Constructing and Using a Personal Numeracy Teaching Model in a Classroom Setting

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This paper describes a process in which a practising teacher developed a personal teaching model for numeracy. Influences included Vigotsky's Zone of Proximal Development, Steffe's counting types, a mathematics education paper which included study of Wright's Mathematics Recovery and Burns' cooperative group teaching, a trial of the Count Me in Too project with subsequent developments in New Zealand, and an explicit strategy teaching model derived from the theory of Pirie and Kieren. The teacher uses her teaching model in a classroom setting. The teacher's espoused teaching model is compared with its implementation. Some implications for assisting teachers to construct personal teaching models are made.

New Zealand participated in The Third International Mathematics and Science Study (TIMSS). Results relating specifically to New Zealand were presented in a number of reports (e.g. Garden, 1996, 1997). These reports indicated that the standard of mathematics learning in New Zealand was below international averages at the age levels tested. One of the consequences of TIMSS was that, in 1997, the Ministry of Education invited proposals for the professional development of teachers in teaching mathematics at the year three level. The successful proposal (Hughes, 1997) in the Auckland area was made on behalf of the Auckland College of Education (ACE). In its contract proposal was a requirement that teachers would enrol in an Advanced Studies for Teachers (AST) 300 level mathematics education paper. In this paper teachers would be expected to interview students from their own class using an assessment based on the Mathematics Recovery frameworks (Wright, 1993). The proposal included an approach to professional development of teachers derived from Guskey (1985). He suggested that "the most significant changes in teacher attitudes come after they begin using a new practice successfully and see changes in student learning". (Guskey, 1985, p. 57). Figure 1 shows the essence of his model of teacher development. Consequently the AST paper contained significant elements where teachers were expected to assess and teach children in their own classrooms. Resulting, in part, from the perceived successful delivery of the AST paper to 175 teachers at ACE, the Ministry of Education instigated a series of New Zealand National Numeracy projects from 2000 to 2003 (Higgins, 2001, Thomas & Ward 2001, Thomas & Ward, 2002)

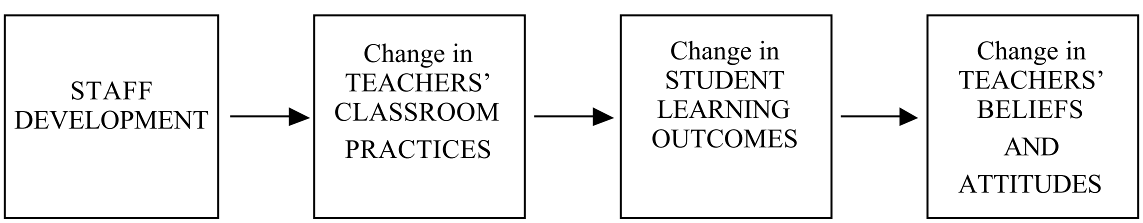

Figure 1. A Model of Teacher Change

Method

Subject

The teacher, whose construction and use of a numeracy teaching model is the subject of this study, was educated in Canada where she was awarded Bachelor of Arts (Hons) and Bachelor of Education degrees. She taught in Ontario primary schools for two years before immigrating to New Zealand. While teaching in an Auckland primary school she enrolled in the AST paper in 1999. Subsequently she was closely involved in the New Zealand National Numeracy projects. She was, in turn, a lecturer in Mathematics Education at ACE, an in-class facilitator for the National Numeracy Project, the co-ordinator of the Numeracy facilitators in the Auckland region, and currently is a senior teacher in a year 3-4 class in an Auckland school. In this study the teacher traces her own development of a numeracy teaching model over time and applies the model to a lesson in her own classroom. She is co-author of the paper.

Data Gathering

In a taped interview the teacher was asked to outline the developmental influences on her teaching in general, and more specifically, the ideas and experiences that influenced the construction of her personal numeracy teaching model. A transcript was made. Her references to research on the tape were added to the transcript where these were known. She was then asked to edit her comments to improve clarity and meaning. This was followed by a videotaped lesson of the teacher teaching a group of children in her own classroom where she attempted to apply her strategy teaching model. A transcript of the actions and words of the students and teacher was made from the videotape. The teacher was then invited to view the videotape, read the transcript, then add her comments to the transcript about her observations of the children and her reasons for the teaching decisions she made.

