On Student Observation and Student Assessment

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Based on cases of classroom events this paper discusses two perspectives on the issue of teacher informal classroom observation as a means of assessing their students' understandings. One perspective emphasizes difficulties and obstacles that might be overcome. The other centres on *hearing through* as an intrinsic characteristic of what it means to understand what someone else is saying or doing. The paper concludes with the suggestion to merge the two perspectives in practice.

The mathematics education community is in the midst of a tremendous attempt to reform student assessment. Traditional assessment focuses on certifying students' attainment at the end of a period of instruction, and on classifying or ranking students. In recent years educators advocate the moving from a sole concentration on summative assessment toward emphasis on formative assessment, whose main purposes are to advance students' learning, and to inform teachers as they make instructional decisions (Black & Wiliam, 1998, 2000; Clarke, 1997; National Council of Teachers of Mathematics, 1995, 2000; Shepard, 2001). An important characteristic of reform assessment is that teachers are expected to assess student understanding not only by a separate activity specifically designed for this purpose. Rather, much of the information on students' performances and achievement needs to be derived by teachers during the process of instruction. The view that assessment should be an integral part of instruction, combined with disappointment from the limited information received by traditional paper-and-pencil mathematics tests, entails the use of new assessment methods, tools and techniques.

One such tool is informal observation during instruction. It is assumed that observing and listening to students would enable teachers to respond better to students' learning needs, and facilitate teachers' ability to make informed instructional decisions (National Council of Teachers of Mathematics, 2000; Shepard, 2001). Several research studies support this assumption (e.g., Cobb & McClain, 1999; Even & Markovits, 1993; Fennema, et al., 1996; Goldenberg, 2000; Simon & Schifter, 1991). Still, research suggests that such a task is extremely difficult for teachers to perform (e.g., Ball, 1997; Coles, 2001; Crespo, 2000; Davis, 1997; Morgan & Watson, 2002; Nicol, 1999; Watson, 2000). These studies have begun to reveal the complexity associated with listening to students and with teachers' attempts to understand what students are saying, showing, feeling and doing—what Ball (1997) calls *to hear students*.

This paper describes two different theoretical perspectives for approaching the problematic associated with *hearing students*. One focuses on difficulties or obstacles that might be overcome; the other centres on *hearing through* as an intrinsic characteristic of what it means to understand what someone else is saying or doing. The paper concludes with pragmatic suggestion for what listening to students and observing them might mean in practice.

Obstacles to Hearing Students

Making Unplanned Changes

The following episode was observed in a seventh grade algebra lesson (Robinson, 1993). Benny, the class teacher, was a novice teacher in his first year of teaching. Benny wrote on the board two expressions: 4a+3 and $\frac{3a+6+5a}{2}$. Then he asked the students to substitute a fraction in both expressions:

T: Substitute $a = \frac{1}{2}$.

S_1: You get the same result.

T: Are the algebraic expressions equivalent?

S_2: No, because we substituted only one number.

S_1: Yes.

S_3: It is impossible to know. We need all the numbers.

S_4: One example is not enough.

And the teacher concludes:

T: We can conclude – it is difficult to substitute numbers in a complicated expression and therefore we should find a simpler equivalent expression.

Clearly, Benny ignored the students' discussion completely. Why did he do so? Examination of Benny's typical way of teaching (Robinson, 1993) suggests that Benny was not able to deviate from his planning of the lesson. He was not at all tuned into hearing students beyond the level whether they were *with him* or not. His hearing seems to be constrained by the fact that he was listening *for* something rather than *to* the students' discussion. Davis (1997) defines this mode of listening *evaluative listening*. When planning his teaching Benny used to study the textbook very carefully and to follow it closely. Analysis of the textbook's approach indicates that the two expressions that Benny wrote on the board (4a+3 and $\frac{3a+6+5a}{2}$) were included in a task that was intended to provide meaning for the all-too-common meaningless requests in algebra lessons to simplify algebraic expressions. The students were to substitute the same fraction in two equivalent expressions, one *complex* and the other *simple*. Since calculations in the former case were more complicated than in the latter, the textbook author expected that students would realize that simplifying expressions is worthwhile, and would thus be motivated to learn to simplify. Benny, who had planned to follow this line, indeed wrote the two expressions on the board, but forgot to mention that they are equivalent. Noticing this omission later, Benny added this information. But instead of just stating the fact, as suggested in the textbook, he asked: "Are the algebraic expressions equivalent?" This question triggered an unforeseen debate of this issue among the students. Clearly, the students were not engaged in the task as had been planned; however, they were engaged, on their own initiative, in a genuine and important mathematical discussion. But the teacher adhered to his original plan: "We can conclude – it is difficult to substitute numbers in a complicated expression and therefore we should find a simpler equivalent expression." Later, in an interview, Benny reflected on his teaching: "I prepare my objective and the exercises I want to give the students, and it is very confusing for me when they suddenly ask something not according to my planning."

