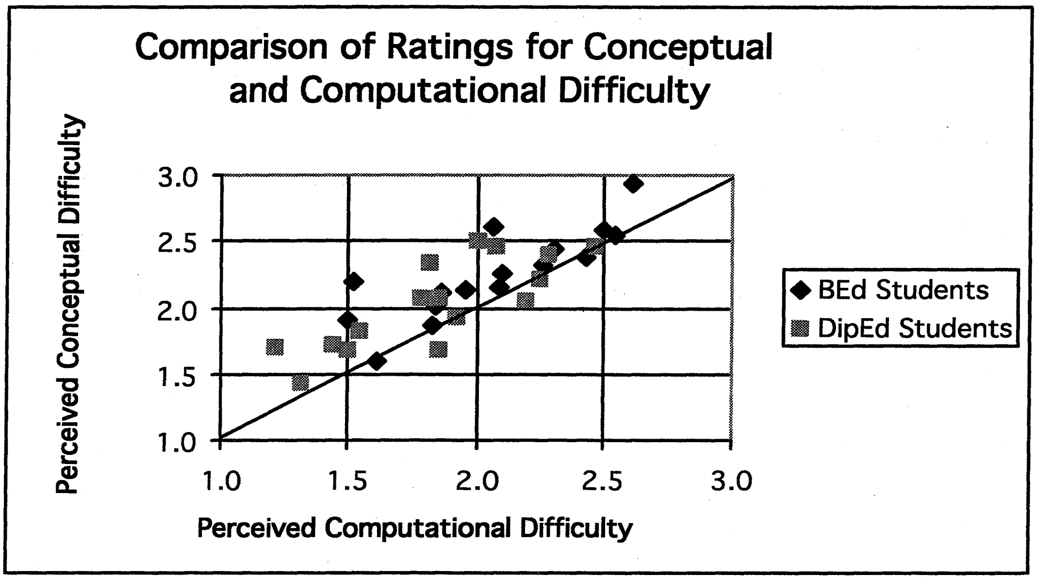

Figure 1. Relationship between the averages of the perceived conceptual and computational difficulties for different items for both cohorts of students.

Figure 1, showing average ratings for the two groups, reveals that students' perceptions of computational and conceptual difficulty were strongly related. Students gave most questions a higher rating for conceptual difficulty than for computational difficulty, with most points in Figure 1 above the line $y=x$. This was particularly true of the decimal comparison and number line items (Questions 1a and 1b), the geometry items (Question 4), and the probability spinner item (Question 5), all of which involved little or no calculation. For Question 10c, which asked students to say which spaceship travels further, the BEd students rated it equally on computational and conceptual difficulty, when in fact no actual computation is required (answers to likely computations were given). Nevertheless, recall that this question proved difficult for the BEd students, suggesting that perhaps they were themselves concentrating on calculation rather than concepts, or that they carried out computations that were not relevant to the question. The DipEd students, in contrast, had a significant difference between their difficulty ratings on this item. The multiplication and division estimation questions (Questions 7 and 8) were the only two questions where computational difficulty rated marginally higher than conceptual difficulty, but only for the DipEd students. Again, this is surprising given that only estimation was expected to have been involved. The largest difference between the average perceived conceptual and computational difficulties was 0.66 for the BEd students on questions 1a and 1b (decimal comparisons), although most differences on items were less than 0.3 for both cohorts.

Perceived Difficulty and Performance

This section examines the relationship between perceived difficulty and actual performance. Not all items were included because it was inappropriate to combine some scores, or because difficulty ratings were not requested. Where a single pair of difficulty ratings was collected for multiple parts, scores were combined. Because of the correlation between the ratings, the computational and conceptual difficulty ratings were added for this analysis. A difficulty value of 2 implies that the question was regarded as easy both computationally and conceptually, and 6 that it was perceived as hard in both aspects.

