Spatial Reconstructions from Primary Children's Drawings

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In this paper geometric eigenproductions of primary school children are analysed in *spatial reconstruction experiments*: A *donor* draws an addressee- and purpose-specific picture of a spatial building, and an *acceptor* tries to reconstruct this object only by means of the partner's drawing. Comparing the original building with its reconstruction, the two partners evaluate the informational substance of the drawing. This paper discusses, in how far the drawings are reconstruction-efficient and how they are improved in the course of the interaction.

Eigenproductions in the Primary Mathematics Classroom

Eigenproductions are documents which children create whilst mastering (in the full sense of the word) a mathematical problem (Treffers, 1991; Selter, 1993). In them children establish their initial statements for the solutions, their strategies and/or results.

Time-based eigenproductions allow recognition of their history of origin; such documents are transcripts or representations which have the intention of presenting a chronological sequence of their elements. However, eigenproductions in which the time sequence of their composition is not intended, can occasionally still be "sequenced", that is, analysed with regard to their history of origin. The study of time-based documents in empirical didactical research frameworks therefore can shed light on the processes of learning mathematics—both with respect to individual learning as well as regarding to interaction processes.

Many of the primary school children's eigenproductions found in didactical analyses have been specifically created for these studies. Eigenproductions relating to the same task often portray a rather confusing variety, making systemisation difficult. In the experience of the author, eigenproductions are in each case *part of a learning history* and hence influenced by its development. Once the conditions of this learning history are considered, the eigenproductions can be interpreted more accurately.

Eigenproductions are messages. They carry information about the fictitious partner a child has in mind and the respective message or information this partner is assumed to be interested in. Children who are not given specific information regarding the addressee and his/her interests, often create eigenproductions for a fictitious partner, whom they sometimes even model as a kind of archetypal copy of themselves.

The author therefore differentiates between *purpose-specific eigenproductions* and non-purpose-specific ones. For example, the response to the task "draw a picture of this building" will in general not be a purpose-specific one, while the response to the task "draw a picture of this building that can be used as a construction plan" will most likely be purpose-specific.

The author further differentiates between *addressee-specific eigenproductions* and non-addressee-specific ones. For example, the verbal response to the task "describe how to start a car" differs depending on whether the person has or has not a drivers license.

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Working environments in which children use purpose-specific and addressee-specific eigenproductions can additionally be provided with a *feed-back option* that can influence further eigenproductions. Thus, reconstruction tasks are created. The organisation of working environments as reconstruction tasks is a fundamental and effective method for both the empirical investigation of eigenproductions as well as for the organisation of classroom working environments.

An Investigation of Geometric Eigenproductions

One research focus of the author places particular emphasis on teaching and learning of spatial geometry in primary mathematics (Wollring, 2001; Wollring & Jungwirth, 2001).

Spatial geometric drawings of children, which are not purpose-focused and not addressed to specific partners have been analysed in a number of field studies. Pars pro toto the classical experiments of Lewis (1963), Mitchelmore (1978) and Ingram and Butterworth (1989) can be named. Lewis analysed children's drawings of a cube with special paintings on three of its sides and classified them by morphological patterns. The main distinction is *schematic versus realistic*. Mitchelmore analysed children's drawings of various solids in a rather behavioristic environment and classified the drawings by *series which characterise developments*. Both studies describe children's methods of encoding spatial objects. Ingram and Butterworth, in addition, investigated how kindergarten and primary school children draw configurations of two cubes. The smaller one could be placed either in front of, within, behind, on, or next to the bigger one. Beside morphologic strategies Ingram and Butterworth additionally described the drawing strategy of *partial occlusion*.

All three studies analysed drawings initiated by the neutral stimulus "draw what you see" which were neither purpose-focused nor specifically addressed. However, these studies provided categories of drawings for a new research project by the author investigating spatial reconstructions from drawings. This study, which provides the background of this paper, addresses the following research questions with respect to children's drawings: How do pairs of grade 3 children (8 to 9 years old) communicate the solution of a spatial geometric reconstruction on the basis of their eigenproduced drawings? Which effects does this communication have on succeeding reconstructions? What patterns of drawings and interaction arise? How successful are the pairs of children in solving the task?

