Hops, Steps and Jumps: Mathematical Progress in the Early Years

Brian Doig
Deakin University
<badoig@deakin.edu.au>

Molly de Lemos
ACER
<de lemos@acer.edu.au>

While curriculum frameworks are major influences on learning, teachers know that children progress at different rates. Sometimes this is evident within a particular topic, and at other times more obvious across different topics. In this paper, we present the hops, steps, and jumps of numeracy learning of some 3000 Australian children. All were assessed using *I Can* Do *Maths*, and their achievements mapped to provide a detailed picture of how children hop, step and jump on their numeracy journey. This mapping provides teachers with information about key hurdles to numeracy learning for Australian children.

The Australian Council for Educational Research project on *Curriculum and Organisation in the Early Years of School Schooling* conducted from 1997–1999 (de Lemos, 1999) collected data on student outcomes in the early years of schooling and involved the assessment of over 3000 children on various measures of early literacy, numeracy and developmental level.

The *I can do maths* assessment instrument, which was developed for use in the project, contained a total of 47 questions designed to assess key learning objectives in the early years of schooling, as outlined in the various state and national documents on mathematics curriculum and objectives. These items covered the three main areas of number, measurement and shape. Because the questions were designed to cover a wide age and ability range, it was necessary to include questions that were relatively easy for children in their first year of school, as well as more difficult questions that were able to discriminate at the Year 2 level.

The *I can do maths* assessment has two levels: Level A suitable for children in their first and second years of schooling, and Level B suitable for children in their second and third years of schooling, and cover the three main areas of early mathematics: number, measurement, and space. They are ordered by increasing level of difficulty and all questions are read to children to avoid their mathematical performance being affected by reading factors.

This paper sets out details of children's responses to the *I can do maths* items, and provides analyses of these responses in non-standard ways that shed light on the learning of young children in mathematics.

The Sample and Data

All the data on *I can do maths* were collected from 84 schools, selected at random from all states and territories, with the exception of Tasmania which did not participate in the study. The sample covered a wide range of schools and included children drawn from different backgrounds throughout Australia. From each of the participating schools, one class at each of the relevant year levels (Pre-Year 1 to Year 2) was selected.

This provided a total sample of over 3000 children (800 children at the Pre-Year 1 level and over a thousand children at the Year 1 and Year 2). Data were collected at the end of the second school term (June 1998), or at the beginning of the third school term (July/August 1998).

Analysis

Responses to the *I can do maths* questions were scored as correct or incorrect. These data were analysed using a Rasch analysis that estimates the difficulty of obtaining higher scores; that is, it is not assumed that all questions are of equal difficulty or show the same "amount" of development. This shows in the scales as uneven spacing between adjacent scores. The four scales constructed are Number, Measurement, Space and Total (the total of the other three scales). These scales are the basis for the reports described in this paper.

Throughout Australia there are different approaches to school entry which have an impact on early years mathematics programs. In some state systems there is a pre-Year 1 year prior to entry to Year 1, while in other systems Year 1 is the first year of school. This results in differences both in years of schooling and in age level between children at the same year level in the different state systems. In order to accommodate these differences as well as provide measures of development over time, *I can do maths* reports are defined for the following distinct school structures:

1. Pre-Year 1 (ACT, NSW, VIC, SA, and NT)
2. Year 1 (QLD, WA)
3. Year 1 (ACT, NSW, VIC, SA, and NT)
4. Year 2 (QLD, WA)
5. Year 2 (ACT, NSW, VIC, SA, and NT

Norms were constructed for each of these groups based on the total sample of children at each of the five school levels.

Results

Hops — Diagnostic Maps. Children often "miss" learning some detail of their mathematics program, and simply "hop" over it. A Diagnostic Map (*Diamap*) (Doig 1992) provides detailed information about children's development and focuses on where such "hops" may have occurred. A *Diamap* is constructed so that knowing a child's total score can indicate their expected pattern of responses (that is, what success they are expected to have on each task) and any variations from this expected pattern provide starting points for more detailed investigations.

