Monitoring Effective Use of Computer Algebra Systems

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Computer Algebra Systems (CAS) potentially offer students a powerful tool. However the mere availability of CAS does not enhance teaching and learning; students (and teachers) need to use it effectively. This paper presents a framework for components of Effective Use of CAS and illustrates its use as a guide for monitoring the progress of students. The results presented highlight the importance of considering the interaction between technical and personal aspects of Effective Use of CAS.

Background

Since their development in the 1970's and their introduction into some tertiary teaching and learning in the 1980's, Computer Algebra Systems (CAS) have been recognised as highly valuable for doing mathematics and potentially valuable for teaching and learning mathematics. In 1989, after reflecting on her experience of observing undergraduate students using CAS for doing and learning mathematics, Heid listed the characteristics of symbolic manipulation systems that could have potential for changing the content and processes of mathematics taught in secondary schools. She summarised these characteristics as follows:

1. Symbolic-manipulation results are exact and free of manipulation errors.
2. Symbolic-manipulation results are quickly generated.
3. A wide range of symbolic capacities is available within a single environment.
4. Symbolic-manipulation systems can handle more complicated problems than most students can do by hand. (1989, p. 411)

Studies in the intervening years, involving both tertiary and secondary mathematics classes (for example Atkins, Creegan and Soan, 1995; Bennett, 1995; Mayes, 1993; McCrae, Asp and Kendal, 1999; Pierce, 1999; Lagrange, 1999) have supported Heid's early findings, but also point out that the mere presence of CAS in a classroom does not mean that its potential benefits will be realised. Students must learn to use both hardware and software effectively. Artigue (2001) calls this process, by which such available technology becomes a powerful tool, 'instrumental genesis'. The learning this process requires presents a new, additional challenge for students. Atkins, Creegan and Soan (1995) expressed concern that students learning new mathematics with new technology may be distracted by the overhead of learning to use the technology while Lagrange (1999) observed that these 'technical difficulties in the use of CAS replaced the usual difficulties that students encountered with paper/pencil calculations' (p. 144).

To benefit from the availability of CAS, students must not only be able but also willing and discriminating in their use of this new technology. Arnold (1995) observed that the participants in his study showed a range of levels of engagement with the technology. He found that students' use of algebra software was sometimes impeded by their beliefs

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about mathematics and their perceptions of what was valuable. Arnold reported that students who viewed mathematics as 'answer-based' dismissed the possibilities for exploration of mathematical ideas that CAS provided. They valued unaided individual effort and devalued the use of technology. Lagrange (1996) also commented that in his experience not all students wanted to use CAS. They did not want to be relieved of pen and paper work and that many, in fact, enjoyed doing routine calculations. The present authors' previous studies (Pierce and Stacey 2001a, 2001b) provide examples of the individual nature of students' responses to the availability of CAS. This is clearly shown in the following students' comments.

> **Student A:** Sometimes I use pen and paper and (later) find you can do it on the computer. Then I prefer the computer but I don't start on the computer first or I get confused. (2001b p.16)

> **Student B:** I think it (CAS) actually helps me learn new things because when there are new things that I'm learning, while I'm finding them difficult, I can use DERIVE and go through the steps. With more practice, and seeing DERIVE go through it, I pick it up myself and then I can feel confident doing it myself without a package. (2001a p.9)

The value of CAS, like any new tool or aid, depends on how effectively it is used. Employing CAS to do mathematics requires the student to become familiar with both the hardware and the software associated with this technology. This presents the student with some learning overhead on top of learning mathematics. Students' success in obtaining benefits from the use of CAS will depend on how effectively they learn to use this technology. The efficacy of their use will depend on both technical and personal aspects: whether the student can operate the program with a minimum of difficulty; their attitude towards the use of CAS; and the manner and purpose of their partnership with CAS. In the light of the earlier literature and our previous experience of students' learning in a CAS environment, the first named author decided to develop a framework which might act as a guide for monitoring students' progress in Effective Use of CAS.

Framework for Effective Use of CAS

The aim of the framework, set out in table 1, is to highlight aspects of CAS use that determine how effectively a student is able to employ CAS both to do and to learn mathematics. The framework may provide a guide for systematically examining whether students are using CAS effectively. It may also provide a basis for planning and reflecting on teaching content and practice. It is designed for use by researchers and teachers, to structure observations about students' progress and to identify features of teaching which contribute to their progress. The purpose of the framework is not to provide a list of categories such that an observation instrument can be created with each item relating to one, and only one category. This may be desirable but does not seem possible.

