What Does it Mean to Teach Mathematics Differently?

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In the past decade, many countries called for changes that result in students knowing a 'different' kind of school mathematics. This is due to a changed view of learning that requires different forms of teaching which are only beginning to be understood. In this paper, findings from research in primary maths classes with 'reform-oriented' culture are presented; examples drawn from these classrooms illustrate differences in teaching that influence the mathematics children learn. Two dynamic dimensions derived from this research are presented.

Two aspects characterize the current attempts to improve mathematics education in many countries over the past decade. First is the influence of the 'cognitive revolution' in psychology that redefined how individuals learn, and second the attempt to change the perception of the discipline of mathematics in school. Currently there is widespread acceptance of the view that learning is an active constructive process. It is thought that as students learn they impose their own interpretation and use these to make sense of the world (Bruner, 1990, 1996). Additionally, rather than conveying a view of mathematics as narrowly consisting of executing well-established procedures, the shift in goal is to present it as a subject that consists of patterns and relationships that are understandable through mental activity that involves mathematical reasoning and logic (Devlin, 1994).

At the center of this reform in mathematics education at the primary level are two interrelated qualities that are thought to underlie children's learning that, when taken into consideration, designate significant changes in the nature of teaching. The first quality is the social nature of children's learning and the fact that rich social interactions with others substantially contribute to children's opportunities for learning (e.g., Bruner, 1990, 1996). The second aspect is the interplay of children's developing cognition and the unfolding structure that underlies mathematics (e.g., Nunes & Bryant, 1996). These qualities suggest that social situations for learning are needed that are different from those that exist in most primary maths classes (Wood, 2001). It follows from this that a different form of teaching is needed.

Research shows that the quality and nature of students' mathematical learning in the 'reform classes' in the United States are significantly different from pupils' learning in conventional classes (e.g., Carpenter, Fennema, Chiang, & Loef, 1989). Cobb, Wood, Yackel, and McNeal (1992) found that, in particular, these classes could be distinguished by the differences in the quality of children's explanations and justifications they were required to give for their solution processes. Furthermore, these differences were found to affect the quality of children's thinking and reasoning about mathematics.

Along with this, there are similar studies that examined teaching in these classes. The research has ranged from studies examining teacher beliefs to descriptions of changes in practice (Fennema & Nelson, 1998; Wood & Nelson, 2001). Yet, our understanding of these forms of teaching are is still not as clear as our knowledge of children's learning in these settings. For past decade, I have examined this teaching from a perspective grounded in the practice of teachers through the examination of the interaction patterns that exist in

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conventional and reform classes in an attempt to provide better understanding. Therefore, the purpose of this paper is to present findings from this research, to reconsider what it means to recreate a class culture for learning mathematics differently, and to offer suggestions about how teachers might accomplish this. Empirical examples of teaching drawn from a research project in primary maths classes are used to illustrate the differences in teaching and the ways these influence children's opportunities for learning. In the end, two dynamic dimensions are offered as a way to reflect on and examine mathematics class cultures which teachers might find useful to examine the interaction in their own classes. In order to examine teaching, it is important to take into consideration that teaching by definition is an interactive activity and teaching in school settings is an endeavor that is conducted as children are participating as members of a group (Wood & Turner-Vorbeck, 2001). Thus, it is essential to not only characterize the teaching that is observed but to also examine the ways teachers create a culture for learning in order to fully understand how pedagogy influences students' opportunities for mathematics learning.

Methodology and Analysis

The data for the analysis of primary maths classes was collected as part of a larger project that examined teaching and teacher learning.[1] The data was collected over a two-year period and consisted of approximately 300 videotaped mathematics lessons gathered over a period of a school year in 15 'reform-oriented' second-and third-grade mathematics classes in the United States. From this data resource, six classes were selected and 30 lessons from each were analysed using a coding scheme developed for the analysis. For further details about the lessons, the coding scheme and methodology see Wood and Turner-Vorbeck (2001). In a secondary analysis, this data along with data from conventional classes were analysed for the purpose of examining the nature of children's mathematical reasoning related to teacher questioning during these lessons.[2]

Mathematics Lessons

The 'reform-oriented' lessons frequently consisted of children working in pairs for 25 minutes followed by class discussions of their solutions for approximately 20 minutes. Lessons from conventional classes consisted of whole class discussion followed by individual seatwork. The aspect of the lesson of interest for the investigation of teaching is the class discussion. This is the most complex lesson event for both teachers and students and therefore provides the greatest opportunity to examine demands of teaching in such situations.

