Pedagogical Content Knowledge Development of Preservice Secondary Mathematics Teachers

Jamie King
Syracuse University
<kjking@syr.edu>

Joanna Masingila
Syracuse University
<jomasing@syr.edu>

This study investigated how pedagogical content knowledge developed while five preservice secondary mathematics teachers were enrolled in a mathematics methods course. In three phases, data collected were participants' concept maps of pedagogy, teaching and learning, and assigned content and related interviews. Themes of adjusting and reflecting often developed during interactions with students' misconceptions or when timing and motivation were pedagogical issues. Implications include the metacognitive benefits of focused reflection on pedagogical content knowledge during preservice education.

Only a century ago, majoring in a subject was considered sufficient for those becoming teachers. There was no concern for how subject matter was transformed from the teacher's knowledge into the content of instruction (Shulman, 1986). This raises questions today about the transition from expert student to novice teacher. A particular transformation of which little is known, is characterised by the development of pedagogical content knowledge. Shulman conceptualized pedagogical content knowledge as, "the ways of representing and formulating the subject that make it comprehensible to others as well as the most useful strategies likely to aid in the reorganization of the understanding of the learner" (Shulman, 1986, p. 9). Clarified by Ball (2000), pedagogical content knowledge includes knowledge of what is typically difficult for students, most useful for teaching a specific idea or procedure, and ways to develop a particular idea.

Researchers have studied many concepts related to pedagogical content knowledge such as teacher beliefs and conceptions (Cooney, Shealy & Arvold, 1998), teacher development (Ball, 2000; Day, 1996), teachers' reflection-in-action (Artzt, 1999; Goodell, 2000; Schon, 1987) and teacher knowledge (Ball 1990; Jones & Vesilind, 1996; Simon, 1995, Sullivan & Mousley, 1994). Results have indicated pedagogical content knowledge develops, in part, due to teachers' experiential processes, the permeable belief structures of teachers, and reflection on and within their practice. Few of the studies, however, are specific to novice secondary mathematics teachers. General and mathematics educators, working from a constructivist theory of learning, have not yet answered the question of *how* pedagogical content knowledge develops. Therefore, what is missing is a way to define the building process of pedagogical content knowledge of preservice mathematics teachers as it is "unique to the profession and relatively undeveloped in novice teachers" (Brown & Borko, 1992, p. 212). In this investigation therefore, we aimed to answer the following research question: How and when does pedagogical content knowledge develop in preservice mathematics teachers? Gaining deeper understanding of teachers' processes can be of great importance to teacher educators. Ducharme & Ducharme (1996) called for the application of such research findings. They claimed that "if teachers could be helped and persuaded to make explicit the knowledge which they implicitly use in their day-to-day work, teacher education could begin to achieve something of the practical relevance and theory-practice integration which it is still accused of lacking" (p. 1031). Through this

study, we attempted to identify practical and applicable aspects of the development of pedagogical content knowledge for mathematics teacher educators.

Theoretical Framework

Teaching can be thought of as a continuous inquiry into content and ways to structure the learners' experiences to best facilitate understanding (Ball, 1990). Janvier (1996) considered preservice teachers' theories of teaching and learning as an important aspect of their evolutionary development. He claimed such evolution occurs through self-reflective processes. Indeed, Artzt (1999) found that individual and social construction of preservice teachers' knowledge changed through ongoing, reflective, self-assessment. We know from these studies that novice teachers' theories shift and change more often when encountering their initial teaching experiences, that is, when they take on the role of the teacher.

Social and interactive processes of communication occur continuously within the classroom as a dimension of teaching practices. The nature of the teacher-student interaction is interdependent but has been outlined by Steinbring (1998) as autonomous also. The teacher cannot be sure the student has comprehended what was the intended new knowledge, and must engage in continuous, informal assessment. As clarified by Steinbring,

> The teacher has to observe and diagnose the students' learning activities and achievements, then change and vary the new learning offers accordingly. In this way, there is an indirect relationship between two autonomous systems: the students' learning process and the interactive teaching process between teacher and student (p. 159).

