Teachers' Conceptions of School Algebra and its Teaching: Preliminary Findings from a Study in Colombia

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This research report is based on data from a larger study whose overall purpose is to investigate the relationship between mathematics teachers' conceptions of their teaching practices —especially of beginning algebra— and of *improving* their practice. Data collected from 13 secondary school mathematics teachers show that although the teachers' conceptions of school mathematics and their portrayed teaching sequences in Grade-8 algebra were very similar, teachers differed widely in their conceptions of their own teaching.

Introduction and Background

In Colombia, and at the international level, one of the most problematic areas of school mathematics learning is algebra (see, for example, Küchemann, 1981; Agudelo-Valderrama, 2000). Over the last two decades we have seen a growing body of research on children's algebra learning, on adults' perceptions of algebra and, at the same time, an increase in theoretical conceptualisations related to approaches that can be followed in the teaching of school algebra. However, little is known about the teaching practices that take place in algebra classrooms or about teachers' conceptions of algebra and its teaching (see for example, Kieran, 1992; Doerr et al., in press).

Because of the interest of this study in how teachers' conceptions of their teaching of algebra relate to their conceptions of change, the term conceptions has been defined to encompass teachers' knowledge intertwined with teachers' beliefs and attitudes. Knowledge assumes certain evidence that beliefs do not assume (Cooney, 2001), and beliefs have an affective factor in them (McLeod, 1992; Pehkonen & Furinghetti, 2001). Attitudes are "stable, long-lasting predispositions to respond to certain things in certain way (Törner & Grigutsch, 1994, cited in Furinghetti & Pehkonen, 2002). Beliefs and attitudes have been included within conceptions in this study because of their "powerful impact on teachers' make up and approach" (Ernest, 1989, p. 25) and, therefore, on teachers' conceptions of their change.

Doerr and colleagues have argued that the lack of research on teachers' knowledge has adversely affected the impact in schools of the research on children's algebra learning. This present study proposes that the knowledge gained on children's algebra learning can be more effective when we gain some insights not only into teachers' knowledge of basic algebra concepts and its teaching but also into teachers' conceptions of their **own** teaching of school algebra. A large number of quantitative studies have focused on teachers' beliefs about mathematics and mathematics teaching in general, but these studies often focus attention on teachers' beliefs "as if they existed in a social and political vacuum drawing fundamentally upon a psychological paradigm which seems unable to account adequately for the difficulties of teacher change" (Gates, 2001, p. 19). The few qualitative studies that have looked at the relationship between teachers' professed conceptions of mathematics and mathematics teaching and their actual practice have not looked at the teachers' conceptions of their own practices. How do teachers see the relationship between their conceptions of mathematics and their conceptions of their own practices (as opposed to how researchers see it)?

In thinking about the possibilities of change of mathematics teaching practice in Colombia —as in any other context— what counts is the teachers' conceptions of their own teaching practice, which must take into account their understanding of what they see as barriers to change in the specific contextual situation. We have very little knowledge about teachers' understanding of their own teaching practices, (e.g. their conceptions of their role as teachers, the contextual situations of their teaching and their change).

The ultimate purpose of the study from which this paper stems is to investigate the relationship between mathematics teachers' conceptions of their practices —especially in the area of beginning algebra— and of *improving* their practices. In order to establish the relationship between mathematics teachers' conceptions of their practices and of *improving* their practices, it is necessary to study the teachers' conceptions of mathematics and mathematics teaching, as these would represent the bases for teachers talking and explaining their own teaching. In this paper we will report findings related to teachers' conceptions of their teaching, arising from data collected through a questionnaire and follow-up interview in Phase 1 of the study. Specific data on teachers' knowledge (e.g., content and pedagogical content knowledge) is not reported in this paper.

Some Background Information on the Teaching of Algebra in Colombia

Until the late eighties, the provision of mathematics education in Colombia took place according to the National Curriculum in force which was prescriptive and followed a "Pure Mathematics model" (Robitaille & Dirks, 1982). Topics were treated in a compartmentalised way, presenting algebra as a package course to be taught in grades 8 and 9. These prescriptions were translated into text books that became the sole focus of teaching and learning all round the country (Mockus et al., 1985), despite the fact that textbooks were not mandated.