Data Interpretation

It would have been possible to analyse audiotapes and videotapes in order to infer the teacher's intentions and analyse her actions. However, it was felt that a reflective teacher could explain more convincingly than an observer her own development of a theory of teaching and the reasons for her actions in an actual classroom setting. Moreover it is reasonable to assume that an actual teacher's voice would appeal to practicing teachers viewing this research.

Results and Discussion

The Construction of a Personal Teaching Model

In an audiotaped interview the teacher recalled important theoretical influences in her teaching from her work towards a Bachelor of Education degree.

> I was highly influenced by constructivist ideas about how learners need to construct their own meaning. I knew that learning was not just a passive process of absorbing my knowledge. The children had to be actively involved. I used Vigotsky's notion of the "Zone of Proximal

> Development" to guide me. I found it helpful because it gave me a way of knowing where the children were now and where they might be with help from me and their social group.

In 1999, while a classroom teacher in New Zealand, the teacher undertook study in an AST numeracy paper at ACE. The readings for this paper included a handout describing a method of small group teaching (Burns, 1990).

> As a result of my teaching background, I was already sympathetic to small group teaching models. I felt a great affinity with the teaching and learning models that were demonstrated in the AST course during the regular part of two and a half hour lectures. The lecturer taught us mathematics as learners not teachers. I was really influenced by the use of many of Burns' suggestions. For example, the lecturer put us in groups of four or five, and set us problems to solve without providing any solution methods. I was impressed by his use of active listening. During the whole class sharing phase of any teaching session, I was intrigued by his method of neither confirming nor denying the problem solutions, nor ever indicating which solution method he thought was the best.

The teacher began experimenting with these ideas back at her school.

> I started creating small groups in my classes and training students in the use of the Burns type of group rules written on a poster. The rules were, and still are, "get in groups of four; listen to the task; give everyone a chance to share their strategy; one person talks at time; everyone must be able to explain each other's answers; be prepared to share group answers back with the whole class or larger group; and a rule for myself – don't offer solution methods to any group and provide scaffolding to a group only when absolutely needed".

The teacher reflected on what active listening had done for her teaching.

> From the modelling in lectures I took active listening to mean that a teacher could repeat children's solution methods without evaluation, criticism, or correction. Of course I thought that scaffolding was legitimate, that is, the teacher could offer help to those who did not understand another child's explanation by modelling it, say, on materials. This model of the teacher as active listener seemed to fill a gap in my constructivist teaching practices. I had never been certain what the children were thinking. But by using active listening, I was more confident that I could understand the conceptions and misconceptions of my children.

The teacher encountered the notion of counting types frameworks in the AST lectures and course readings (Steffe, von Glasersfeld, Richards, & Cobb, 1983, Wright, 1993).

> I immediately liked the idea from Steffe and Wright that children use increasingly sophisticated counting types. I was used to framework-based language models such as Reading Recovery. So applying a framework to numeracy appealed to me.

Part of the AST paper requirements was an in-school component in which children were withdrawn from class. Assessments based on the Mathematics Recovery programme (Wright, 1993) were modelled by the lecturer. Then, course participants interviewed a number of children themselves. Subsequently the teacher began using this assessment tool with her own students with consequent implications for her understanding of their learning needs.

> I knew the methods of assessing I had been using, based on timed pencil and paper tests or algorithms, were inadequate. Students were being assessed for teaching on the basis of outcome not thinking. But what was the alternative? Now the counting type assessment gave me a way of creating groups of like-thinking students which made much more sense to me. And, as far as student learning went I could see that I was being much more effective in planning for, and aiding their learning needs.

In 2000 the teacher was a lecturer in Mathematics Education at the Auckland College of Education, and a facilitator with teachers in classrooms in the New Zealand Count Me In Too (CMIT) trial. (Thomas & Ward, 2001). The teacher was happy to use the distinction between knowledge/recall and strategy/process used in the Numeracy Project teacher material (Hughes, 2002, p. 350) in part due to experiences in 2001 and 2002 modelling and observing teaching. She felt teachers already effectively used their existing models for teaching number knowledge.