Because Benny was a novice teacher we may conclude that Benny' s problem is related to his lack of experience. It is reasonable to assume that with time, Benny would be able to hear his students and deviate from his plans to better respond to unexpected class events.

Lacking Knowledge About Common Students' Conceptions and Their Possible Sources

The following excerpt is also from Benny's class. It took place in a lesson that was conducted a little later in the year, when Benny was teaching his students how to simplify algebraic expressions (Robinson, 1993). Benny writes the expression 3m+2+2m on the board and asks,

T: What does this equal to? Add the numbers separately and add the letters separately. Let us colour the numbers [colours 3m+2+2m]. We get [writes] 5m+2.
S1: And what now?
S2: 7m.
T: [Rather surprised] No! 5m+2 does *not* equal 7m. The rule is "add the numbers separately and add the letters separately". Here is another example: 4a+5-2a+7. We colour the numbers [colours 4a+5-2a+7]. What do we get? 2a+12. Let us write the rule [dictates]: In an expression in which both numbers and letters appear, we add the numbers separately and add the letters separately. Repeat out loud.
S's: [Repeat the rule out loud.]
T: Let's take another example: 6x+2+3x+5 = . We add according to the rule and get 9x+7.

The rest of the lesson is devoted to work on similar exercises. The students continue to experience difficulties. Towards the end of the lesson the students are working on simplifying the expression: 3+2b+7.

S1: 12b.
T: No!
S2: I got 10+2b. Why isn't it 12b?

The teacher does not respond to the last question. Did he hear his students? In his reflection on the lesson Benny explains that he senses there is a problem but he does not understand its sources; he does not really understand what his students' difficulties are. Benny hears that the students have difficulties but he does not hear the nature of these difficulties. In a study (Tirosh, Even & Robinson, 1998) that investigated teachers' awareness of students' tendency to conjoin or *finish* open expressions (e.g., to write the expression 2x+3 as 5x or 5), we found that although Benny's students tended to conjoin or finish algebraic expressions, he was not aware of this tendency nor of possible sources for it (e.g., conventions in natural language, expectation that the *behaviour* of algebraic expressions be similar to that of arithmetic expressions, the dual nature of mathematical notations: process and object).

We may conclude that not being familiar with common students' misconceptions, such as their tendency to conjoin or finish algebraic expressions, prohibited Benny from hearing his students. It is reasonable to assume that being an attentive and devoted teacher, Benny would learn about common students' misconceptions and then he would be able to hear his students and better plan and conduct his teaching. This conclusion is supported by findings related to some of the experienced teachers in the study (Tirosh, Even & Robinson, 1998).

Attributing Value to Students' Ways of Thinking

Ahuva had 20 years of teaching experience in elementary school, 12 of them in fifth grade. At the year of the study (Goldenberg, 2000) she participated in a six-month in-service workshop that focused on the use of portfolio as a means for student assessment. One of the tasks she discussed with the workshop participants was Ron's solution to the following problem: $\frac{3}{5}$ of a number is 12. Calculate the number. Explain your solution.