The framework divides Effective Use of CAS into two aspects; the students' ability to handle the *technical* aspect of their CAS and the *personal* aspect, encompassing their attitudes and manner of use of CAS. Each of these is further divided into elements, which are discussed below and then illustrated by a set of common instances. Whereas the aspects and elements are intended to be generally applicable across levels of mathematics studied and types of students and courses, the common instances are specific to the setting where the framework is being used. The common instances of table 1 were developed for teaching an introductory functions and calculus course.

Table 1
Effective Use of CAS Framework

Aspects	Elements		Common Instances	
1. Technical	1.1	Fluent use of program syntax	1.1.1	Enter syntax correctly
			1.1.2	Use a sequence of commands and menus proficiently
	1.2	Ability to systematically change representation.	1.2.1	CAS plot a graph from a rule and vice versa
			1.2.2	CAS plot a graph from a table and vice versa
			1.2.3	Create table from a rule or vice versa
	1.3	Ability to interpret CAS output	1.3.1	Locate required results
			1.3.2	Interpret symbolic CAS output as conventional mathematics
			1.3.3	Sketch graphs from CAS plots
2. Personal	2.1	Positive attitude	2.1.1	Value CAS availability for doing mathematics
			2.1.2	Value CAS availability for learning mathematics
	2.2	Judicious Use of CAS	2.2.1	Use CAS in a strategic manner
			2.2.2	Discriminate in functional use of CAS
			2.2.3	Undertake pedagogical use of CAS

Elements of the Technical Aspect

The *technical* aspect relates to students' ability to use their own CAS to achieve mathematical goals. *Fluent use of program syntax* can be summarised as the changing of conventional mathematics to CAS syntax and knowing 'which buttons to push' or 'which commands to execute' when using the program to perform mathematical calculations. Common Instances of fluent use are seen when students enter syntax correctly or use a sequence of commands and menus proficiently. *Ability to systematically change representation* can be seen when students use CAS to, for example, plot a graph from a given rule, plot a graph from a table. Changing representations is highlighted because, in common CAS, it involves moving from one module of the software to another, whereas the element fluent use of program syntax applies within a module. *Interpreting CAS output* can cause difficulties that will lessen the Effective Use of CAS. In this paper, 'interpret' refers to the specific understanding needed when the format of results is different from the conventional pen and paper presentation, or is constrained by the limitations of the screen, rather than being able to make mathematical or real-world deductions from an output

Elements of the Personal Aspect

A *positive attitude* towards the use of CAS is seen when students value the availability of CAS as a tool for both doing and learning mathematics. Students with a positive attitude could be expected to persist with CAS use and overcome initial technical difficulties due to lack of familiarity with the peculiarities of the program. Students with a negative attitude towards the use of CAS for doing mathematics are more likely to avoid its use. The manner in which students approach the use of CAS can be described as being anything from a passive observer to fully involved participant, who displays *Judicious Use of CAS*. This is seen when students plan to use CAS in a strategic manner, rather than randomly "trying things out". They will selectively choose to use CAS for more difficult or time consuming problems, doing other problems by hand so that their *functional* use (aiming to get answers to problems) is discriminating. Furthermore, they will undertake *pedagogical* use, by using CAS to explore relationships or patterns in order to increase their own understanding of mathematics.

The Study: An Illustration of Use of the Framework

A study was undertaken to examine and monitor students' use of CAS in both doing and learning mathematics. The group studied consisted of 21 undergraduate students taking a 15-week (13 weeks teaching) introductory calculus course, where CAS (DERIVE 2.55) was available for teaching, learning, and assessment tasks. It is fully reported by Pierce (2001). The aim was to use the framework as a basis for considering whether students' Effective Use of CAS improved during the course and, if there was differential change amongst the students, which students improved in what aspects and why. Surveys, interviews and observation were used to collect data for class results and detailed case studies.

Technical Difficulties and *Judicious Use of CAS* surveys were administered at the conclusion of laboratory classes in weeks 1, 7 and 13. Students were asked to reflect on their use of CAS during that laboratory session and respond to a series of statements. The statements relating to possible difficulties (see table 2) had the option of 'not applicable' as well as a 5-point frequency scale from 'never' to 'every time'. The Judicious Use statements (see table 3) were presented as a single multiple-response item. In week 1 extra questions were asked concerning students' prior use of technology for mathematics and in week 13 they were asked to respond to statements concerning when they choose to use CAS and if they find CAS helpful. These responses, with those from interviews conducted in weeks 13-15, provided basic information about students' attitude towards using CAS.