Lessons were collected on a daily basis during the first four weeks of school. These *sequential lessons* were primarily used to examine the manner in which each teacher initiated and established the social norms that underlie the interaction that occurs during the mathematics class. Additional lessons were collected twice a month for two consecutive days and thereafter throughout the remainder of the school year. Another set of lessons was drawn from this latter data source that consisted of those discussions in which all the teachers used the same instructional activity. These *anchor lessons* were used as contrast lessons for the purpose of analysing the patterns of interaction and discourse that were established between the teachers and their students. The lessons for the conventional

[1] This project was sponsored by the National Science Foundation.
[2] This project was sponsored by the Spencer Foundation.

classes were also collected during the first weeks of the school year and contained similar mathematical content.

The methodology and analysis follows a qualitative research paradigm in which observations of lessons were the data source for the interpretative procedures used. Initially, the methodology and analysis used were similar to that of Strauss and Corbin (1990) in which categories were developed and coded. In this approach, the process of coding is seen to underlie analysis. That is, interpretation is an integral part of the coding process. Following this analysis, micro analytic interpretative procedures similar to those described by Voigt (1990) and Krummheuer (personal communiqué March 1996) were used to analyse selected lesson discussions.

Each lesson was videotaped and fieldnotes taken. Following the videotaping, a log of each lesson was made which served as a detailed record to be used for later analysis. The technique of logging captured the nature of the interaction that occurred during the lesson but it did not contain the precise detail of a transcript. The logs were recorded in a line-by-line format and coded according to the scheme developed.

A set of coding categories was developed for coding the discussions in a line-by-line manner. Each line of the log was coded individually by three members of the research team and then discussed as a group, and any points of differences in coding were further deliberated among the members until consensus was reached. One set of coding categories was used to analyse the norm statements made by teachers. Another set of coding categories was used to analyse the teachers' questions or statements made during class discussion. The codes then served as the starting point for describing the patterns of interaction and the discourse that occurred during the class discussions. Initially, the codes served as a common notation for reacting to each sentence in the log which allowed not only for the identification of patterns in the interaction and discourse but also for the initial interpretative conjectures used to guide the micro analytic interpretative process.

Micro Analytical Interpretation

Following the coding, the typical patterns in the interaction and discourse were identified and used to establish the episodic sections of the discussion. In addition, from the data resource, specific lessons were selected and transcribed. These lessons were analysed using micro analytical interpretive procedures. These procedures continue the line-by-line examination of the dialogues but provide in depth analysis of the events that occurred in the episodic sections. This process allowed interpretations to be made about the meanings held by the teacher and students in each situation. Interpretative memos were written which contained detailed descriptions of each event.

Results

In this work, the nature of teaching is broadly described in relation to opportunities created for students' reflective thinking and reasoning revealed in empirical analysis of the students' talk. The categorical patterns of interaction served as the basis from which to identify the similarities and, more importantly, the differences in the classroom mathematics practices that represent the ways in which the culture of the classrooms differed. The findings from the analyses confirmed that reform-oriented classes are fundamentally different from traditional classes. In addition, the analysis revealed that among reform-oriented classes the classroom mathematics practices do not consist of a singular form of practice, but rather consist of distinctive environments or contexts. The distinctions in classroom environments also illustrated the augmented demands for more sophisticated types of pedagogy for teachers.

Following the analyses of the empirical data, the results were condensed into generalizable patterns of interactive and communicative exchanges from which the theoretical framework was created as shown in Figure 1.

Conventional Class Culture

Discussion Context	Mathematics Thinking	Explainers (student)	Listeners	
			teacher	students
Report Correct Answers	Recall answers & procedures	Tell answers Tell procedures	Evaluate Ask test questions	Pay attention

Reform Class Cultures

			Responsibility for Participation		
Strategy Report	Compare Contrast	Tell different ways	Responsibility for Thinking	Accept Elaborate	Solve same way
Inquiry	Reason to clarify or question	Clarify solutions Give reasons		Ask questions Provide reasons	Way makes sense Ask questions
Argument	Reason to justify or challenge	Justify Defend solutions		Disagree Make challenges	Disagree Make challenges

Figure 1. Theoretical Framework for Teaching and Learning.

The findings revealed differences in students' thinking and reasoning could be attributed to the type of questions teachers asked. In addition, interaction patterns that were created by teachers with their students for participation influenced the nature of participation. The differences that exist in the class cultures, thus, could be attributed to specific variations in the expectations for participation the teacher establishes and the cognitive demand teachers' questions place on students' mathematical reasoning. Two dimensions, participation and student thinking, form the theoretical framework used to conceptualize differences among reform-oriented classroom into three separate contexts, *strategy reporting, inquiry*, and *argument.*

Thinking Dimension and Axis

Therefore, in the Theoretical Framework, one axis is the *Responsibility for Thinking Axis* (represented by the vertical black arrow in Figure 1) that represents conjectures made about the relationship of the three types of classroom practices, in particular the questions asked, to the possibilities for students' reflective thinking. It is conjectured that the observed differences in children's thinking that occur in the three types of environments

indicates an increase in the demand for reflective activity. Thus, the inclusion of the Responsibility for Thinking Axis shown in Figure 1 enables connections to be made from the categorical changes in demand for children's thinking to theoretical cognitive explanations of conceptual understanding. Thus, the inclusion of the Responsibility for Thinking Axis enables links to be made from the categorical changes in children's thinking to the shifts in teachers' activity. Finally, the assumption that underlies the Thinking Axis is increased responsibility for student thinking is an increase in student autonomy in learning.