It is in the relationship among the students' and teachers' systems where the in-action development of pedagogical content knowledge takes place (see Figure 1). Teachers have to analyse in-action students' mathematical knowledge construction, comparing those understandings to what was intended for students to learn.
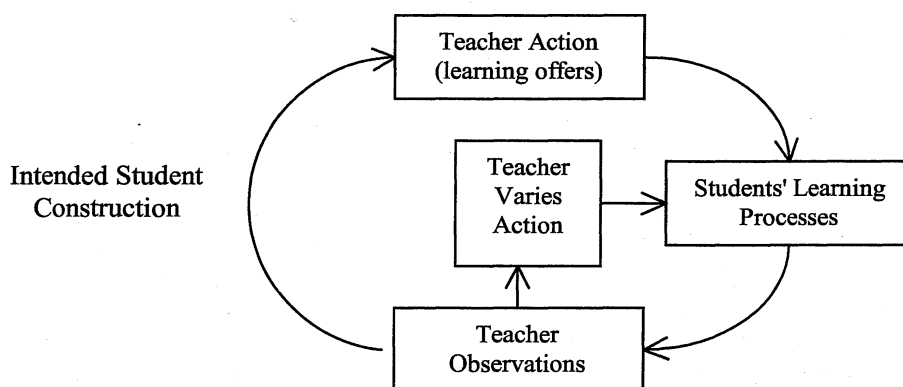

Figure 1. Adaptation of Steinbring's teaching and learning as autonomous systems.

It was precisely the observations and concurrent thoughts causing the varying action that we hypothesised to characterise pedagogical content knowledge in this study. In order to study the actions and possibly subsequent varying actions of preservice teachers, it was essential to be able to clearly document and analyse their metacognitive structures by utilising what cognitive psychology offers via concept mapping.

The study of mental events primarily concerned with the human mind (i.e., knowledge, beliefs) and the mental processes in which people engage (i.e., thinking, problem solving, planning), is what cognitive psychology encompasses (Brown & Borko, 1992). An important and relevant assumption of the cognitive psychologist is that within the human mind, knowledge is organized and stored in structures. "Research on learning to teach must examine how teachers learn to translate the knowledge stored in their teaching schemata into operational plans or agendas for classroom action, and how they learn to carry out these agendas in the classroom" (Brown & Borko, 1992, p. 212). Gaining access to such schema can be achieved by studying the concept maps of preservice teachers, which emerged from the seminal work of Novak in the early 1970's. One purpose of using concept maps was to provide a schematic summary by learners of what has been learned (Novak & Gowin, 1984). Since then, it has been adopted to diagnose and test, design curriculum and instruction, and be used as a metacognitive aid to help students learn. Concept maps reveal not only what the learner knows but also the ways he or she organises such knowledge. We used concept maps to serve as a tool for understanding preservice teachers' structural knowledge, especially the development of the novices' pedagogical content knowledge.

Methodology and Data Analysis

Our study involved a voluntary group of five preservice secondary mathematics teachers over a period of four months during their enrollment in a methods course and a concurrent, mid-semester, seven-week, student teaching experience. There were four undergraduate juniors: three white males and one white female, and one white female graduate student. During the seven-week student teaching experience, the two females were assigned to an urban high school where classes were block scheduled. They were assigned to remedial algebra classes and a geometry class. All three males were assigned to urban middle schools in 8th grade classrooms. The preservice teachers were asked to draw three concept maps during three phases. These three phases are outlined as follows: Phase One - *Preactive*, during the middle of the first six weeks in class and before student teaching, Phase Two - *Interactive*, during the middle or near to the end of seven weeks of student teaching, and Phase Three - *Postactive*, during the last three weeks in class after seven weeks in student teaching. Participants studied the concept map in course readings and were taught and given an opportunity to practice drawing a simple concept map prior to Phase One. Their maps represented the structure and organization of their knowledge in three areas: (a) theories of teaching and learning, (b) pedagogy, and (c) content they had been assigned for student teaching. During the second and third phases, the previous map the preservice teachers had drawn was returned. Participants were asked to modify or redraw any part of the map while being interviewed about their revisions. Each 60 to 90 minute session was audiorecorded and transcribed.