Ahuva admitted that she expected her students to solve the problem the way they have learned at class, $12 , \frac{3}{5} = \frac{12*5}{3} = 20$. But, because of the emphasis during the workshop on different solutions, she was looking for them in her students' answers. Ahuva explained that she had problems with Ron's solution: $12 * 2 = 24$

$$24 \div 6 = 4$$
$$24 - 4 = 20$$

She said: "He reached a correct answer but I didn't understand what he did. It didn't seem right". Ahuva explained that Ron is an average-good student, who usually has difficulties with homework. She decided to mark Ron's solution as wrong. After several days, Ron came to Ahuva and told her that he checked the solution with his father, and they think that his solution is correct. Ahuva told Ron that she would check it and decided to ask the workshop participants to help her. The teachers tried to understand Ron's way of thinking and reached the conclusion that it is indeed correct. But Ahuva was not convinced. She did not believe that Ron really solved it the way the workshop teachers suggested, because she thought their interpretation was too complicated for him. She therefore embraced the suggestion to interview Ron. Ron explained to her that if $\frac{3}{5}$ is 12 then 24 is $\frac{6}{5}$ and this is more than one whole. Then he calculated the value of $\frac{1}{5}$, $24 : 6 = 4$. From 24 he subtracted the value of $\frac{1}{5}$, $24 - 4 = 20$, reaching the correct answer of 20.

Ahuva's case suggests that in order for teachers to hear their students, teachers need to be tuned to do this. They need to believe that there is something to hear and to be aware of the fallibility of their sense making. Teachers need to adopt a mode of *listening,* which Davis (1997) terms as *interpretive listening*. It is reasonable to assume that with the support of adequate professional development, such as the workshop in which Ahuva participated, Ahuva would become more tuned to hear students and would attribute value to their ways of thinking.

Hearing Through…

The analysis of the cases above focused on identifying difficulties or obstacles to hearing students and on suggesting ways to overcome these obstacles. We now take a different approach to the nature of hearing students and to the complexity of understanding what students are saying and doing. To do this we use the case of Ruth (Wallach & Even, 2002) – an elementary school teacher with 11 years of teaching experience. As part of the requirements in an in-service workshop in which she participated, Ruth observed two of her fourth grade students – Sigal and Ore – solve the following problem (see Figure 1): "Because the number 15 is odd and the number 4 is even, then it's impossible" was Sigal and Ore's immediate solution. What might this mean? There are various possible interpretations. One is that the girls know how to solve this problem by removing four players from the entire team, dividing the remaining players into two equally sized groups, and finally, adding the four players to one of the groups. They understand that in this specific case the division is not possible because 15 is odd, and when they subtract 4 from 15 the difference is 11, which is an odd number, and an odd number is not divisible by 2. A different viable interpretation is that the girls understand the generalization of this method – that the division is possible only when the number of players is even. Because only in this case the removal of four players would leave an even number of players, which can then be divided into two equally sized groups. Another possible interpretation is that the girls use verbal hints from the text. They know that the required division is impossible because this is stated in the text: "The task in front of you does not have a solution". They also know and recognize that 15 is odd and that 4 is even. So they link these two pieces of information, and without any logical argument state that "Because the number 15 is odd and the number 4 is even, then it's impossible". Of course, there may be other possible interpretations.

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The Rishonim school team players meet for practice once a week. The players stand in line according to the numbers on their shirts. Each week their coach divides them into two groups differently.

The task that you are presented with does not have a solution:

Divide the 15 children in the line into two groups, so that in one group there are 4 players less than in the other group.

Explain why there is not such a division __________

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Figure 1. The "Shirts and Numbers" problem.

What did Ruth hear when the girls said, "Because the number 15 is odd and the number 4 is even, then it's impossible"? When asked to describe how the students solved the problem, Ruth recounted their explanation as "… If you take away an even number from an odd number, then you are left with an odd number, which you cannot divide by 2". And she explains, "They said: 'Oh, for sure it's impossible. Odd minus even, it's impossible to divide it by 2'". Obviously, Ruth hears things that were not said by the students—she *over-hears* them.

Why does a teacher *hear* what was not said or done by the students? Why does she *mis-hear* what was said or done by them? Hearing students, like any other kind of understanding, *cannot* be an accurate reflection of what actually was said or done. Teachers hear students *through* various factors, such as, the teachers' own knowledge of mathematics, their conceptions of the solution of the problem at stake, their beliefs about the nature of mathematics learning and knowing, their understandings of the nature of mathematics teaching, their dispositions toward the teacher's role, their feelings about their students, expectations of their students, the context in which the hearing takes place, and so on.

