# The Victorian Curriculum and Assessment Authority (VCAA) Mathematical Methods (CAS) Pilot Study Examinations, 2002

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Victorian Curriculum and Assessment Authority Mathematical Methods (CAS) Units 1–4 were accredited in February 2001 for a pilot study from 2001 to 2005. As a consequence of this decision, the VCAA has produced sample examinations and supplementary materials and set examinations for Units 3 and 4, with the first cohort of 78 students taking these examinations in November 2002. This paper provides some initial commentary on student performance, including common items from the Mathematical Methods Units 3 and 4 examinations.

The use of technology in the senior mathematics curriculum, as well as in end of secondary schooling mathematics examinations in Victoria, has evolved over the last several decades as different technologies have become more widely available and integrated into mainstream teaching and learning practice. For end of senior secondary mathematics examinations in Victoria, scientific calculators were permitted from 1978, graphics calculators were permitted from 1997 (with assumed student access from 1998), and access to approved CAS assumed for a pilot study from 2001.

Mathematical Methods (CAS) Units 1-4 is an accredited pilot study of the Victorian Curriculum and Assessment Authority for the period from January 2001 to December 2005. Details of the pilot, including the study design for Units 1–4 (Years 11 and 12), sample 2002 examinations, 2002 assessment reports, and other teacher resources, can be accessed from the CAS pilot VCE Mathematics Study section of the VCAA website (see VCAA, 2003).

The first phase of the pilot study 2001–2002, involved students from three Stage 1 volunteer schools, and was implemented in conjunction with the *CAS-CAT project 2000–2002*, a research partnership between the VCAA, the Department of Science and Mathematics Education at the University of Melbourne, and three calculator companies (CASIO, Hewlett-Packard, and Texas Instruments). The CAS-CAT project has been funded by a major Commonwealth Australian Research Council (ARC) Strategic Partnership with Industry Research and Training (SPIRT) grant (see DSME, 2002).

In November 2002, 78 students from the three Stage 1 volunteer schools sat end-of-year Mathematical Methods (CAS) Unit 3 and 4 examinations (Semesters 1 and 2 of Year 12), for which student access to an approved CAS calculator (TI-89, CASIO ALGEBRA FX 2.0, or HP 40G) was assumed. This paper reports on results from these examinations, provides preliminary commentary from the panel setting chairs and chief assessors based on VCAA examinations data. Given the small cohort size, from three pilot volunteer schools, these comments must necessarily be tentative in nature, however, they do indicate areas for further research. While examinations are an important source of data, this data needs to be considered in conjunction with data from various different forms of assessment; qualitative research data illustrating how CAS is used by teachers and students of different levels of CAS expertise and experience in the teaching and learning process; research data from the literature; and available data from other systems using CAS.

The second and third stages of the expanded pilot study 2001–2005, incorporate the original three schools that are implementing Mathematical Methods (CAS) Units 1 and 2 from 2001 and Units 3 and 4 from 2002, and include two additional groups: nine Stage 2 volunteer schools implementing Units 1 and 2 from 2002 and Units 3 and 4 from 2003, and a further seven Stage 3 volunteer schools implementing Units 1 and 2 from 2002 and Units 3 and 4 from 2004. The schools in the expanded pilot include co-educational and single sex, metropolitan and regional schools from government, Catholic and independent sectors, using a range of different CAS. Thus, there will be just over 250 students enrolled in Units 3 and 4 from 11 schools of the expanded pilot in 2003. This will include students using the CAS TI *Voyage* 200, *Derive* and *Mathematica* in one school for each of these CAS. Progress of the pilot study has been reported in Leigh-Lancaster (2002a) and Leigh-Lancaster (2003).

From curriculum and pedagogical perspectives, a range of potential benefits from the use of CAS are typically advanced, including the following: the possibility for improved teaching of traditional mathematical topics; opportunities for new selection and organisation of mathematical topics; access to important mathematical ideas that have previously been too difficult to teach effectively; as a vehicle for mathematical discovery; extending the range of examples that can be studied; as a programming environment ideally suited to mathematics; emphasising the inter-relationships between different mathematical representations (the technology allows students to explore mathematics using different representations simultaneously); as an aid to preparation and checking of instructional examples; promoting a hierarchical approach to the development of concepts and algorithms; long and complex calculations can be carried out by the technology, enabling students to concentrate on the conceptual aspects of mathematics; the technology provides immediate feedback so that students can independently monitor and verify their ideas; the need to express mathematical ideas in a form understood by the technology helps students to clarify their mathematical thinking; situations and problems can be modelled in more complex and realistic ways.