Design of the Spatial Reconstruction Experiment

The study was organised as a set of eight experiments characterised by different working environments. All of these experiments, designed and directed by the author, were organised as a series of clinical interviews. The interviews were conducted by student teachers acting as teacher-researchers in interpretative classroom research (for details with respect to this approach see e.g., Wittmann, 1985; Peter-Koop & Wollring, 2001).

The special experiment reported in this paper, consisted of 16 clinical interviews with pairs of children from third grade classrooms. Each of the four student teachers involved in this part of the study conducted four of these interviews.

Each individual interview consisted of eight episodes in the form of *spatial reconstruction tasks*.

Each episode was designed as follows: One child acted as *donor*, the other child as *acceptor*. At the beginning only one child, the donor, was present, while the other child

was waiting outside the room. The donor was given an *original spatial building* with the task to draw this building, bearing in mind that the partner, once entering the room would be confronted with the task to rebuild this object with given material merely on the basis of this drawing. Once the donor had completed the drawing, the original building was covered before the acceptor was asked to come into the room. The drawing was shown to the acceptor who was then asked to produce a *reconstruction*, that is, a copy of the original spatial building. During this reconstruction process the donor was present, however had to remain silent. After the acceptor had finished the reconstruction, it was compared with the original. The partners then assessed whether the drawing was reconstruction-efficient, discussed possible deficits of the drawing and, where necessary, jointly developed suggestions for improvements to be used in the drawings of the subsequent episodes. After this, the episode was completed. Before a new spatial building was introduced in the next episode, the two children changed roles (i.e., the donor became the acceptor and vice versa).

In the 16 interviews of this experiment, the spatial buildings were dice configurations similar to those used by Ingram and Butterworth (1989) in their experiments.

Each interview was documented by a video recording of the children's interaction and the original drawings of the children. On the basis of these documents the interviews were analysed in a quantitative and in a qualitative dimension:

Quantitatively, the reconstruction success was evaluated noting "1" if the reconstruction was successful and yielded a complete copy of the original spatial building and "0" if it was not.

Qualitatively, the interviews were analysed on the basis of *episode plans* arising from the video documents. For every episode, the episode plan portrays the roles of the partners, the original building, the drawing and the reconstruction (see Tables 1 and 2). Sections of special interest were additionally documented by transcripts. This data basis yielded information with respect to several aspects: The structure of the drawings was analysed and compared with the patterns reported in the literature. The sequences of the drawings were analysed with respect to specific developments and regarding the respective interaction between the two partners.

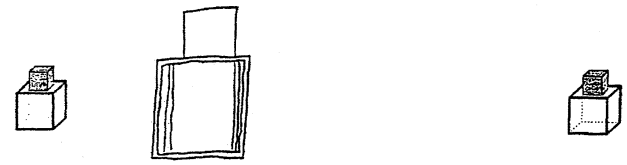
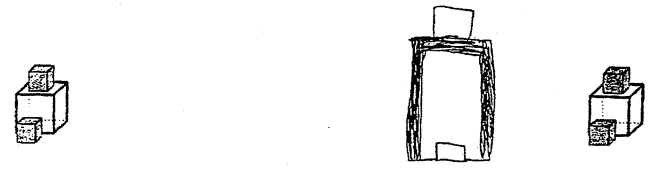
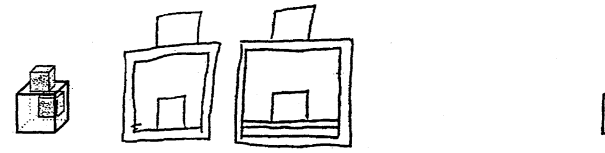
Discussion of Selected Results

Key results of this study can be summarised both with respect to quantitative as well as qualitative categories. In this paper, primary focus is given to qualitative results describing the formation and the structure of the children's communication. However, the quantitative results were largely unexpected and are therefore briefly introduced.