> I observed teachers generally teaching knowledge/recall well through the use of whole class warm ups, paired games/activities and individual maintenance number work which catered for the different levels of number learning of the students. And student progress was obvious and measurable.

But this success in knowledge teaching was in marked contrast to the teaching of number strategies that she observed.

> It became clear to me that teachers needed help in teaching strategy development. I found that it was complex stuff to plan, organise and teach. And in fact we did not know how to do it in a simple way that teachers could understand and use. We needed to develop a strategy teaching model where there was no obvious route to its construction. So a few facilitators and lecturers began discussing this strategy teaching challenge.

She was part of a strategy teaching model that began to emerge.

> I was reading and discussing P-K theory (Pirie & Kieran, 1994) with colleagues. It had strong constructivist connections that appealed to me. And I knew the New Zealand mathematics curriculum document (MoE, 1992) was, at heart, constructivist, recommending things like having learners discuss, interpret, make connections and search for their own personal understandings in mathematics. In particular P-K notions of property noticing and formalising eventually became *Using Properties of Numbers* in the New Zealand strategy teaching model (Hughes, 2002).

The teacher felt that a paper in circulation within the mathematics education department (von Glasersfeld, 1992) was important. Significantly for her von Glasersfeld wrote:

> Mathematics is the result of abstraction from operations on a level on which the sensory or motor material that provided the occasion for operating is disregarded. In arithmetic this begins with the abstraction of the concept of number from acts of counting. Such abstractions cannot be given to students, they have to be made by the students themselves. The teacher, of course, can help by generating situations that allow or even suggest the abstraction. This is where [materials] can play an important role, but it would be naive to believe that the move from handling or perceiving objects to a mathematical abstraction is automatic. The sensory objects, no matter how ingenious they might be, merely offer an opportunity for actions from which the desired operative concepts may be abstracted; and one should never forget that the desired abstractions, no matter how trivial and obvious they might seem to the teacher, are never [obvious] to the novice. (p.6)

The teacher reflected on this comment.

> When in classrooms in schools I had observed that year 1 – 3 students spent most of their time doing mathematics using concrete materials but not going any further. Then, in years 4 – 6, concrete materials were abandoned. Something was missing. While valuing materials, von Glasersfeld told me that mathematics was inherently and necessarily abstract. But there was no obvious bridge to help move students from materials to abstractions.

In 2001 the teacher became an in–class modeller with 70 teachers and the regional co-ordinator for the New Zealand Numeracy Project in the Auckland region. She was in a position to trial the strategy teaching model in classrooms and to observe teacher reaction to its use. By this time the idea of imaging was being discussed and trialled.

> By introducing *Using Imaging* to the strategy teaching model we seemed to have found a sensible, simple, understandable link from materials to abstraction. The model gave me purpose and focus in the use of materials and how to move beyond them. And now I had a model which defined Zones of Proximal Development where I could recognise where each child's construction was at, and know how to challenge each of them to reach towards abstractions. For many teachers the strategy

teaching model was a challenge to their current teaching practices. However, after initially struggling to move beyond the use of materials with their students many of these teachers realised that their ultimate goal, and success, as a teacher of mathematics was to move students beyond *Using Materials* and imaging to the final abstractions.

The teacher returned to a fulltime classroom position in 2002. The teacher summarised the major components of her current model for strategy teaching with her year 3-4 students.

> I now see myself as trying to apply a constructivist style of teaching based on small groups, with feedback to the larger group. I don't see myself as the provider of solutions, but as the repeater of student solutions where I scaffold those children who don't understand a solution while repeating a solution method. I apply the strategy teaching model. Then I use it as a way of judging how to move forward with each learner as I observe the success, or otherwise, of each child at the various materials, imaging, and abstraction stages.