Micro analytical interpretive analysis of the interaction patterns reveals notable differences in opportunities for students thinking. But more importantly the comparison of the micro interaction patterns with students' thinking considered as collective or group thinking reveals differences. Specifically, the analysis reveals the extent to which students are engaged in mathematical reasoning that is the heart of which is abstraction and generalization in the construction of mathematical knowledge. Moreover, it is the demand for *reasoning of justification* that separates these class cultures (Samarapungavan & Wood, 1998).

Participation Dimension and Axis

In the Theoretical Framework the other axis is the *Responsibility for Participation Axis* (represented by the horizontal black arrow in Figure 1) that illustrates the increasing responsibility of students to participate in the ongoing discussion. Micro analytical interpretive analysis of the class discourse reveals some distinction in the behaviour of the explainer. But, the important difference among the class cultures resides with the expectations for what to do if a listener. The origin of these differences is found in teachers' normative statements and the manner by which they establish meanings for these expectations at the beginning of the school year.

Illustrative examples of class cultures. The following dialogue of an example of a Strategy Reporting culture is taken from a second-grade class (7 year olds).

Teacher: How about 36 plus 36? (Teacher writes on the overhead, 36 + 36 = __). Sheila.
Sheila: 66.
Teacher: And how did you get 66?
Sheila: Well, 3 plus 3 is 6.
And then there are two 6's and a plus. Then I just got 66.
Teacher: What did you do with these two 6's?
Sheila: I just added them up.
Teacher: You added them up.
And what did you get when added 6 and 6?
Sheila: Well, I got 12, but it didn't work.
Teacher: 12 didn't work.

Sheila: I think it is 12 (inaudible), I think it is 72.
Teacher: Did anybody do it a different way?

The detail of the children's descriptions varies depending on the extent to which teachers, through their questioning, demand comprehensiveness and clarity in the explanations from the children. This situation can be depicted as one in which students "tell how they solved the mathematics problem."

The following dialogue example of Argument culture is taken from a second-grade class (7 year olds).

Teacher: What did you get for an answer for this problem? [52 – 33 =___]
Fred: 25.
Sara: 19.
Adam: 21.
Teacher: Any other answers? Okay. Fred tell us how you got 25.
Fred: We used the unifix cubes. 52 and then we took away 33. First I took away the tens. 42, 32, 22. Then I counted back the ones, 21, 20, 19.
Karen: But you said it was 25.
Fred: I know, but now I think it is 19, because I counted it again with the cubes.
Teacher: John what do you want to say? (He has his hand raised).
John: I went back to 52 take away 30 is 22 (points to second problem on the paper). And I took away 3 more and that was 19. So I think it is 19.
Teacher: Okay. But why did you take away 3?
John: Because 52 take away 30 is 22, and 33 is 3 more than 30 so it was 19.
Teacher: How did you know that it was 19?
John: Because if 52 take 30 is 22, 52 take away 33 is 3 more than 30, so then I had to take away 3 more from 22, and that would be 19.
Teacher: That makes sense. Sarah what would you like to say?
Sara: Well if you take 30 from 50, then you would have 20. Then you would have 2 and that 3, so you could take 1 from 20, and that would be 19.
Mark: This is too confusing for me. Sarah, I don't understand why you took the 1 from 20.
Sara: Because you have 2 minus 3 and so you need 1 off the tens.
Mary: But if you took 1 from the 20, what happened to the 2 and the 3?
Sara: I took 1 from the tens and added it to the 2 to make 3. [Then] 3 minus 3 is 0. So then I had to take 1 from the tens-20 and that makes the answer 19.
Ryan: Well if you check it by adding 19 and 33, you get 52, so 19 is the answer.
Karen: I think the answer must be 19, because we did it so many different ways to figure it out. And we got 19.
Class: Agree. It is 19.

Discussion and Implications

As adults, with experience in traditional mathematics pedagogy, it is not easy for us to envision how teaching mathematics could be different. Thus, it is somewhat difficult to transform the calls for change into the reality of our everyday teaching. We might begin to reconsider and recreate a mathematics class culture for learning by examining interactive situations that occur during whole class talk. To do this, the two dynamic dimensions discussed above might provide a means for reflection on the interaction that occurs in whole class talk. By focusing our reflection and examination on the students, what they do to participate, and what they reveal of their thinking, as teachers we can begin to examine the culture created in our mathematics classes in light of the changes proposed for student learning.

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