Quantitative Results

- In the entire study 120 clinical interviews of spatial reconstruction experiments were conducted and video-taped. 16 of these interviews form the data basis of the reconstruction experiment using a specific working environment which is similar to that of Ingram and Butterworth (1989). In a "coarse" scoring system, in which successful reconstructions were counted as "1" and unsuccessful ones as "0", the pairs of children reached an average reconstruction success of 63% (standard deviation 10%). In a finer scoring system in which partial reconstruction success was scored "> 0", the pairs reached an average success of 78% (standard deviation 8%).

Table 1
Richard and Joachim (3rd grade) Reconstruct Dice Configurations According to their Eigenproduced Drawings. the Large Die is Transparent, the Small Ones are not. Left: Donor's Perspective, Middle: Donor's Drawing, Right: Acceptor's Reconstruction

Episode 1 2.34 – 5.41 *Donor:* Jo. *Acceptor:* Ri. Successful rec.	
Episode 2 6.06 – 8.16 *Donor:* Ri. *Acceptor:* Jo. Successful rec.	
Episode 3 8.41 – 12.17 *Donor:* Jo. *Acceptor:* Ri. Successful rec.	
Episode 4 12.43 – 14.53 *Donor:* Ri. *Acceptor:* Jo. Successful rec.	
Episode 5 15.18 – 18.50 *Donor:* Jo. *Acceptor:* Ri. Successful rec.	
Episode 6 19.20 – 21.15 *Donor:* Ri. *Acceptor:* Jo. Successful rec.	
Episode 7 21.40 – 27.00 *Donor:* Jo. *Acceptor:* Ri. Unsuccessful rec.	
Episode 8 27.25 – 29.27 *Donor:* Ri. *Acceptor:* Jo. Successful rec.	
Episode 9 29.48 – 32.34 *Donor:* Jo. *Acceptor:* Ri. Successful rec.	

Similar results were found in the other spatial reconstruction experiments employing different working environments. (However, the finer scoring systems have to be closely adapted to the specific material used in this environment.) In general, the pairs of children were less successful in the first two reconstructions of a sequence than in the following ones. This indicates that a special starting phase is needed for effective communication.

Selected Qualitative Results: Two Case Studies

With respect to the qualitative results of the study the following two case studies were chosen in order to document typical effects. These are related to the formation of the drawings and their development in the children's interaction. Tables 1 and 2 show the episode plans of the reconstructions of two teams—Richard/Joachim and Sirkka/Martin.

The documents indicate that the drawings are for the most part *reconstruction-effective.* They enable the acceptor to produce an exact copy of the original dice configuration. Furthermore, the partners obviously learn through comparative feed-back. The donors organise their drawings according to schematic patterns, which in most cases the acceptor can correctly interpret. The picture sequences during this process indicate two forms of mutual understanding:

- The two children each develop their own drawing strategy but increasingly learn to understand those of their partner as in the case of Richard and Joachim.

Significant elements of the drawing strategies are mutually adopted by the partners – as in the case of Sirkka and Martin.

In the opinion of the author, however, a most remarkable finding is the fact that the series of the children's drawings do not develop in the sense of an increasingly realistic visual presentation, for example in the form of slanting pictures. Rather the contrary is the case. The drawings show an increasing tendency towards formal abstraction. Similar to parts of a mathematical formula, the donor only represents those elements which are considered necessary for effective interpretation by the acceptor. The drawings hardly show any redundancies.

Richard for example, from the second drawing onwards, only draws the "frame" formed by the front edges and in addition only the rear bottom edge. This he uses in order to designate the position of the smaller die unmistakably. Joachim learns to interpret this way of drawing, although he himself draws differently.