The Use of a Personal Teaching Model in Classroom Setting

The teacher taught a lesson to a group of students from her year 3-4 class which was designed to move a group of students from using "counting–on" to using "part-whole" reasoning. Only 12 minutes of a 40-minute videotape are directly reported because of space restrictions. This selection attempts to represent fairly the main aspects of the lesson.

The teacher drew the lesson from the teacher material in the New Zealand Numeracy Projects (MoE, 2003b, p 26). The stated objective for the student's learning given at the start of the activity is "I am learning to subtract by splitting numbers into parts instead of counting down". The materials needed were a steel board with tens frames drawn on it, pre-printed tens frames, and a set of magnetic counters.

The teacher had grouped seven children by their strategy stage.

> These children were all counting on, which created a group of like thinkers. My intention was to challenge them to connect to part-whole thinking with problems involving two digits minus a one digit number.

The teacher organised the rest of the class into working individually, or in small groups, to practice knowledge/recall.

> I had set the other two groups to work quietly and independently of me where they were re-enforcing their knowledge from a variety of activities involving playing number games in small groups, working from cards placed around the room in boxes, or practising on worksheets.

The teacher reminded the group of seven children on the mat in front of her of the small group rules by showing them a poster and reading the rules out loud. Then the teacher briefly checked their recall of basic addition facts.

> I had to check combinations to ten, and the ten plus a digit gives a "teen" number as these are essential to encourage children to abandon counting down. Most of them had good recall but I was concerned that a couple had little immediate recall of this required knowledge. But I decided to keep them in the group despite feeling part-whole thinking would be beyond them on this basic fact failure alone.

The teacher now showed 14 magnetic counters as ten and four on a metal board with tens frames drawn on it and said, "I am going to give a problem to think about in your groups of four or groups of three. Using the ten frame how could you solve: you have 14 chocolates and you eat 6 of them." A discussion about what to write on the portable newsprint board occurred, then the teacher wrote 14 – 6 on the board. The children now split into two groups. The children began discussions in their groups.

> I subdivided them into a group of four and a group of three and set them going. I did not provide any prompts initially for the problem. I listened in and thought the group of three was making good progress. In the four I could see that Paridnya and James were on task. But Maree and Lewis were struggling. So I tried to provide a scaffold for those two to prompt part-whole thinking. But Lewis and Maree still counted backwards. I could not help them connect. Meanwhile, I could see that Elaine, Isobel, Vaine in the group of three were actively discussing moving the counters in groups. Paridnya and James were discussing a different strategy of removing the six counters. So these five were making good progress.

The teacher then clicked her fingers, which was their standard signal for groups to join together.

> At this point I decided to stop the sub-groups and pull the students together in a larger group since I felt Lewis and Maree were not benefiting from any part-whole interactions. At this point I used active listening to let a number of students explain their solutions. I set some further similar problems to be done in their groups. And I could see Lewis and Maree were still not progressing.

The teacher then checked to see that students understood how to construct two digit numbers greater than 20. She created 24 using counters on tens frames.

> Lewis, James and Maree took some time to see the number 24 as being made up of the two tens and four ones. This alerted me to the fact James was going to have trouble with part-whole thinking. This was also an indication that these three would not be able to move on to imaging partitioning effectively. The other four had no trouble. I asked some more similar questions where the pattern of who understood and who did not continued. Despite this I decided to try to move on to imaging.

The teacher recorded the problem 24 – 5 on the whiteboard and made the number 24 using the tens frames. She briefly showed the two tens and four ones then hid the counters from view. The teacher asked, "What I want you to know is 24 by imagining, just by picturing the counters in our heads. Think about the tens frames in your head, but I'm not going to show it to you. For this question, what I want you to talk about is what you see and what you would move."

> I was now pushing them to image if they could. It was clear to me straight away that Elaine, Isobel and Vaine in the one group along with Paridnya in the other group were imaging the 24 counters as they were talking animatedly about what 5 counters they would remove. However, Lewis, Maree and James were using their fingers as well as grabbing for the tens frames that I had hidden. They were not imaging. I asked Maree and Lewis to discuss things with me. They still struggled, and I thought James was not gaining anything by talking with Paridnya.