To illustrate this let us look at Ruth's own solution to the problem. At the workshop, before presenting the problem to her students, Ruth solved the problem by subtracting 4 from 15 and receiving 11. Then she stated that the division of the result by 2 does not give a whole number, which means that the problem has no solution. Ruth also created a *short version* of this solution: "15 is odd and 4 is even. It is not possible to divide 11 by 2". And then she claimed that "15 is odd and 4 is even" is enough as it actually represents the more elaborate solution. Thus, it is not surprising that when later the girls stated: "Because the number 15 is odd and the number 4 is even, then it's impossible", Ruth over-hears them saying: "… If you take away an even number from an odd number, then you are left with an odd number, which you cannot divide by 2".

Obviously, it is not only Ruth's own conception of the solution to the problem that contributed to her over-hearing what Sigal and Ore said. Analysis of Ruth's concern for her students' success, Ruth's view of knowledge use, Ruth's acquaintance and familiarity with Sigal and Ore, and the context in which Ruth's interpretation of the students' work took place (Wallach & Even, 2002) show how the interactions of various personal factors influenced the nature of Ruth's hearing of the two students.

Hearing Students: *Deficient* Versus *Hearing Through*

When a teacher practice does not comply with theoretical expectations, we often conceive the situation as if we ought to identify the difficulties or obstacles that interfere with the application of the theory. We assume that teachers have some deficiencies; that they lack essential knowledge and skills. We presume that the problem can be fixed and suggest ways to overcome these obstacles, usually by teaching teachers the required missing knowledge and skills. This perspective frames the analyses of the three excerpts from Benny's and Ahuva's classes presented in this paper. Each analysis revealed a different teacher deficiency. One case shows a novice teacher who finds it overwhelming to pay attention to students' unexpected discussion. Another case presents the same teacher having difficulties to hear his students because he was not familiar with common students' conceptions and ways of learning in algebra. The third case exhibits an experienced teacher who, at first, did not try to understand how her student solved a problem because she did not expect him to come up with an original solution. All these problems have the potential to be fixed. With more teaching experience and participation in adequate formal and informal professional development activities, Benny may learn to be more open to unexpected events in the classroom. He may also learn about students' common misconceptions. Ahuva may learn to attribute more value to original students' solutions and to pay attention to their processes of solving problems.

But Ruth's case implies a completely different perspective to hearing students. Instead of looking for obstacles that could be overcome, and paying attention to what teachers *do not* – for example, do not listen to students, do not change their plans, do not know about students' common conceptions, do not understand the mathematics – Ruth's case suggests that teachers always *hear through* various personal factors; that it is unrealistic to expect an *accurate* teacher understanding of what students are saying and doing. Findings of several other studies support the adequacy of this perspective that recognizes the intrinsic nature of *hearing* students as *hearing through*. Ball (1997) shows, for example, how the teacher's own ways of understanding the subject matter and her commitment to her students influence her hearing her students. Morgan and Watson (2002) show that when teachers assess students' mathematical performance they rely on their personal resources. For example, on their personal knowledge of mathematics and the curriculum, on their feelings toward mathematics based on their personal mathematics history, on their expectations about how mathematical knowledge can be communicated, and on their expectations of students and classrooms in general, and of individual students. They further show how different teachers interpret similar student work differently.

Hearing through is not something that could be overcome. Rather, it is a fundamental characteristic of one person's understanding of what another person is saying, doing, and feeling. Thus, hearing students is always influenced by the teacher's knowledge, understandings, beliefs and dispositions regarding mathematics in general and the specific piece of mathematics being dealt with, learning and teaching mathematics in general and the specific piece of mathematics with the specific students involved, the teacher role in general and in the specific context, and so forth. Current rhetoric that advocates reform formative assessment does not seem to pay attention to this aspect. Understanding what students are doing and saying is usually presented as unproblematic. However, as researchers in mathematics education we know that this is not so; that understanding what students are doing and saying is not an easy task, nor certain.