At present, and according to central policy issued in 1994, schools are encouraged to adapt the curriculum to suit their own school needs 'to cater for the educational needs of different communities'. As it reads in the new legislation, "teachers are expected to have an important input in this process that aims at gaining the necessary flexibility of the curriculum". Nevertheless, and as it could be expected, current studies show that the same textbooks continue to be the only source for teaching and learning in schools (see Perry et al., 1998; Agudelo-Valderrama, 2000).

Data collected in the Colombian context on the learning of beginning algebra (Romero et al., 1998; Agudelo-Valderrama, 2001) and on its teaching (Agudelo-Valderrama, 2000) point to the great need to focus on the curriculum of this area, where students' lack of motivation for work is a general problem which, consequently, produces the highest rates of failure and school drop out. Comparison of performance in mathematics of students from 42 countries in the TIMSS[1] has ranked Colombian students in the lowest places.

Current Study

The data collection —which took place over a six-month period— was divided into two phases. In Phase 1, whose aim was to identify a variety of conceptions from an initial group of teachers in order to select case studies, data were collected from a group of 13 mathematics teachers, who taught at six different (state and private) schools in Bogotá. The participating teachers varied in ages and teaching experience and were teaching in Grade 8 during the academic year of 2002, when the data collection took place. In Phase 2, a multi-case study (a maximum variation sample (Patton, 1990)) with the participation of nine selected teachers was carried out.

Data Collection

In Phase 1, data were collected through the use of two questionnaires and an interview, which took place in the following sequence: Questionnaire 1, Questionnaire 1 follow up-interview, and Questionnaire 2. In phase 2, data were collected through classroom observation, interviews (where a concept map activity was included), examination of curricular materials and a focus group session. The data presented in this paper are based on data from one of the sections of Questionnaire 1 and the follow-up interview.

Questionnaire 1 and follow-up interview. Questionnaire 1, which was developed using the format of the Values and Mathematics Project[2], was composed of three sections: Section A contained items related to the purpose of teaching algebra in schools, teaching style, type of classroom pupils' work and forms of pupils' assessment. With the exception of question A1, related to the purpose of algebra, questions in this section were organised so that respondents could make a distinction between their preferred style and what they actually do in their classrooms. Space was provided for respondents to add their own ideas with the corresponding rankings, if they wished, as can be seen in the following example where questions A4 and A5 related to teaching styles and types of classroom work are presented.

A4. The types of classroom activity that I would **prefer** to see in my Grade-8 classroom are:

Ranking

- Pupils developing efficiency in algorithm-routine practice. ☐
- Pupils working at the board, especially when they have difficulties in applying algorithms. ☐
- Pupils engaged in the creation of algorithms and formulae. ☐
- Pupils discussing ideas and working systematically. ☐
- Pupils solving closed word problems. ☐
- Pupils using calculators to assist their learning and to use their working time more efficiently. ☐
- Pupils posing open problems and working on developing ways to solve them. ☐
- ☐

A5. The types of classroom activity that **actually** take place in my classroom are:

List of descriptors, as above, inserted here.

If the choices you made for question A4 are different from those of question A5, please express the reasons why they are different. ----------

Descriptors used in Questionnaire 1 were developed assuming that teachers' conceptions of the nature of mathematics can be very different, and can range from a traditional perspective of a collection of unrelated facts, rules and skills as in the "instrumentalist" view of Ernest (1989) to one which emphasises problem solving and understanding as central in mathematical activity (e.g., Cockcroft Report, 1982).

The follow-up interviews were carried out in order to explore the teachers' reasons for their answers in Questionnaire 1. Each interview lasted more than 50 minutes.

Data Analysis

Data analysis was conducted in the language (Spanish) of the data collection. The analysis was based on information related to the purpose of the teaching of algebra, the content to be taught, teaching style and types of pupils' work (i.e., method).