To read a *Diamap* a child's Total score is located on the scale at the left of the *Diamap*, and a line is drawn across the *Diamap* where the items are ordered according to difficulty on a Rasch scale (Adams & Khoo, 1996). The tasks that the child has answered correctly are then circled or highlighted. Some of these tasks may be above the Total score line and some below it. Once the Total score line is drawn and each task marked, there are four conditions of diagnostic interest represented:

1. tasks expected to be correct and are correct;
2. tasks expected to be correct and are *not* correct;
3. tasks *not* expected to be correct and are correct; and
4. tasks *not* expected to be correct and are *no*t correct.

Specific strengths are shown by correctly answered questions above the Total Score line. When these questions are all within a particular curriculum area, such as Measurement, this may indicate a general strength in that area. On the other hand, isolated correct responses or clusters of correct responses within an area may reveal particular strengths within an area. For example, a particular strength in simple addition within number, but not strength in number more generally. Specific weaknesses are shown by questions answered incorrectly lying below a child's Total Score line. Again, when these questions are all within a particular curriculum area such as Space, then this may indicate a general weakness in that area. Individual questions on the other hand may reveal particular weaknesses within an area. For example, a particular weakness in money within Measurement, but not in Measurement more generally.

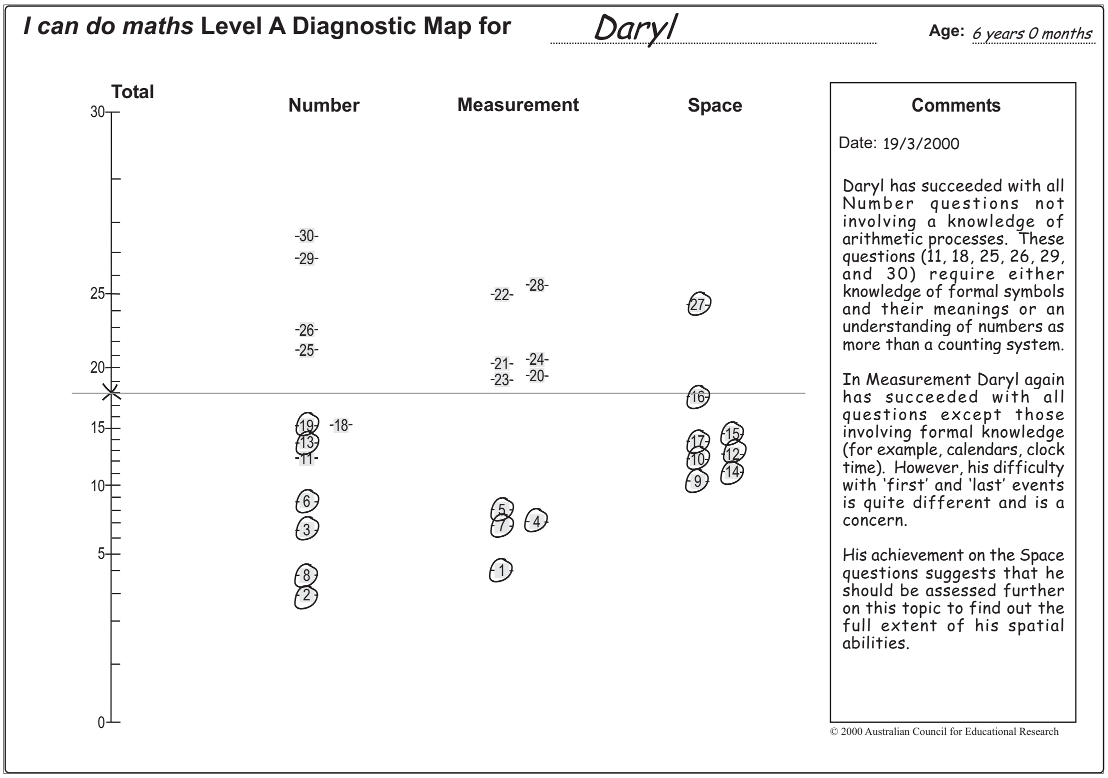


Figure 1. Daryl's diamap.

When interpreting a *Diamap* it must be remembered that it is a consistent pattern of strength or weakness that should be taken as an indicator for further investigation and assessment. Random "hops" in children's patterns of response are expected and should be interpreted with caution.