For systems, these potential benefits need to be considered along with the various concerns about potential negative effects that are also expressed by academics, teachers, parents and students, including those who are nonetheless positive about the overall benefits of CAS: the extent to which the use of CAS may reduce students knowledge and skills with important and valued conventional by hand or mental techniques; how students, including those who may be less mathematically inclined, will cope with a more conceptually demanding curriculum; a potentially diminished role for teachers in terms of traditional (and valued) pedagogy; and, whether appropriate cognisance has been given to the role of by hand approaches in the development of important mathematical concepts, skills and processes. Each of these benefits and concerns can form the basis of propositions for research with respect to the pilot study context in particular, and for CAS use in general.

A principled and coherent response to the natural questions of *What mathematics*? (selection from discipline and domain knowledge, theory and application); *For whom*? (subsets of the cohort); *How*? (curriculum and assessment study requirements and related advice on possible pedagogies); and *Why*? (rationale and purpose), is central to the responsibilities and work of curriculum and assessment authorities. These responsibilities and this work are informed by various considerations including the views, knowledge and research of various stakeholders and interested parties, such as mathematics educators, mathematicians, parents and the universities (see, for example, Drijvers, 2000; Herget, Heugl, Kutzler, & Lehmann, 2000; Monaghan, 2001; Gardner, 2002; Leigh-Lancaster 2002b; Asp, Ball, Flynn, and Stacey, 2002; and Garner, 2003) as well as responding to government policy directions on ICT, access, equity, and flexibility of curriculum and assessment. The experience of other systems, boards, and authorities is also drawn on, in the context of international benchmarking, for the development and review of senior secondary mathematics studies.

Consultation with universities and the Victorian Tertiary Admissions Centre (VTAC) took place throughout the development and accreditation of the pilot Mathematical Methods (CAS) study, and in March 2001, VTAC informed the VCAA that the pilot study design had been approved by all universities for prerequisite purposes from 2003. For the first two years of pilot examinations, VTAC agreed to scale the pilot study in the same manner as for Mathematical Methods.

## Mathematical Methods (CAS) Examination 1, 2002

Student exam responses showed that the paper was accessible and provided opportunities for them to demonstrate what they knew and could do, with achieved scores ranging from 12 to 49 out of a maximum of 50 available marks. There were excellent papers presented by several students, with around 13% of students scoring 90% or more of the available marks. The mean mark for the paper was 31.5, comprised of a mean of 18.8 marks (or approx 70%) on the multiple choice section (27 available marks) and a mean of 12.7 marks (or approximately 55%) on the short answer part (23 available marks). Of the pilot cohort in 2002, 72% of the students scored over half of the available marks for the paper. In general, students made good use of mathematical symbols, notation, and conventions with only limited use of CAS distinctive syntax. Overall, the symbolic facility of the CAS was used well, particularly for the multiple-choice questions. This was shown in particular for the following multiple-choice questions.

Question 14: If $y = \log_e(\cos(2x))$, then $\frac{dy}{dx}$ is equal to …

The correct answer of $-2\tan(2x)$ was obtained by 73 of the 78 students. Although it is not clear as to exactly how many students used CAS, it is likely that the good practice of the calculators of putting brackets immediately after a chosen function name and checking that all brackets are entered correctly contributed to this result. Similarly, for Question 20:

Question 20: If $f'(x) = 2\cos(5x)$ and $c$ is a real constant, then $f(x)$ is equal to …

The correct answer of $\frac{2}{5}\sin(5x)$ $+ c$ was obtained by 70 of the 78 students. The most popular distractor was $-10 \sin(5x) + c$. Access to CAS in these instances improved the reliability and accuracy of symbolic calculation.

It was anticipated that Question 9 might have been similarly well done, but this was not the case:

Question 9: The linear factors of $x^4 + x^3 - 3x^2 - 3x$ over $R$ are …

A. $x, x + 1, x^2 - 3$ B. $x, x + 1, x + \sqrt{3}, x - \sqrt{3}$ C. $x, x + 1$

D. $x + 1, x + \sqrt{3}, x - \sqrt{3}$ E. $x + 1, x^3 - 3x$

This was answered correctly by 54 of the 78 students. The most popular distractor was A. Students either failed to pick up the *linear* in the question stem, or if using a symbolic facility, did not specify the correct field over which to factorise. Access to CAS functionality does not *automatically* confer increased reliability, without sound conceptual understanding.

Question 19 was a new general type of question where a more subtle approach to understanding relationships is tested through one of the functions involved not being specified explicitly by rule, and was reasonably well done.

Question 19: Let $g(x) = e^{f(x)}$. If $g'(x) = -2xe^{-x^2}$ , then a rule for $f$ is …

The correct answer of $-x^2$ was obtained by 67 of the 78 students. Of the distractors, the most popular was $e^{-x^2}$ , indicating a lack of understanding of function notation.