The purpose of the teaching of algebra. All teachers made it clear that algebra is for all pupils. For five teachers, the most important reason for the study of algebra in school was: 'pupils need algebra knowledge for the next mathematics level'. For four teachers, it was: 'algebra is an important tool for problem solving'; and for four other teachers: 'algebra is important for the development of critical thinking'.

The content to be taught. The teachers' descriptions of the work they organise in their lessons in grade 8, the content emphasised and the sequence of topics followed showed a common teaching-learning sequence that can be summarised as follows: once the unit on Rational numbers has been covered, algebra work starts with the presentation of algebraic expressions and the definition of algebraic expression. After focusing on parts of an algebraic expression, types of algebraic expressions and ordering algebraic expressions, work moves on to collecting like terms and operating with expressions. After 'factorising' comes 'linear equations, systems of equations' and then the unit on word problems.

The teaching style and types of pupils' work. To present a general idea of the teachers' priorities in relation to teaching styles and pupils' work a summary of their mean rankings to pairs of corresponding questions is presented in Table 1.

Table 1
Mean rankings for descriptors of teaching style and pupils' work

Preferred practices	*Mean ranking**	***Portrayed practices***	*Mean ranking*
Questions A2 & A3: Teaching style			
Giving clear explanations of procedures to follow in assigned tasks	2.0	Giving clear explanations of procedures to follow in assigned tasks	1.9
Developing communication skills	2.0	Developing communication skills	2.0
Providing work that allows differentiation	2.2	Promoting connections between concepts	2.4
Promoting connections between concepts	2.4	Providing work that allows differentiation	2.7
Promoting discussion & systematic work	2.7	Promoting discussion & systematic work	3.4
Assigning repetitive exercises	4.0	Assigning repetitive exercises	3.6
Testing pupils at the end of each activity	5.0	Testing pupils at the end of each activity	3.8
Questions A4 & A5: Pupils working on			
Open problems	1.8	Algorithm-routine practice	1.8
Discussion & systematic work	1.9	Correction of exercises at the board	2.1
Correction of exercises at the board	2.2	Discussion & systematic work	2.2
Algorithm-routine practice	3.2	Open problems	3.1
Formulae construction	3.3	Closed problems	3.5
Closed problems	3.0	Formulae construction	4.0
Use of calculators	4.2	Use of calculators	4.5

* Respondents had opportunity to give equal rankings to given descriptors. Highest ranking value is 1.

The rankings shown in Table 1, in conjunction with information collected at the interview, provided clearer ideas about the teachers' conceptions of mathematics and mathematics teaching. In the following section a short account of the meanings the teachers assigned to certain descriptors is presented. This account will be followed by the teachers' explanations of their responses to specific questions.

Teachers' meanings assigned to given descriptors. Certain expressions in descriptors had specific meanings for different teachers. Nine teachers associated 'Open problems' with the set of word problems corresponding to the last unit of the program of study. All teachers took 'Providing work that allows differentiation' to mean "decreasing speed of work for the whole group". 'Promoting connections between concepts' meant "linking yesterday's work with today's work" for most, but four teachers referred to connections between some algebraic work and some geometric concepts. For nine teachers,

'Developing communication skills' meant "having pupils verbalise the procedures that they are applying"; the other four teachers referred to the capacity to "explain and justify what they did".

Teachers' explanations for their rankings of preferred and actual practices. Asking teachers at interview to explain what they meant by the descriptors and the reasons for their rankings provided information that allowed greater insight into their thinking about specific aspects of their practice. Distinct patterns of responses were identified by focusing on the teachers' first priorities for *preferred* and *actual* practices. Two examples of the patterns of their responses to questions about types of classroom pupils' work are shown in Table 2. Priorities in *preferred* and *actual* practices show two groups of teachers who provided reasons for the differences in rankings. When there was no difference between rankings, the teachers made specific comments and gave explanations about the results of their teaching, as they normally felt that their pupils' performance was not good.