Figure 1 is an example of a completed *Diamap* for a six-year-old assessed mid-year in his Pre-Year 1 class. The *Diamap* "score line" shows a reasonably clear distinction between questions that he could and could not answer successfully, a pattern expected in most *Diamaps*. (Details of all questions are in a later section).

For the Number questions below the score line (expected to be easy for Daryl) he has all correct except questions 11 and 18. Question 11 asked him to find the sum of 5 and 4 and question 18 asked that he identify the number 65.

The Number questions above the score line (expected to be too difficult for Daryl) he has had no success with. All these questions require a knowledge or awareness of numbers as operators rather than simply a counting mechanism.

Daryl's Measurement successes all lie below his score line as expected. Those questions above the score line (expected to be too hard for him) were too difficult. Four of these questions require knowledge of mathematics conventions. Questions 24 and 28 focus on time, while question 21 deals with money (giving change) and question 23 deals with reading a simple graph. Daryl's achievement on spatial questions is better than is expected of a child with his overall score, and more assessment to gauge the full extent of his spatial abilities would be sensible.

The overall picture is that of a child well skilled in counting and measurement, but not as yet familiar with the conventions of mathematics that enable success with more formal work. For a child in the mid-year of a Pre-Year 1 class, the most likely reason is that of curriculum timing. Daryl's *Diamap* alerts us to a possible problem with "first" and "last", but, more importantly, shows that a total score alone does not provide a complete picture of his abilities. Daryl's Number score of 6 out of 12 and Measurement score of 4 out of 10 fail to identify his underlying strengths in these areas and fail also to provide suggestions for future learning programs.

Steps — Individual Profiles. We expect that as a child progresses through school, they will "step" up in their performance. An Individual Profile, shown in Figure 2, shows a picture of these "steps" and enables a comparison of any child's score to an Australia-wide sample of children at the same "step". These normative comparisons provide a guide to the performance expected of the middle 80 per cent of children at particular levels of schooling but allow for variation in performance between children stemming from individual differences.

In the example, Daryl is achieving as expected. He is in Pre-Year 1, but his overall score (18) puts his achievement slightly above average for children in Pre-Year 1. His achievement in space (8) is comparable with high achieving Year 2 children, while his number achievement (6), is average for his year level. In measurement Daryl's score (4) is at the low end of the expected range for Pre-Year 1 children. The comment that he be assessed further to determine the extent of his spatial abilities is clearly appropriate for Daryl.