Data were analyzed using both qualitative and quantitative methods. The concept maps analyzed quantitatively were scored using a modified version of Novak and Gowin's (1984) method. One point was assigned for the each superordinate concept (S), taken as an indication of the extent of conceptual knowledge. Five points were given for hierarchies (H), indicating instances of knowledge subsumption. One point was given for each example (E) of a hierarchical or superordinate concept, representing specificity of knowledge. Finally, ten points were assigned for cross-links (CL), depicting integration of conceptual knowledge. See Figure 2, in flowchart form, a partial concept map of Rick's

pedagogical knowledge, indicating the labeling method and subsequent scoring. The weighting of the scores indicated that (a) cognitive structure is hierarchically organized, from superordinate or general to subordinate or specific, (b) concepts undergo progressive differentiation, gaining greater specificity or more examples, and (c) integrative reconciliation occurs when two or more concepts are related (Novak & Gowin, 1984).
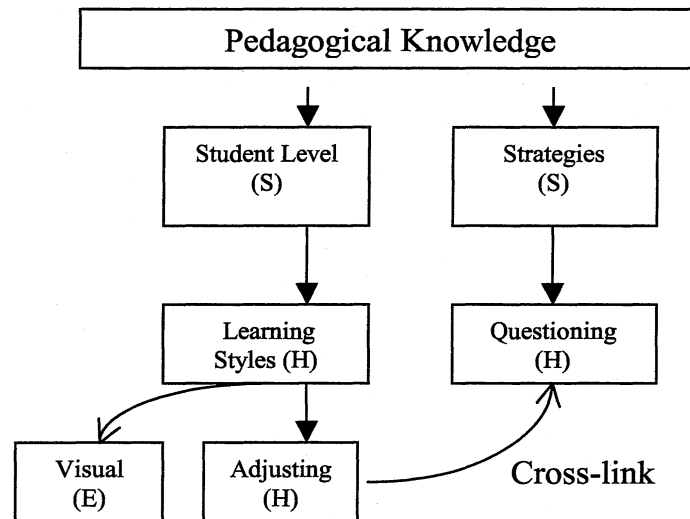

Figure 2. Partial concept map of Rick's pedagogical knowledge.

Analyzing the phases of the study included quantitative examination between-phase maps and within-phase maps for each of the four categories outlined above to first determine when changes occurred in the thinking of the preservice teachers. Qualitative analysis of the maps produced emerging themes of descriptive concepts between and among participants, illuminating how pedagogical content knowledge developed. The interviews provided deeper insight into such themes as the preservice teachers explained their rationale for including concepts as well as the concepts' importance and place in their respective hierarchical schema.

Discussion of Results

Quantitative analysis of the concept maps of the domains of pedagogy and theories of teaching and learning showed further evidence of depth of hierarchical subsumption. In the case of participants' theories of teaching and learning analyzed across phases, all five showed the greatest number of additions of superordinate concepts, hierarchical levels, and examples during the interactive phase of the study. Across that phase they collectively increased the number of cross-links but it was in the postactive phase that they collectively connected levels of hierarchies the most, using cross-links. Participants collectively gained holistic and tighter connections between these domains of content and their emerging theories of teaching and learning.

It was in the extended analysation of pedagogy maps that we were able to clarify the depth of the changes that took place during the interactive phase as well as across all phases. Using aggregated data of the five preservice teachers across the three phases, the weighted subscores were used to generate the descriptive statistics in Table 1.