Table 2
Summary of responses to questions A4 and A5 about types of classroom pupils' work

Group 1	Preferred practices	Actual practices	Reasons for differences
María	Open problems	Algorithm-routine practice	Pupils' motivation
Alfi	Open problems	Algorithm-routine practice	Pupils' preferences
Nora	Open problems	Algorithm-routine practice	Time constraints
Sandra	Open problems	Algorithm-routine practice	Pupils' motivation
Luis	Open problems	Algorithm-routine practice	My knowledge for teaching
			Teachers' specific explanations
José	Open problems	Open problems	My mathematics knowledge
Mario	Open problems	Open problems	Time constraints for teaching
Alvaro	Open problems	Open problems	Pupils' self concept
Group 2			
Gladys	Correction of exercises at the board	Correction of exercises at the board	Pupils' abilities
Juan	"	"	Pupils ways of learning
Nacho	"	"	Pupils ways of learning
Mario	"	"	Pupils ways of learning
Sandra	"	"	Pupils' motivation

In Group 1, from the five teachers whose actual practice privileged an algorithm-routine practice but preferred having pupils working on open problems, three teachers attributed this difference to the pupils' behaviour, and one teacher, to the nature of algebra work in grade 8. But Luis, who thought of open problems as "problem-situations where there is not an established procedure to follow", attributed the difference to his lack of "knowledge for teaching". The three teachers for whom there was no difference in preferred and actual practice rankings (i.e., José, Mario, Alvaro) pointed at difficulties they had when working on "open problems". Although for José open problems meant closed tasks (i.e., routine exercises) "where pupils are allowed to discuss their work", he also felt that low results were due to his limited knowledge of mathematics, which did not allow him to organise better work.

The five teachers in Group 2, for whom 'open problems' were the closed word problems of textbooks, justified their priorities (i.e., correction of exercises at the board, an algorithm-routine practice) in terms of factors related to the pupils.

While the conceptions of mathematics and mathematics teaching of most of the teachers were very similar, the conceptions of their own teaching show great variation. In

order to illustrate this variation more fully, data from one teacher from each of the two groups is presented now in the following sections.

Luis (Group 1)

Luis had taught secondary mathematics school for eight years and, in 2002, was teaching Grade-8 algebra for the fourth time. He was also teaching Calculus in a tertiary institution. According to Luis, "mathematics is a structure of concepts", and algebra is a set of concepts and algorithms that are to be learned in order to be able to solve mathematical problems. "It is important to learn algebra because it develops formal thinking and it represents key knowledge in the solution of problems that belong to higher level of mathematics".

Although at the school where he teaches it is not permitted to ask pupils to buy a textbook, for algebra teaching Luis followed the sequence established in Grade-8 textbooks:

> After I do the work on rational numbers, algebraic work commences with algebraic expressions and all that is related to that: coefficient, literal part, type of algebraic expressions, etc., and then we move on to addition and the rest of the operations with polynomials.

He characterised his pupils' work in the class as 'closed tasks' and 'routine practice', although he would prefer them to engage in 'open tasks' and 'discussion'. Consequently, his teaching was based on giving clear explanations about the procedures to follow in a list of exercises, and the most common way of assessing pupils' work was by frequent quizzes. But Luis seemed to be dissatisfied with his teaching. After declaring that he did not do any planning for his lessons and that he just taught in the traditional way, he explained:

> The majority of pupils don't like mathematics (...) . Because I have read about constructivism, I consider it important to organise activities for the classroom, so that pupils become engaged in the solution of problems and have the opportunity to interact and discuss what they are doing. I have tried to do that but I do it, maybe, once a month. That's not the common way of work. It is difficult.

Without being asked, Luis went on further to explain that *the teacher* was one of the main reasons for the problem with the teaching and learning of mathematics in his school:

> The reason why pupils don't like mathematics is the type of work the teachers propose. The teacher is the main problem! All teachers do it. They give the pupils a list of exercises because we have not planned an activity for the lesson (...) .

Luis seemed to be struggling with his awareness of the reasons why pupils don't like mathematics and his difficulty in changing his teaching. He identified *his knowledge* as a key factor influencing his teaching. When asked why the teacher was one of the main problems, he explained: "the problem is in the teachers' knowledge. Teachers do what they know (...) ; one can attend seminars, read, reflect on what one does but when it comes to putting it into practice, it is very difficult(…) . Teachers have not been trained for that".