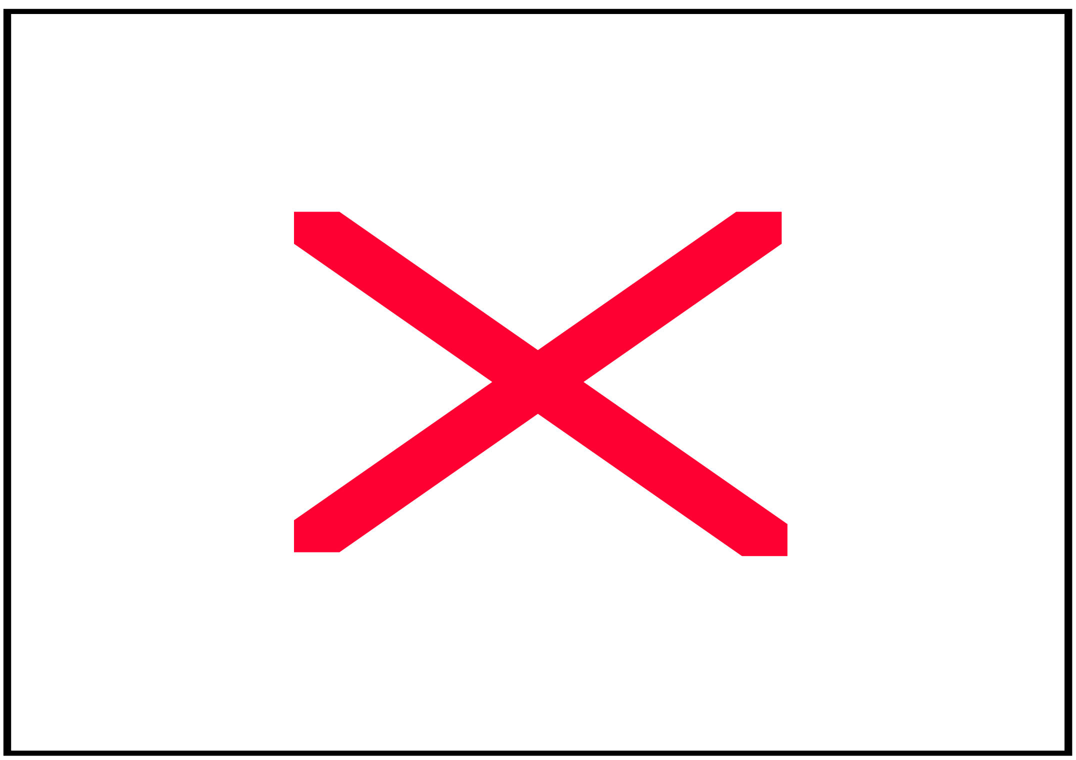


Figure 2. An Individual profile.

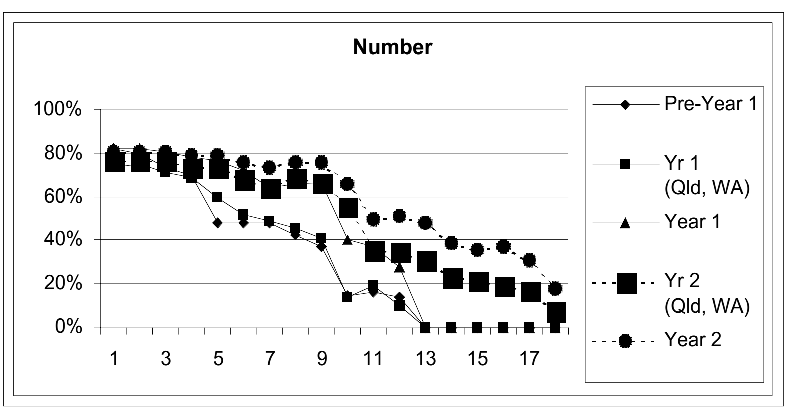
Jumps — Learning Trajectories. A picture of children's overall development such as that provided by norm-referenced Individual Profiles is important, but for planning purposes it is also important to know those specific aspects of children's mathematics abilities that are at odds with curriculum structures. That is to say, understanding "jumps" in children's learning trajectories. These may be either "jumps" ahead, or "falls" behind, what curriculum designers have considered appropriate.

One way of finding such jumps in learning trajectories is to arrange children's performance by the test items' difficulty order. This approach has two immediate consequences: learning trajectories can show increased performance for higher year levels, and all year levels should have lowering performance as question difficulty increases.

Number. The Number learning trajectories (Figure 3) are striking by the clear depiction they give of performance decreasing as questions become more difficult. Moreover, jumps between year level trajectories are also clearly seen: for example the Pre-Year 1 and Year 1 trajectories, similar to one another, jump "away" from the higher year levels by the fifth question.

Below is a description of each of the Number items.

1. Put a tick on the picture with more things in it. (5 or 6)
2. Count how many butterflies there are in this picture. (11 butterflies)
3. Put a tick under the 10 cent coin.
4. Put a tick under the runner who is second in the race.
5. Tom had 5 gumnuts and found 4 more. How many now?
6. Jan had 6 lollies and ate 2 of them. How many now?
7. Put a tick beside the number sentence that matches the picture.(7+3 =10)
8. Put a tick on the number sixty-five. (65)
9. Put a tick beside the number sentence that matches the story. (8–3 = 5)
10. Write the next number in the counting pattern (13, 15, 17, _)
11. Put a tick beside the number sentence that matches the picture. (4×5 = 20)
12. Put a tick beside the number nine hundred and fifty. (950)
13. Con had 24 marbles in his bag. Write how many marbles are left in the bag.
14. Write the answer to this problem. 14 + 31 =
15. Jill has 36 pencils in her pencil-case. She puts 17 pencils on the table. Write how many pencils are in the pencil-case now.
16. Write the answer to this problem. 25 + 18 =
17. Write the answer to this problem. 37 – 14 =
18. Write a fraction that is bigger than one half.