The teacher clicked her fingers and pulled the groups together again.

> I had heard Vaine successfully discussing what she could image so I decided to have her elaborate on her thinking for the whole group in one last effort to see if Lewis, Maree or James could follow part-whole thinking by imaging 24 minus 5.

Vaine said, "Ten over here and ten over here and four over here" as she drew on the empty tens frame board with her finger. The teacher asked Vaine, "How did you think about taking five away?" Vaine said, "You take the four and move one away and that would make 19." As the student described her thinking she was drawing her finger along the empty board as if to push away counters.

> It was obvious that Vaine was successfully imaging to solve the subtraction problem. Also Isobel, Elaine and Paridnya seemed able to follow her thinking. But, even when I folded back to Using Materials on tens frames, Lewis, Maree and James were completely lost. I now thought I should do

> more imaging problems with Lewis, Maree or James. But time was running out. So I decided to push out to larger numbers in the *Using Number Properties* phase for the sake of Paridnya, Elaine, Isobel, and Vaine. I could return to Using Materials and Using Imaging with Lewis, Maree, and James next time.

The teacher said, “Let’s try one last one. No counters, nothing. You can imagine the numbers if you want but I am going to make the numbers pretty big. What if I made it 63. Think about not counting back. Think about what you would push away. And you want to take away four. Just the numbers.” The teacher wrote 63 – 4 on the board, and then told the students to, “Talk to your group”. The group of three discussed the solution independently of the teacher. The teacher watched, then interacted with, the group of four. Then the teacher snapped her fingers, and the two groups gathered together. She asked Lewis, Paridnya, Vaine, and Elaine to explain how they got their answers.

> Lewis counted down as I expected. Each of Paridnya, Vaine, and Elaine basically took away three from 63 then one more. So they understood abstract part-whole reasoning well. I did not ask Maree, James, and Isobel to explain but I was confident that I knew what they were thinking from listening and interacting with them in the smaller groups.

The teacher then described her conclusions from the lesson and stated her future plans for teaching them.

> The last part of the lesson really confirmed what I suspected would happen. Vaine, Elaine, Isobel, and Paridnya all gave good abstract part-whole explanations of 63 – 4. Obviously I need to follow up with similar examples in a later lesson to reinforce their thinking. But Maree, James, and Lewis did not really make it. And I don’t think they are really anywhere near doing so. So I am going to have to go back to revise their lack of instant recall of basic facts. I will attempt to push them towards part-whole thinking later, but meanwhile they will remain in their current zone of comfort where I expand on their ability to use counting methods - for example counting down, skip counting by five for multiplication problems and so on.

Conclusions

The design of the AST paper included a requirement that teachers involve themselves in some new classroom practices. In the case of the teacher in the study she experimented with small group teaching, used active listening, and assessed student thinking against number frameworks. The teacher reported feeling that she was teaching more effectively because she felt her students were learning better. She incorporated these new ideas into her teaching which reflected changes in her beliefs and attitudes. The teacher’s self-described process of professional development fits well with Guskey’s model (Figure 1). Its success offers encouragement for the use of this model in mathematics education tertiary study.

In the lesson the teacher’s teaching corresponded closely to her espoused personal teaching model. The teacher implemented Burns’ group teaching rules effectively. An impressive feature was she did not bind herself to follow the rules rigorously when the situation demanded a change. One of the teacher’s rules is that groups should work in fours. Noting that Maree and Lewis were not benefitting from group discussion she broke up the group of four in order to try to scaffold the learning of these two students. She consistently applied active listening in the lesson where she sought children’s solution methods and offered scaffolding to assist other children in the large group to make connections. The use of the strategy teaching model in the lesson with its Using Materials, Using Imaging, and advancing to Using Number Properties provided the teacher with an effective tool to structure her lesson, and then assess the future learning needs of every child.

Teachers wanting to improve their teaching practice may well benefit from reading a study such as this where a teacher's voice should appear understandable, real, credible, and accessible. Also teachers might profit from engaging in a similar process of professional development as this teacher went through, even though most teachers probably would not be as analytical as she was in reflecting on, and articulating their own practice.

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