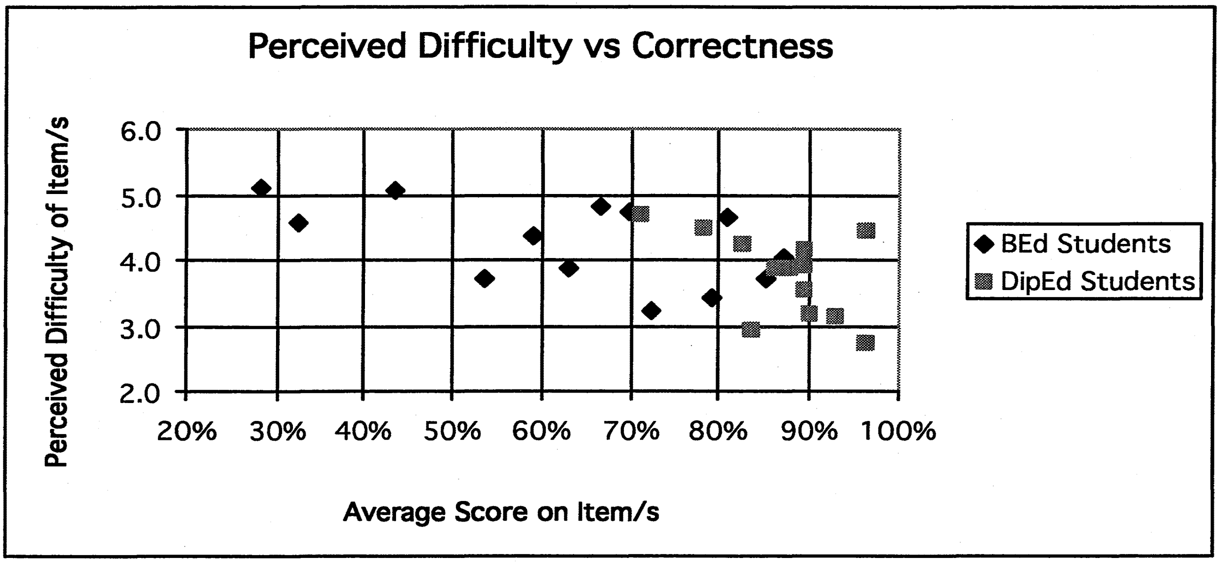

Figure 2. Relationship between the perceived difficulty of the items and average performance for both cohorts of students.

As can be seen in Figure 2, there was a general association between the students' average performance on questions and the average perceived difficulty of questions, with students perceiving as easier those questions on which they scored better (hence the negative slope of a trend line). There are a few interesting outlying values. Apart from one student who omitted it, the DipEd students scored perfectly on question 3c. They recognised the item's conceptual and computational difficulty, however, by giving it a relatively high difficulty rating of 4.45. It was similarly regarded by the BEd students, but they did not perform as well (Score=67%, Difficulty=4.80). Interestingly, my own opinion was that items 3a to 3c increased in difficulty, which was reflected in both cohorts' perceptions and the BEd cohort's performance, but the DipEd students actually improved in their performance over the three parts. Question 10c, requiring students to identify which of the two spaceships travels further, was similarly done well but regarded as moderately difficult by the DipEd students (Score=90%, Difficulty=4.17). In contrast, the BEd students also recognised the difficulty of 10c, but performed very badly (Score=28%, Difficulty=5.09). Question 5, ranked easiest by the BEd students, was not their best-performed question (Score=72%, Difficulty=3.21). The DipEd students also found question 5 easy, but did not perform particularly well on it (Score=84%, Difficulty=2.93).

Question 7 required students to select an estimated value for 19.7×3.52. This was regarded as relatively easy by the BEd students, although their performance was not good (Score=54%, Difficulty=3.70). The DipEd students performed better, but had a similar difficulty rating (Score=90%, Difficulty=3.54). It is interesting to compare these results with those of Question 8, which was similar but involved division instead of multiplication. Both cohorts regarded the division question as much harder than the multiplication item (BEd Difficulty=5.08; DipEd Difficulty=4.24), and yet each cohort's performance does not reflect this as much as might be expected (BEd Score=44%; DipEd Score=83%). The DipEd students, however, performed better than the BEd students.

Discussion and Conclusion

First and foremost it is acknowledged that, as it stands, the questionnaire has some shortcomings. Nevertheless there is scope for refining it so that it better measures the depth of understanding of fundamental mathematics and is also able to address pedagogical content issues, such as how to explain particular mathematical results to students. Future components of the study will involve others who might be called upon to teach middle school mathematics (e.g., science teachers), and pre-service teachers in other countries.

As can be seen from the results, the DipEd students perform significantly better on the aspects of fundamental mathematics used in the study. The area with the smallest gap was geometry, which may indicate that teacher education needs to address this for DipEd students in our state. The better content knowledge of the DipEd students was also reflected in the generally lower ratings of difficulty that they gave to the items.

While there were differences between the average rating levels of perceived conceptual difficulty and computational difficulty, it is not clear that these distinctions are clear in the minds of pre-service teachers, since some of the responses are difficult to explain. Future versions of the questionnaire need to consider this more carefully. This version, for example, did not provide participants with a good opportunity to distinguish between the two (e.g., there was no question that was obviously hard computationally while being easy conceptually, although there were questions that were the reverse). We think most mathematics educators believe that there is a difference between the computational and conceptual difficulty and that identifying it is important, but perhaps we need to think more about what we mean by it and how best to convey it to our pre-service teachers.

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