• *Figure 3.* The Number trajectories.

Although simple addition is not usually taught in the first year of school, the small addends (5 and 4) involved have enabled between forty and sixty percent of these children to answer question 5. It is surprising then, that the later year level children

Counting patterns pose a problem for children at all year levels, although particularly in the first year of schooling, as can be seen from the jump "down" in all learning trajectories.

The later questions deal with operations on whole numbers, and it is interesting to note that the Year 2 (QLD, WA) learning trajectory is only about a ten percent jump "down" from that of Year 2 in other states, despite a years difference in school attendance.

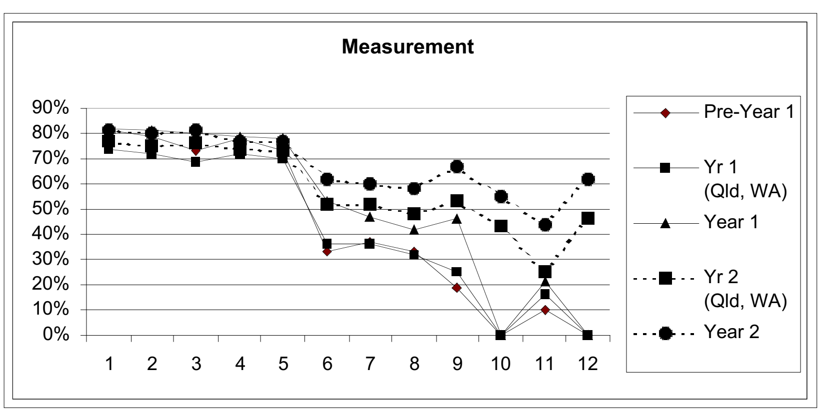
It is also noteworthy that this group (Year 2 in QLD and WA) perform in a similar manner to the Year 1 group in other states except for question 10 (counting patterns) and question 12 (identify 950) where they do better. (The Year 1 group appear to score 0% from 13 on-ward, but this is because they did not answer these questions.)

Measurement. The Measurement learning trajectories (Figure 4) show more uneven-ness than those for Number, as shown in the diagram below. The descriptions of the Measurement items are below the diagram. The learning trajectories for measurement are disturbed by two questions. Questions 10 and 12 were attempted only by the Year 2 level children, and so are marked at zero for all others.

Looking beyond this there is the expected variation between year level trajectories, with the jumps between Year 1, Year 2 (QLD, WA) and Year 2 increasing as the questions become more difficult. The first year of school (Pre-Year 1 and Year 1 (QLD, WA)) trajectories follow similar paths that are quite different from later year trajectories. This is clear from question 6 (complete a graph) onwards.

Below is a description of each of the Measurement items.

1. Put a tick on the smallest star.
2. Put a tick on the rug that covers the largest area.
3. Put a tick under the coin you need to buy the pencil. (50 cent coin)
4. Put a tick on the longest snake.
5. Put a tick on the shortest snake.
6. A group of children ate some fruit. Later two more apples were eaten. Colour squares on the graph to show the fruit they had eaten now.
7. Put a tick on the picture that shows the first thing to happen.
8. Put a cross on the picture that shows the last thing to happen.
9. Put a tick beside the clock that shows twelve-thirty. (12:30 on a digital clock)
10. Which line is 10 centimetres long? Use your ruler to measure the lines.
11. Rosa has $1. She buys some fruit for 85 cents. Put a tick on how much change she gets. (15 cents)
12. Circle the third Wednesday of May on this calendar. (17th)



• *Figure 4.* The measurement trajectories.

Interestingly there is the jump “down” for all Year 2 children at question 11 (giving change from a dollar). It is curious that many younger children can succeed on this task while fewer children from higher year levels than expected do so.

A further point to note is the jump “up” from the usual downward trend as questions become harder, by both Year 2 groups. This is very evident for question 12 (finding a date on the calendar). It is possible that the lower year levels may have been able to do well on this question too.