Gladys (Group 2)

Gladys has taught mathematics for 28 years, and Grade-8 algebra for 24 years. For Gladys, algebra is a set of definitions about algebraic expressions and the procedures to operate with them. Her declarations that "pupils have to learn the formal definition of terms like 'algebraic expression', 'parts of an expression', 'types of algebraic expressions', etc., so that they will be better equipped to do operations with polynomials", further reveal her conception of the nature of algebraic knowledge. She explained that she did not have to do lesson planning because there was a general plan for the eleven Grade-8 groups they had in the school, and that they knew what they have to do, making more evident her conception of mathematics as a fixed body of rules and facts to be learnt in order to carry out pre-

specified exercises.

Gladys described typical patterns of her teaching as consisting of a sequence of "my explaining at the board definitions and procedures to follow and then pupils doing exercises" is in sharp contrast with her statements that the first reason why algebra is taught is the development of critical thinking in students, and that algebra is for all pupils. She noted that "learning algebra is important for pupils because it develops the logical thinking that prepares them for life" and that "if they develop logical thinking, they will be critical thinkers".

However, her conceptions of her pupils' ways of learning and their abilities contradict her "professed purpose of algebra teaching". She noted that pupils start their work in Grade 8 with high motivation because algebra is "something new for them", but pointed out that she had been trying to work with them in a way "to see if they can construct their own knowledge" but she was disappointed with the results she had obtained:

> On many occasions, I have concluded that pupils cannot construct their own knowledge, because, unfortunately, pupils (in this school) are lazy when it comes to thinking. Often they don't pay attention. There have been very few exceptions in which the pupils try to analyse, try to produce their own knowledge. Due to this, for my teaching approach, I have disregarded the idea that pupils can construct their own knowledge.

So "when a large number of pupils fail", she knows she has "to explain again and give more exercises. After they clarify and do more exercises the situation improves".

Gladys explained her decisions in terms of the determinants of her practice. The pressure from a group of colleagues was identified as an important factor for keeping her traditional practice in place. When describing the type of work she did when she reached the conclusion that pupils cannot construct their mathematical ideas, she pointed out that "doing things in a different way (from her usual teaching patterns) is problematic", mentioning the case of her colleague "who is always in trouble (with the group of colleagues) because he does different things in his teaching, nobody fails with him and yet when pupils move on to the next grade they don't know what they are supposed to know".

Similarities and differences in conceptions of the two teachers

Unlike Gladys, Luis' portrayal of teaching is consistent with his stated purpose for teaching. Despite this difference, what counts as mathematical knowledge was very similar for both teachers. They both felt responsible for delivering the set of definitions and rules to operate with algebraic expressions, then moving on to the set word problems. Although both teachers' portrayals have pupils involved in repeated practice, they have different conceptions of their teaching, of themselves and of their pupils' abilities and motivations for learning mathematics. Furthermore, they seemed to have different attitudes towards mathematics teaching. While Gladys has rejected the idea that she should search for other teaching approaches, Luis is open to the idea that he needs to keep learning.

Concluding Comments

The data presented in this paper provide a first sketch of these teachers' conceptions of mathematics, mathematics teaching, and their own teaching. The data show that the teachers conceptualised school mathematics as a set of topics to be studied in a strictly sequenced order, where algebra is the block that is studied in grades 8 and 9. Most of the teachers followed the same pre-specified teaching sequence and emphasised formal definitions and routine-procedural skills that had no meaning, as they conceptualised

knowing algebra as mastering a set of rules to manipulate given expressions without concern for understanding the basic concepts involved.

Of concern is the finding that despite the fact that the great majority of teachers reported dissatisfaction with their pupils' performance, they attributed the situation to external factors, mainly to the pupils themselves. Further analysis of cases, where conceptions of individual practices vary, may reveal important aspects to be considered in order to start looking at the relationship between teachers' conceptions of their practice and *improving* their practice.

Notes

1. Third International Study in Mathematics and Science (1991-1995).

2. Refer to [http://www.education.monash.edu.au/projects/vamp/].

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