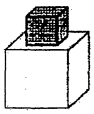
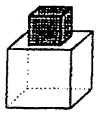
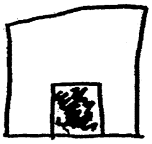
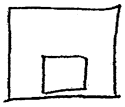
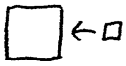
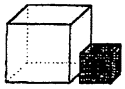
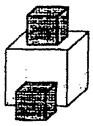
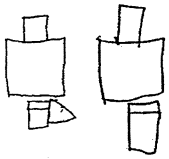
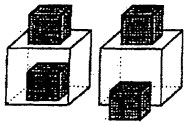
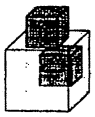
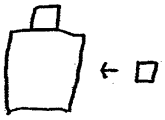
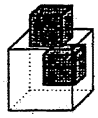
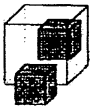
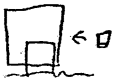
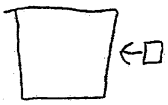
Sirkka "invents two signs" as she explains in the interview—the dark shading to signify "inside" and the arrow for the mental displacement to designate "behind this". Martin learns to read these signs and then adopts them with increasing success in his own drawings.

The findings of these two case studies correlate with the overall findings of the study. In their drawings all teams create elements of a situation specific pictorial language. In this, they generally do not only use optimised visual realistic pictures, they also invent formal elements similar to mathematical formulae and negotiate the meaning of these formal components. The correspondence itself always follows one of two different patterns:

- parallel development of individual encoding strategies accompanied by the understanding of the partner's strategy; or

development of a common encoding system by exchanging invented "strategy elements" during the interaction process.

Table 2
Sirkka and Martin (3rd grade) Reconstruct Dice Configurations According to their Eigenproduced Drawings. All dice are Non-Transparent.
Left: donor's perspective, middle: donor's drawing, right: acceptor's reconstruction

Episode 1 2.04 – 3.32 *Donor:* Si. *Acceptor:* Ma. Successful rec.				
Episode 2 3.53 – 5.57 *Donor:* Ma. *Acceptor:* Si. Unsuccessful rec.				
Episode 3 6.16 – 8.57 *Donor:* Si. *Acceptor:* Ma. Unsuccessful rec.				
Episode 4 9.10 – 12.12 *Donor:* Ma. *Acceptor:* Si. Successful rec.				
Episode 5 12.29 – 13.50 *Donor:* Si. *Acceptor:* Ma. Successful rec.				
Episode 6 14.12 – 15.15 *Donor:* Ma. *Acceptor:* Si. Successful rec.				
Episode 7 15.34 – 17.25 *Donor:* Si. *Acceptor:* Ma. Unsuccessful rec.				
Episode 8 17.42 – 18.59 *Donor:* Ma. *Acceptor:* Si. Successful rec.				

Conclusions

The children's drawings document genuine self-organised mathematical learning processes—the interactive development of an efficient means of communication in a geometric working situation.

Working environments such as those used in this study not only yield important research results, they also serve as models for working environments in the classroom.

Even if, in many cases, the expressions and procedures conventionally used in mathematics are not found or invented, it can still be assumed that children who have worked in environments as described above become increasingly sensitive to respective problem posing and, influenced by the need of invention, they also approach the corresponding conventions differently.

While it is certainly not suggested that the entire primary mathematics curriculum can be organised according to such environments, the main idea behind these experiments can inform classroom practice. With respect to a constructivistic understanding of mathematical teaching and learning, hence the teacher should be sensitive to children's eigenproductions and foster social learning by encouraging purpose- and addressee-specific working environments. Thus, children's original approaches and productions are not overwhelmed by norms too early.

In this context, the analyses of eigenproductions carried out by the student teachers involved in this project can serve as effective elements of a teacher education programme, since they yield foundations rather than simple action patterns for didactical decision-making, provide encouragement and self-confidence, reinforce design-competence and can in some cases even lead to a more positive re-evaluation of mathematics course contents (Wittmann, 1985).

Eigenproductions are communicable. In interactive learning processes they are optimised and support systems of communication in a self-organising fashion. Hence, especially geometric eigenproductions seem to be suitable in order to stimulate and to further develop a culture of interaction in the classroom. With respect to teacher education, the study of efficient interactive eigenproductions can provide an authentic constructivistic perspective on classroom discourse.

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