In general, the Mathematical Methods (CAS) cohort performed comparably, or better, than the Mathematical Methods cohort on common multiple-choice questions. Table 1 summarises the difference in proportion of correct responses to the 20 (out of a total 27) common questions. In Table 1, a positive difference indicates that a higher proportion of CAS pilot students selected the correct response. The questions have been classified as technology independent (I); technology of assistance but neutral with respect to graphics calculators or CAS (N); or use of CAS likely to be advantageous (C). Items likely to be answered efficiently by conceptual understanding, pattern recognition or mental and/or hand approaches have been indicated by a tick (✓).

Table 1

*Summary of Differences Between Proportions of Correct Responses to Common Examination 1 Multiple Choice Items: Number of Items (Question Number/S)*

| | Negative difference* | | | Positive difference* | | |
|---|---|---|---|---|---|---|
| Item type | ≥ 20% | 10 to 19 % | up to 9% | up to 9 % | 10 to 19 % | ≥ 20% |
| I | | 1 (22) | 1 (26) | 3 (7, 17, 18) | 3 (6, 11, 13) | |
| N | | | 2 ✓ (4, 24) | 4 ✓ (1, 3, 5, 25) | 1 ✓ (27) | |

| | | |
|---|---|---|
| C | 1 ✓ | 1 ✓ |
| | (16) | (14) |
| | 2 | 1 |
| | (9, 15) | (20) |

* there were no items with zero difference proportions

The short answer part of the exam consisted of 6 questions, of which Question 5 was common to both CAS and non-CAS papers, and Question 6 on the CAS paper was similar to a question on the non-CAS paper. The type of question, maximum available, mean Mathematical Methods (CAS) cohort and mean Mathematical Methods cohort scores were respectively Question 5a (I, 1, 0.31, 0.24)—specifying a sequence of transformations to produce a given function rule; Question 5b (I, 2, 1.33, 1.11)—stating the domain and range of the transformed function; and question 6b (N, 1, 0.56, 0.49)—finding a numerical value for a derivative. An important consideration for students is how they decide when to use by hand skills, CAS, or some combination of both. Question 1, a CAS only question, involved a probability density function that could have been tackled using either approach, or a combination of both.

Question 1:The life of a light globe, in hours, can be modelled by the random variable X with probability density function

$$f(x) = \begin{cases} \frac{c}{x^2} & if \quad x > 100 \\ 0 & if \quad x \le 100 \end{cases}$$

Find the value of *c*. Find the median life of a light globe according to this model.

There were only a small number of attempts at either part of this question by hand. Using CAS, it is fairly simple if formulated correctly. For this question in particular, a number of students gave a correct formulation, but then made no attempt at calculating an answer. The available and mean scores were (2, 1.07) and (2, 0.99) respectively.

Question 4, another CAS only question, required students to find the rule of a cubic polynomial function with undetermined coefficients using a combination of conditions involving the values of the function and its derivative. The three parts of the question involved formulation as a set of simultaneous linear equations (2, 1.56), their representation in matrix form (2, 1.39), and solution (by any method) to find the rule explicitly (2, 1.13). No student attempted to do this question by hand, apart from formulation stage and determining the value of one of the coefficients, $c = 0$. Some students did not select the appropriate calculator functionality with respect to exact/approximate mode, and gave answers that were approximate instead of exact as required, and often gave the coefficients without stating the rule.

Question 6 asked students to solve an equation involving circular functions over a specified domain. This is an example of a question that can easily be done using CAS, but the student must understand how to operate the CAS efficiently. Some CAS will give one solution only, while other CAS provide a general parametric form of the solution. The student then has to use this information to identify the required solutions. While access to CAS supports a renewed emphasis on exact values, in the case of equations with circular functions, the specification of families of multiple solutions, including parametric representations, still needs to be soundly understood, independently of CAS. In broad terms, the same sorts of conceptual understanding and mental skills apply whether a graphics calculator or a CAS is used to identify an initial solution.

While there is evidence that access to CAS improves engagement, perseverance, reliability, and accuracy for student work on a range of problems, there is certainly the opportunity for further developments in effective CAS use; and for related research to take place in partnership with practitioners. In particular, students need to be able to make efficient choices about when to use mental, by hand, CAS, or a combination of these approaches; work in exact or approximate mode as required or appropriate and specify numerical answers to a required accuracy; and pay particular care to the form of, and constraints on, the solutions of equations—especially those involving circular functions.