Conclusion

A simplistic conclusion from the above might be that if we can never truly understand what another person is saying and doing, then we should not expect teachers to conduct formative assessment as an integral part of regular instruction. This is not at all what we propose to conclude here. Nor do we suggest that teachers cannot improve their ability to understand what their students are doing and saying. Our main suggestion calls for practically merging the two perspectives presented in this paper. Conducting informal observations and listening to students during regular instruction is an important teaching skill. As indicated earlier, teachers can learn to value and to aim at understanding their students' conceptions and ways of thinking and to become better in doing that. At the same time, understanding what students are saying and doing should neither be regarded as an unproblematic task nor as something that can be certain. Furthermore, although the research mentioned above provides important information on this issue, more research is needed to better understand the interrelations among teacher attention to students' talk and actions, the nature of hearing students (i.e., hearing through), and students' learning.

References

Ball, D. L. (1997). What do students know? Facing challenges of distance, context and desire in trying to hear children. In B.J. Biddle, T. L. Good, & I. F. Goodson (Eds.), *International Handbook of Teachers and Teaching,* (pp. 769-818). Dordrecht, The Netherlands: Kluwer.

Black, P., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education: Principles, Policy, and Practice,* 5(1), 7-74.

Black, P., & Wiliam, D. (2000). *Inside the black box*. Retrieved December 20, 2002, from King's College, university of London Web site: http://www.kcl.ac.uk/kings_college/depsta/education/publications/blackbox.html

Clarke, D. (1997). *Constructive assessment in mathematics: Practical steps for classroom teachers.* California: Key Curriculum Press.

Cobb, P. & McClain, K. (1999). Supporting teachers' learning in social and institutional context. In F. Lin (Ed.), *Proceedings of the 1999 international conference on mathematics teacher education* (pp. 7-76). Taipei, Taiwan: National Taiwan Normal University.

Coles, A. (2001). Listening: A case study of teacher change. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th annual conference of the International Group for the Psychology of Mathematics Education* (Vol 2, pp. 281-288). Utrecht, The Netherlands: Freudenthal Institute, Utrecht University: PME.

Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education*, 3, 155-181.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education*, 28(3), 355-376.

Even, R., & Markovits, Z. (1993). Teachers' pedagogical content knowledge of functions: characterization and applications. *Journal of Structural Learning*, 12(1), 35-51.

Fennema, E., Carpenter, T., Franke, M., Levi, L., Jacobs, V., & Empson, S. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education,* 27(4), 403-434.

Goldenberg, G. (2000). *The portfolio as a means for changing teacher assessment of student knowledge of positive rational numbers*. Unpublished master thesis. Tel Aviv, Israel: Tel Aviv University (in Hebrew).

Morgan, C., & Watson, A. (2002). The Interpretative Nature of Teachers' Assessment of Students' Mathematics: Issues for Equity. *Journal for Research in Mathematics Education*, 33(2), 78-110.

National Council of Teachers of Mathematics (NCTM) (1995). *Assessment Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Nicol, C. (1999). Learning to teach mathematics: questioning, listening, and responding. *Educational Studies in Mathematics,* 37, 45-66.

Robinson, N. (1993*). Connectedness in teaching: Equivalent algebraic expressions - by expert and novice teachers.* Unpublished master thesis. Tel Aviv, Israel: Tel Aviv University (in Hebrew).

Shepard, L. A. (2001). The Role of Classroom Assessment in Teaching and Learning. In V. Richardson (Ed.), *Handbook of research on teaching*. (pp. 1066-1099). Washington, DC: American Educational Research Association.

Simon, M. & Schifter, D. (1991). Towards a constructivist perspective: An intervention study of mathematics teacher development. *Educational Studies in Mathematics*, 22, 309-331.

Tirosh, D., Even, R., & Robinson, N. (1998). Simplifying algebraic expressions: Teacher awareness and teaching approaches. *Educational Studies in Mathematics,* 35, 51-64.

Wallach, T. & Even R. (2002). Teacher hearing students. In A. D. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th International Conference for the Psychology of Mathematics Education* (Vol 4, pp. 337-344). Norwich, England: PME.

Watson, A. (2000). Mathematics teachers acting as informal assessors: Practices, problems and recommendations. *Educational Studies in Mathematics*, 41, 69-91.