Table 1

Pedagogy Concept Map Using Aggregated Data, Mean ± Standard Deviation

	Preactive Phase	Interactive Phase	Postactive Phase
Superordinate Concept	5.4 ± 0.9	6.4 ± 1.1	7.2 ± 1.3
Examples	8.4 ± 3.7	14.0 ± 4.9	18.8 ± 8.9
Hierarchy	21.0 ± 8.2	50.0 ± 26.2	69.0 ± 27.9
Cross-Links	10.0 ± 10.0	18.0 ± 10.9	38.0 ± 31.1

Since the data of interest within this table was *when* the greatest number of changes occurred, the large spread was considered important only to identify the range within each of the phases. The data indicated that the greatest initial number of changes in preservice teachers' knowledge structure occurred between the preactive and interactive phases for hierarchical subsumptions. This growth remained consistent between the interactive and postactive phases. For the cross-link connection between hierarchies in their schemata, the greatest change in preservice teachers' knowledge structure occurred between the interactive and postactive phases. This suggested that during the student teaching experience, the preservice teachers' schema consisted of detailed knowledge of concepts but lacked integration among them until the postactive phase when they reflected on the interconnectedness of their hierarchical structures.
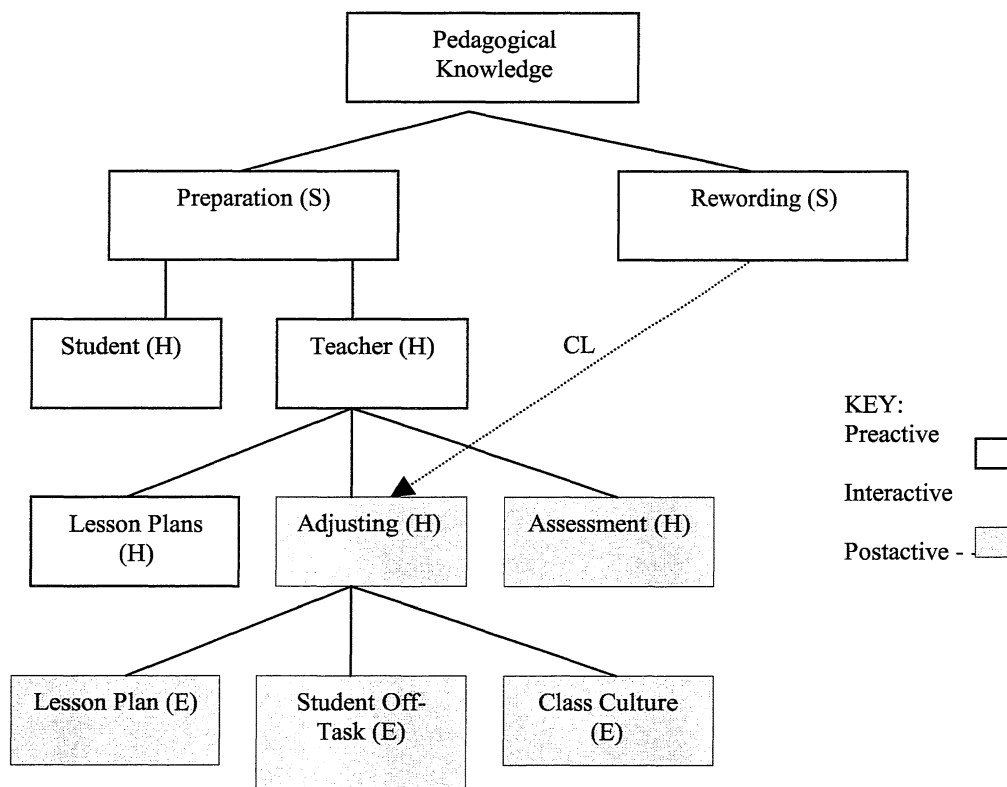

Figure 3 Don's partial pedagogical map illustrating "adjusting".