## Mathematical Methods (CAS) Examination 2 - 2002

The number of students presenting for Mathematical Methods (CAS) Examination 2 in 2002 was 78. Student responses showed that the paper was accessible and provided opportunities for them to demonstrate what they knew and could do, with achieved scores ranging from 7 to 55 out of a maximum of 55 available marks. There were excellent papers presented by several students, one student achieved a perfect score and 13% of students achieved a score of over 80%. The median and mean mark for the paper was 32, 58% of the students scored over half of the marks for the paper and 80% of the students scored over 40%. There was only one student who scored under 20%. On the whole, the symbolic facility of CAS was used well. This was shown in particular in the following questions:

> Question 1c.i: Solve the equation $k(1.1 - 0.5 \log_e(x)) = T$ for $x$, where $k$ and $T$ are positive real numbers.

This was a CAS only question, and the correct answer of: $x = e^{11/5 - 2T/k}$, was obtained by 50 of the 78 students and the answer in this or algebraically equivalent forms is indicative of CAS use. Similarly for the following question (common to both papers):

> Question 4e.i : Find an expression for $\frac{dy}{dt}$, where $y = 15 + e^{0.04t} \sin(\frac{\pi t}{3})$ for $0 < t \leq 60$.

Sixty-five of the 78 students obtained the correct answer of: $\frac{\pi}{3} e^{0.04t} \cos(\frac{\pi t}{3}) + 0.04\, e^{0.04t} \sin(\frac{\pi t}{3})$. This indicates a sound capacity for students using CAS to enter correctly complicated functional expressions that require the use of multiple parentheses for unambiguous expressions, and interpret corresponding results accordingly. This was a common question, and the corresponding available marks and mean scores were (2, 1.75 - CAS cohort, 1.07 -non-CAS cohort).

> Question 3a.
> i: The polynomial $2x^4 - x^3 - 5x^2 + 3x$ can be factorised as $x(2x - 3)(ax^2 + bx + c)$. Find the values of $a$, $b$ and $c$.
> ii Find the exact value solutions of the equation $2x^4 - x^3 - 5x^2 + 3x = 0$.

In this question 63 of the 78 students obtained the correct answers of $a = 1$, $b = 1$, $c = -1$ and $0, \frac{3}{2}, \frac{-\sqrt{5} - 1}{2}, \frac{\sqrt{5} - 1}{2}$, respectively. For part i., the correct answer can be obtained by factorising over the rational field $Q$, which appears to have led to a higher automatic response rate than Question 9 on Examination 1.

In some cases, students did not have their calculator set in exact mode and obtained answers with numerical approximations. For example, in Question 1a.iii, the student was required to find the rule for the inverse of the function with rule $f(x) = 1.1 - 0.5 \log_e(x)$. There are several acceptable forms for the exact answer, including $f^{-1}(x) = e^{2.2 - 2x}$. Some students gave the answer $f^{-1}(x) = 9.02501e^{-2x}$ which was not expected by the assessors. Students would have used the *solve* facility of their calculators to determine the rule for $f^{-1}(x)$, thus, they need to be familiar with the relevant aspects of the operation of CAS when entering coefficients/constants in decimal form.

Some of the calculators gave unusual forms for expressions. For example in Question 3, $e^{t/25}$ was sometimes written with superscript $\sqrt[25]{t}$. Students and teachers need to be aware of these properties of their CAS and be familiar with relating particular CAS representations to more common written forms.

Graph sketching was in general not done well, with only 18 students achieving full marks on Question 1, and this was common to both CAS and non-CAS cohorts, who would have used similar graphing functionalities on both CAS and graphics calculator technologies:

Question 1a.i: Sketch the graph of $f:(0, 5] \to R, f(x) = 1.1 - 0.5 \log_e(x)$

Neither cohort did especially well on this question (3, 1.61, 1.53) and this could be attributed to poor use of the graphing facility. It was clear that the graphing window was not set well. Calculators (CAS or graphics) do not deal well with asymptotes and students were expected to be able to sketch graphs showing their asymptotic behaviour as applicable, and this typically needs to be demonstrated independently of the technology (although thoughtful use of either technology can be used to illustrate asymptotic behaviour of functions graphically). Similarly, student use of a *trace* facility, or similar, to find intersections or intercepts is neither accurate nor quick and this was exemplified in a number of student responses to Question 4, which required the numerical solution of equations**.** The area in which there was minimal difference between the two cohorts was the use of graphing functionalities. Thus there are clearly aspects of graphical analysis which continue to require further work to make the most efficient use of a suitable combination of mental, by hand and technology-based approaches.

There were also indications that, in general, candidates for the CAS paper were more successful in questions that required extended analysis. That is, the capacity for students to continue to engage in questions requiring extended analysis, and on which they might otherwise falter without access to CAS, seems to have been the case here, as reported in other contexts in the literature (see, for example, Dunham, 2000).

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