Table 2
Statements from Technical Difficulties Survey with links to Effective Use of CAS framework

Possible area of difficulty	**Framework Element**
1. Authoring , using /, ^ , *, +, - symbols	1.1
2. Using brackets to force the structure expressions	1.1
3. Using syntax for commands eg FIT, VECTOR, F(x)	1.1 1.2
4. Using sequences of commands eg to substitute	1.1
5. Interpreting the results of the soLve command	1.3
6. Obtaining exact or approximate solutions	1.3
7. Working from the screen to ordinary maths symbols	1.3
8. Moving to a graph window	1.2
9. Obtaining a window which shows the graph required	1.2
10. Zooming to show key features of a graph	1.2
11. Copying from the screen for sketch graphs	1.3
12. Working out the scale of a graph	1.3

Table 3

Judicious Use of CAS survey items with links to Effective Use of CAS framework

Judicious Use of CAS survey statements	Framework Instances
Please tick as many of the statements below as apply.	
Today I have used DERIVE to:	
Find answers if the computer was suggested.	2.2.2
Explore problems if the computer was suggested	2.2.3
Explore variations on the set problems.	2.2.3
Explore, other than when directed but on the same topic	2.2.3
Explore other mathematics not on today's topic	2.2.3
Find answers I could 'easily' have done with pen and paper.	2.2.2
Find answers to 'hard' questions	2.2.2
Find answers to time consuming questions.	2.2.2

Observations of CAS use were recorded during class sessions and during examinations. The teacher/researcher made observations notes during laboratory classes, noting the way students were using the CAS and the nature of the questions they asked. The aims of this observation were to determine: if students had technical difficulties in using CAS; the 'manner' in which they approached its use; if they demonstrated discriminating functional use; and the extent to which they used CAS for pedagogical purposes. Discussions between the teacher/researcher and students during class sessions also provided information about their attitude to the use of this technology. The observation data were used to confirm the classification of students' functional and pedagogical use and to establish their profile of 'manner of use'. The extent to which students used CAS in a strategic manner was determined from their comments and the researcher's observations. Consideration of the students' survey responses and these observations provided cross validation.

Results and Discussion

All but one student had studied mathematics in year 12 but 7 only took a lower level course (V.C.E. Further Mathematics). All students had previously used scientific calculators and forty-one percent had also used graphical calculators for school mathematics. While thirty -eight percent had made some use of computers for school mathematics, only one student had previously used a Computer Algebra System.

Technical Aspect

It had been expected that students' ability to adapt to the use of this 'new' technology may relate to their general computing skills but no student experienced very high levels of technical difficulty. Only a student who claimed to have 'excellent' general computing skills showed any obvious advantage in using this 'new' software. Students' level of self-reported (table 2) and observed technical difficulty fluctuated during the course. Sufficiently similar results were found for each of the three elements of technical difficulty that, in this paper, they will be considered together. Total technical difficulty was scored out of 30 (A composite score equally weighting 1.1, 1.2 and 1.3 where zero indicated no difficulties i.e. low scores were better). At the end of week 1 half the group scored between 0 and 10 and the rest 10 to 15 and by mid course all scored below 10 and most below 5. However late in the course while half the group scored below 5 the other half scored between 15 and 20. Two factors may explain this result. First there were new difficulties to be overcome each time new mathematics was introduced that required new commands and syntax. Functions and then calculus required similar technical skills but trigonometry (taught late in the course) presented new syntax and interpretation hurdles. Secondly some students had few difficulties earlier in the course because they had avoided using CAS. In this short course those 'late starters' were experiencing difficulty in the last weeks. Observation showed that, in general, those who practised using CAS overcame technical difficulties and thus the personal aspect of Effective Use of CAS impacted on the technical aspect.

Personal Aspect

Based on their comments in class throughout the course and their responses to surveys and at interviews, the group could be divided into those with a positive and those with a negative attitude towards the use of DERIVE 2.55 for learning mathematics. Students who began the course with an open mind and an even mildly positive attitude showed improvement during the course in their Effective Use of CAS. As they gained confidence in using the program, they widened the range of use of the program and, as a consequence, their attitude to the place of CAS in their learning became more positive. On the Post-course survey even the three students who initially had a strong negative attitude towards the use of CAS each suggested that they had gained some appreciation for the place of CAS in doing some problems quickly. However, their attitude could only be described as 'less negative' than early in the course. These students with a negative attitude did not necessarily experience greater technical difficulty but they did make more limited use of CAS. It should be noted that a positive attitude did not preclude technical difficulties. Some students with positive attitude still reported experiencing a wide range of technical difficulties during the course. To attain a high level of Effective Use of CAS each student must learn to operate the program correctly. Feeling good about its use is not sufficient.