Qualitative analysis of Don's schema of pedagogical knowledge over time indicated that the concept of adjusting was an important characteristic for him as the teacher.

As we analysed concepts across and between pedagogical maps, we gained deeper insights into participants' schema. Figure 3 above, shows that two examples of superordinate concepts from Don's preactive pedagogical map were "teacher preparation" and "rewording". Don continued by adding the example concepts "adjusting for students off task", " adjusting for the classroom culture" and "adjusting lesson plans". Don's conception of teaching became more refined as he practiced the role of teacher in the classroom. Additionally, during the postactive phase, Don revealed that his concepts of "adjusting" and "rewording" should be connected thus indicating a cross-link. His integration of these concepts occurred through reflection-on-action and indicated that he learned that what he said and did in the classroom were always interdependent, contextual and subject to refinement. He and his supervisor discussed that the timing of a lesson can - affect whether or not students fully engage with content:

I: So what have you been learning about lesson planning?

D: First of all, you learn that the lesson doesn't go as you planned.

I: What is it that's important about that?

D: I think being able to adjust in certain situations.

I: OK, like what?

D: I thought I had a good plan when the supervisor came. I saw him before the lesson and told him I was having discipline problems. He said maybe I was giving too much time for tasks. And that was what was happening, we were taking too much time for tasks. He suggested taking a problem out of the book and telling them, "take five minutes".

I: So how do you think about timing during planning? This will take so long...

D: Everything is fine in theory, but when you get in the battle zone...

I: Battle zone! (laughs)

D: Yeah. Anything can change with one off-the-wall question and then you're off task.

Don paid particular attention to the time management of his lesson after some coaching by his supervisor. He found that adjusting his lesson plan was necessary when encountering students off task. Other factors that may have contributed to such knowledge growth were the classroom culture or the level of the mathematics task. As we found next, with Rick, misconceptions or skill gaps occurred during his lesson and he also needed to adjust his plans:

I: So you have your lesson plan and then you have what happened.

R: (laughs) I noticed that. I am *constantly* changing. Today we went to the auditorium for an activity. They went in three groups. They still had trouble with measuring and I wasn't expecting that. So when the next group came down, I checked to make sure we knew what we're doing. It seemed that everything that was wrong was altered. So by the time we were in the last group, it was amazing how the lesson went, not content-wise.

I: The pedagogy...

R: Yeah.

I: Does that happen every day, in every lesson?

R: Yeah, definitely.

Rick understood that feedback from students' interactions with content could force him to be reflective about his pedagogical style. That is, he varied his actions (learning offers) as necessary to best facilitate understanding of the content. Rick's expectations of student readiness were unfulfilled but he adjusted his strategy in order to teach the prerequisite skills for his intended lesson.

All five preservice teachers added the concept of "adjusting" as hierarchical knowledge during the interactive phase on the pedagogical maps. It seemed that regardless of their classroom setting or assigned curricular content, the need to vary their learning offers to students was encountered in action. The reflective process used by participants was described by Schon (1987), as "thinking on your feet" in the case of reflection in-action or constructing a new theory about an event after the fact as reflection on-action. For this group of novice teachers, when pedagogical content knowledge development occurred, it was not limited to particular realms of teachers' activity either in class or out of class. Via either kind of reflection, participants could describe classroom impediments to their intended outcomes. Rick seemed to simultaneously develop insight that he would need to perceive student meaning and be able to informally assess student knowledge based on his listening skills. He said, "It's almost like you have to think how they'll see it." Thus, for Rick it seemed that pedagogical content knowledge developed because he was thoughtful about his interactions as well as his plans and when he encountered skill gaps.

For Lauren, encountering unmotivated students gave her an opportunity to reflect on her efficacy. She realized that her questioning techniques regarding students' difficulties allowed her to assess what she could do differently to present the content more clearly and in more easily understandable ways:

L: I'm surprised more than I thought, that they would take a zero when they didn't complete homework. I thought it would be some but not so many. I would ask them, 'what do you have difficulty with?' These questions would assess both the students and myself.