Space. As in other areas of mathematics, the spatial performance of children increases as they proceed through school (Figure 5). However, on *I can do maths* spatial questions there is less differentiation between year levels than with other topics. Unusually there appears to be little difference in children’s learning trajectories; that is, the harder questions are difficult for all, not just the lower year levels.

The most difficult question involved distinguishing left from right (question 9), a task that many adults also find difficult.

Below is a description of each of the Space items.

1. Put a tick under the triangle shape.
2. Put a tick on the cone.
3. Put a tick on the shape with all straight sides.
4. Put a cross on the rectangle shape.
5. Put a tick on the shape with all curved sides.
6. Put a tick on the shape that makes the side of the cube.
7. Put a tick on the cylinder.
8. Colour in the sphere.
9. Put a tick on the shape at the left of the square.

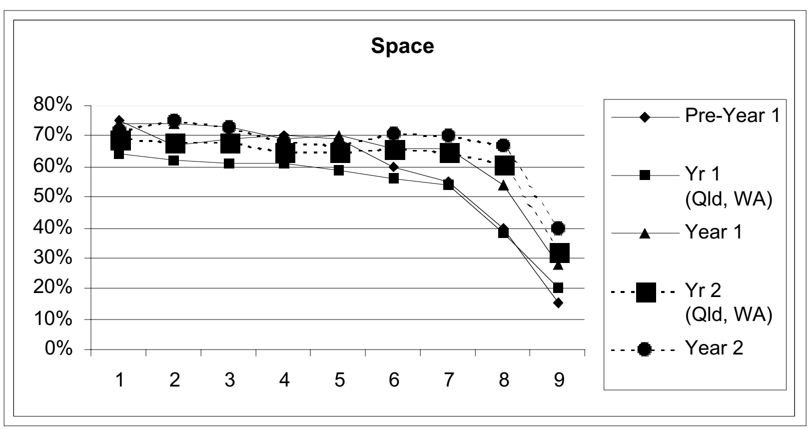


Figure 5. The space trajectories.

The similarity in learning trajectories at all year levels raises the question of appropriateness of curriculum content. Questions 7 and 8 are curriculum-related as children needed to know the objects that matched the words “cylinder” and “sphere” to answer them. Apart from these two questions, children’s performances between Pre-Year 1 and Year 2 differed little, raising the issue of whether curriculum under-estimates younger children’s spatial abilities. Perhaps Pre-Year 1 and Year 1 children can “jump” further than we realise?

Conclusion

The hops, steps and jumps of both individuals and groups as they grow as mathematicians can either be obstacles or gateways. What we have tried to demonstrate in this paper is that good assessment combined with good analysis may throw light on more than children's achievement of curriculum content. Whether the analysis is at the individual or group level, detailed analysis of children's responses to well-constructed assessment tasks is a key factor in providing the learning environment necessary for mathematical development.

In busy classrooms, however, time may be a constraining factor that prevents the collection of detailed information on children and the necessary analysis of this information. It is in this situation that appropriate assessment instruments, with well-designed reporting formats, can be of great assistance in providing a guide for planning for both individuals and groups of children.

Teachers who know and understand the hops, steps and jumps of children's learning are better able to help children overcome obstacles and pass more easily over them.

References

Adams, R. J., & Khoo, S.-T. (1996). *Quest: The interactive test analysis system, Version 2*. Camberwell: The Australian Council for Educational Research.

de Lemos, M., & Doig, B. (1999). *Who am I?*. Melbourne: The Australian Council for Educational Research.

de Lemos, M. (1999). *Curriculum and Organisation in the Early Years of Schooling* Melbourne. The Australian Council for Educational Research.

Doig, B. A. (1992). DIAMAPS-Self-scoring kidmaps. *Assessment Matters* **5**(1).

Doig, B., & de Lemos, M. (2000). *I can do maths*. Melbourne: The Australian Council for Educational Research.

Vincent, D., Crumpler, M., & East London Assessment Group. (1996–97*). Reading Progress Tests, Stage 1*. London: Hodder & Stoughton.