No student wanted to replace all by-hand mathematics with the use of CAS. Each student said that they preferred to do 'easy' problems by-hand. However, perceptions of what constituted an'easy' problem differed from student to student. There was also observable change during the course. Students sometimes used CAS to search for a general pattern by doing a series of examples in which they systematically changed the value of one parameter. Once a 'rule' had been established they would move to doing this processing mentally or by-hand. Most students became discriminating in their functional use of CAS, choosing to use CAS when they thought the problem would be time consuming or difficult. For example as they gained speed and confidence with differentiation they used CAS to differentiate rational functions but not polynomials. It was interesting to see that the majority of students liked to try to work problems by hand and would often choose to work in parallel with CAS, using its facility to check their by-hand work. In these ways students increased their Judicious Use of CAS.

Observation and students' survey responses indicated change during the course in both students' intent (*functional* or *pedagogical*) and manner of approach (*passive* – just observing others, *random* – trying anything, *responsive* –blindly following directions or *strategic* – following their own reasoned plan). As might be expected, students initially used CAS only for functional purposes and this use was non-discriminating. In fact they used CAS for simple problems which they might easily have done mentally or by -hand. At this stage most students were learning to use the program rather than learning new mathematics. In order to check their CAS results, students often chose to use CAS for questions that they felt they could confidently do by-hand. They wanted to be sure that they were using the CAS program correctly. All students progressed to become more discriminating in their functional use (as described above) and to undertaking some pedagogical use of CAS, at least when this was explicitly suggested. Some students took the initiative to use the facility to follow their own variations on problems in order to further their own understanding, taking a strategic approach and extending their pedagogical use of CAS.

Early in the course 40% of the group were passive or random in their approach to CAS and of the 60 % who sometimes used it strategically only 25% sometimes initiated such planned used without teacher direction. By emulating examples (for course details see Pierce, (2001)) and with teacher encouragement, by late in the course 90% of students used CAS in a strategic manner and 67% of these students would sometimes initiate this planning without teacher guidance. Only 10% persisted with a passive or random approach to the use of CAS. By late in the course most students were operating judiciously.

The detailed case studies emphasised that student response to the availability of CAS, and hence their Effective Use of CAS, was quite individual. The following brief examples illustrate this complexity. Sam and Jocelyn provide us with examples of students with a negative attitude towards CAS. Both avoided the use of CAS. Sam had considerable technical difficulty throughout the course and did not progress to using CAS in a strategic manner. Jocelyn was honest and open about her feelings when she said that computers scared her. In fact she experienced a low level of technical difficulty when she did use CAS but this did not change her attitude. Jennifer valued the availability of CAS and practised her CAS use even though she experienced technical difficulties each time new mathematics was introduced. Her difficulties with CAS syntax were linked to her difficulties in using conventional mathematical notation. Louise had a positive attitude towards CAS use but was not serious in her engagement with the mathematics and so continued to use CAS in a mindless random manner throughout the course. Stephen could do most of the work easily by hand and so used CAS less. He talked about being frustrated by technical difficulties but observation suggested that he actually experienced few problems. Stephen was positive about using CAS to extend his learning and to do harder problems. These examples suggest that the various elements of the framework should not be considered in isolation and that both Aspects must be addressed if CAS is to become a useful tool.

Conclusion

The purpose of this paper has been to describe what constitutes Effective Use of CAS, to present these constituents in an organised framework and to demonstrate the value of the framework by using it to describe the evolution of the Effective Use of CAS of a group of students in a functions and calculus course. The division of Effective Use of CAS into two basic aspects, the *technical* and the *personal*, seems robust. The intentions of these two aspects seem readily accessible and data can be gathered on each of these aspects separately. Even in one class, students varied considerably on each of the dimensions. The data showed some independence of the technical and personal and that students with positive attitude could be technically strong or weak. On the other hand, the study has shown that the personal and technical aspects influence each other over time: students with positive attitudes, for example, practise using CAS and so their technical ability improves. The data relating to the technical aspect were more 'user friendly' interfaces and hand-held CAS calculators, but, based on more recent teaching experiences, we still expect that the range of technical difficulty within a class is potentially wide. Overcoming technical difficulties is still a major consideration when teaching with CAS.

The division of the personal aspect into two elements has been successful. Once again, the two components are independent in the sense that the study found students with positive attitudes who were strong in their Judicious Use of CAS, using it strategically, and with discrimination while, on the other hand, a student with a positive attitude often used it mindlessly and randomly thus exhibiting low Judicious Use of CAS. The framework was also useful to point out to the teacher/researcher how her teaching could be altered to further emphasise the development of Effective Use of CAS.

The results of this study only serve to emphasise that a CAS, new in the box, merely has potential. The process of a user realising its capacity and going further to make it a sophisticated instrument in their hands (the process Artigue (2001) calls "instrumental genesis") still needs attention. In this study the framework was sufficiently successful in providing a structure for recording and analysing important components of the process to recommend its use in larger studies.

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