I: How would your knowing what they had difficulty with assess you?

L: If one student said, I'm having difficulty, then I could work with that person one on one. If 50% or more of the class said, 'I have no clue', then I'd say, OK, I've got to go back and do this again. And I've got to find a different way to approach it because what I thought would be something they'd pick up by the method I used didn't work. So now I have to find a different way of teaching this lesson.

Lauren, although frustrated with student apathy, recognised her responsibility as a pedagogue to search for the most comprehensible ways to present the content for students, varying (or adjusting) the learning offers.

Analysis of these concept maps offered that pedagogical content knowledge developed within participants' cognitive framework of reflection in and on their interactions with students' learning processes and ongoing, informal assessment. Instances of considerable awareness on the part of the preservice teachers often occurred when skill gaps, timing, and motivation were issues of relevance in classroom experiences.

Conclusion

While planning lessons, preservice teachers should be prepared to adjust in-action for classroom idiosyncracies. Novice teachers may, over time, be able to reflect on-action in a firm grounding in knowledge, enabling reflection in anticipation rather than in response (Cooney et al., 1998). Future teachers must learn to anticipate students' learning processes, which requires ongoing assessment of the teaching and learning setting. In our study, reflective processes were critical components to the conceptualisation of teaching issues for the preservice teachers. We now know that a fruitful site for growth of the novices is in making adjustments while learning to teach. Since they often do so "on the fly", or in those stream of decision making instances so common in the complexity of classroom dynamics, we suggest that preservice teachers need to engage in continual reflective practice both in action and on action throughout their preparation program. Such metacognition aids in the reorganisation of ongoing constructed knowledge about teaching and learning for novice teachers. We suggest preservice mathematics teachers be required to write, speak and listen to each other in discussion that continuously reexamines and questions their knowledge and beliefs about specific instances of pedagogical content knowledge.

References

Artzt, A. (1999). A structure to enable preservice teachers of mathematics to reflect on their thinking. *Journal of Mathematics Teacher Education, 2*, 143-166.

Ball, D. L. (2000). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education, 51*(3), 241-247.

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449-466.

Brown, C., & Borko, H. (1992). Becoming a mathematics teacher. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 209-239). New York, NY: MacMillian.

Cooney, T., Shealy, B., & Arvold, B. (1998). Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education, 29*, 306-333.

Day, R. (1996). Case studies of preservice mathematics teachers' beliefs: Emerging and evolving themes. *Mathematics Education Research Journal, 8*(1), 5-22.

Ducharme, E., & Ducharme, M. (1996). Needed research in teacher education. In J. Sikula (Ed.), *Handbook of Research on Teacher Education* (pp. 1030-1046). New York, NY: Simon & Schuster MacMillan.

Goodell, J. (2000). Learning to teach mathematics for understanding: The role of reflection. *Mathematics Teacher Education and Development, 2*, 48-61.

Janvier, C. (1996). Constructivism and its consequences for training teachers. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer (Eds.), *Theories of Mathematical Learning* (pp. 449-463). Hillsdale, NJ: Lawrence Erlbaum Associates.

Jones, M., & Vesilind, E. (1996). Putting practice into theory: Changes in the organization of preservice teachers' pedagogical knowledge. *American Educational Research Journal, 33*(1), 91-117.

Novak, J., & Gowin, D. (1984). *Learning how to learn.* Cambridge: Cambridge University Press.

Steinbring, H. (1998). Elements of epistemological knowledge for mathematics teachers. *Journal of Mathematics Teacher Education, 1*, 157-189.

Schon, D. (1987). *Educating the reflective practitioner: Toward a new design for teaching and learning in the professions.* San Francisco, CA: Jossey-Bass.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4-14.

Sullivan, P., & J. Mousley, J. (1994). Quality mathematics teaching: Describing some key components. *Mathematics Education Research Journal, 6*(1), 4-22.